The latter question was part of [Kir97, Problem 4.84], due to Teichner. Note that closed, spin 4-manifolds that are homotopy equivalent have the same Kirby–Siebenmann invariant, so that invariant cannot be used here.

(3) Jim Davis [Dav05] proved that if the fundamental group is π and the map $\kappa_2\colon H_2(\pi;\mathbb{Z}/2\mathbb{Z}) \to L_4(\mathbb{Z}\pi)$ is injective, then homotopy equivalent 4-manifolds with that fundamental group and the same Kirby-Siebenmann invariant are stably homeomorphic.

(4) There are counterexamples known for non-spin 4-manifolds, even assuming they have the same Kirby-Siebenmann invariant [Tei97].

Proposed for K3 by: D. Kasprowski
Scribed by: M. Powell

Problem 4.53. *Does there exist an algorithm that takes as input a closed, triangulated 4-manifold, and outputs in finite time whether or not that 4-manifold is homeomorphic to S^4?*

Remarks.

(1) This is called the *recognition problem*. We say that we can *recognize* an n-manifold X if there exists an algorithm that takes as input a closed, triangulated n-manifold, and outputs in finite time whether or not that 4-manifold is homeomorphic to X.

(2) We assume that the input 4-manifolds are represented by the finite data of a triangulation, and hence they are smooth, since triangulated 4-manifolds are smooth. We are not assured that the input 4-manifold is simply-connected.

(3) The corresponding question has a positive answer in dimensions $\leqslant 3$ [Tho94], [Rub95], and a negative answer in dimensions $\geqslant 5$ [VKF74].

(4) Markov [Mar58] showed that, for some integer k, the corresponding question for the connected sum of k copies of $S^2 \times S^2$ has a negative answer. This raises the question of the minimal k for which this holds. It was shown that k could be taken to be 14 in [Sht05], 12 in [Gor22], and 9 in [Tan23]. See also the exposition in [Kir20].

(5) By Markov [Mar58], there exist infinite lists of group presentations $\{P_i\}$ such that there is no algorithm taking as input one of the P_i, and outputting in finite time whether or not the group $G(P_i)$ presented by P_i is trivial. One possible strategy to solve the problem is to construct a corresponding list $\{X_i\}$ of closed, triangulated 4-manifolds, whose 2-skeleta give rise to the P_i, so in particular $\pi_1(X_i) \cong G(P_i)$, with the property that X_i is homeomorphic to S^4 if and only if $G(P_i) = \{1\}$. The forward direction clearly holds, but it is not clear how to find $\{P_i\}$ and $\{X_i\}$ such that the backwards direction holds.

This strategy does work for $\#^k(S^2 \times S^2)$ in place of S^4, and was the basis for the proofs of [Sht05, Gor22, Tan23] for $k = 14, 12, 9$ respectively.

(6) One can also ask the analogous *smooth recognition problem* with 'diffeomorphic' in place of 'homeomorphic'. This is also open for S^4. As described in [**Kir20**], there exists a k such that $\#^k(S^2 \times S^2)$ is not smoothly recognizable, but there are no known upper bounds on the minimal k for which this holds.

Scribed by: M. Powell

PROBLEM 4.54. *The* quadratic 2 type *of a 4-manifold M is the data*

$$(\pi_1(M), \pi_2(M), \lambda_M, k_M)$$

of the fundamental group $\pi_1(M)$, the second homotopy group $\pi_2(M)$ considered as a $\mathbb{Z}[\pi_1(M)]$-module, the equivariant intersection form λ_M, and the k-invariant in $k_M \in H^3(B\pi_1(M); \pi_2(M))$.

Which quadratic 2-types are realized by closed, oriented topological 4-manifolds? Which are realized by closed, oriented, smooth 4-manifolds? Can this problem be solved for specific fundamental groups, for example for certain families of non-cyclic finite groups?

REMARKS.

(1) This is in [**Kir97**, Problem 4.1].
(2) We can also ask the question with the data of the Stiefel-Whitney classes w_1 and w_2. Which Stiefel-Whitney classes are realized within a given quadratic 2-type?
(3) The problem builds on the geography problem for simply connected topological 4-manifolds, where it restricts to the question of which intersection forms occur. In the smooth category, this is answered by *Donaldson's diagonalizability theorem* along with a positive resolution of the *11/8-conjecture*.
(4) For the topological case, Freedman [**Fre82**] showed that every nonsingular symmetric bilinear form is realized by a closed simply connected 4-manifold, so the problem is solved when $\pi_1 = 1$. A similar result is known for $\pi_1 = \mathbb{Z}$ [**FQ90**].

    For other good fundamental groups, a possible strategy in the topological category was introduced by Hambleton-Kreck in [**HK88**, Lemma 4.1], the paper where the quadratic 2-type initially arose. First, classify the quadratic 2-types that are stably realizable, meaning that they are realizable by topological 4-manifolds after taking the orthogonal sum with the quadratic 2-type of $S^2 \times S^2$. If a quadratic 2-type is stably realizable then it is realizable unstably by a topological 4-manifold, which can be shown using the sphere embedding theorem [**FQ90, BKK$^+$21**].

    Hambleton-Kreck [**HK88, HK93a**] used this strategy to solve the realization problem for finite cyclic groups, in the topological category.
(5) For non-simply-connected, smooth, oriented 4-manifolds, Donaldson's diagonalization theorem holds without any assumption on the fundamental group, so definite integral intersection forms must be diagonalizable. The sphere embedding theorem cannot be applied in the smooth case, so the

strategy described above of stably realizing and then destabilizing is not currently viable.

(6) The existence problem has a closely related uniqueness analogue. For finite cyclic groups [HK88], abelian groups with at most two generators [KPR24], dihedral groups [KNR22], and aspherical 3-manifold groups [Hil23], we know that the quadratic 2-type determines the 4-manifold up to homotopy equivalence. Hence if we could also understand the image of the invariants in one of these cases, i.e. the realization problem, we would have a fairly complete homotopy classification, at least modulo the algebraic problem of being able to reliably distinguish or identify given quadratic 2-types.

(7) Kirk and Livingston's paper [KL09] contains many related references and its own list of interesting related problems.

Scribed by: M. Powell, J. Van Horn-Morris

PROBLEM 4.55. *Let M and N be closed, orientable, connected 4-manifolds with isomorphic quadratic 2-types. If $\pi_1(M) \cong \pi_1(N)$ are finite, are M and N homotopy equivalent?*

REMARKS.

(1) This is known for π_1 trivial, finite cyclic groups [HK88], dihedral groups [KPR24], and abelian groups with at most two generators [KNR22]. It is known to be false in the nonorientable case, due to Kim-Kojima-Raymond [KKR92].

(2) A potentially interesting case is $\pi_1(M) = \mathbb{Z}/2 \times \mathbb{Z}/2 \times \mathbb{Z}/2$. As noted in [KPR24], in this case we can have torsion in $\mathbb{Z} \otimes_{\mathbb{Z}[\pi_1(M)]} \Gamma(\pi_2(M))$, which by [HK88] leads to *polarized* homotopically inequivalent Poincaré 4-complexes with the same quadratic 2-type. Are they homotopy equivalent? Do the homotopy types contain topological 4-manifolds?

(3) Let $L_{p,q}$ and $L_{p,q'}$ be lens spaces that are not homotopy equivalent. Then $S^1 \times L_{p,q}$ and $S^1 \times L_{p,q'}$ are also not homotopy equivalent but they do have isomorphic quadratic 2-type. So the problem is not true in general for infinite fundamental groups.

Proposed for K3 and scribed by: M. Powell

4.6. Cobordisms

The problems here fit under the theme of cobordisms, either 4-dimensional cobordisms between 3-manifolds, or 5-dimensional cobordisms between 4-manifolds.

PROBLEM 4.56 (4D s-cobordism conjecture). *Let $(W^4; M_0^3, M_1^3)$ be a smooth 4-dimensional s-cobordism between closed 3-manifolds. Is W diffeomorphic to $M_0 \times [0,1]$?*

REMARKS.

(1) Matumoto and Siebenmann found counterexamples to the topological analogue in [MS78], where both M_0 and M_1 are $\mathbb{RP}^2 \times S^1$ and W is not known to be smoothable. Specifically that paper showed that the s-cobordism theorem fails either in dimension four or five, providing specific s-cobordisms that would fail to be products. Later work of Freedman and Quinn [FQ90, Theorem 7.1A] showed that the potential 5-dimensional candidate is indeed a product. So the 4-dimensional candidate of Matumoto and Siebenmann must fail to be a product. Cappell and Shaneson later provided further topological counterexamples where M_0 and M_1 are orientable. In a subsequent paper [CS87b] they asserted these examples were smoothable, but this claim was later retracted [CS87a].

(2) A weaker version of the question asks whether s-cobordant, or possibly even simple homotopy equivalent, 3-manifolds are necessarily homeomorphic. Kwasik and Schultz showed, assuming geometrization, that every topological h-cobordism between closed, orientable 3-manifolds is an s-cobordism, and that simple homotopy equivalence implies homeomorphism for closed, orientable 3-manifolds [KS92, Theorem and Theorem 1.1]. The questions appear to be open in the nonorientable setting; in particular geometrization is not yet known for nonorientable 3-manifolds. Whether there exists an h-cobordism between 3-manifolds with nontrivial Whitehead torsion appeared as Problem 4.9 on [Kir97].

(3) The s-cobordism theorem for dimensions 6 and higher is true in the smooth, piecewise linear, and topological settings [Sma62a, Bar63, Maz63, Sta67, KS77] (see also [Mil65, RS72]). The s-cobordism theorem in dimension five is false in the smooth (and equivalently piecewise linear) settings, by work of Donaldson [Don87a], and known to be true in the topological setting for good fundamental groups [FQ90, Theorem 7.1A] (see also [OPR21a]). See Problem 4.46.

Scribed by: A. Ray

PROBLEM 4.57. *Let X and Y be closed, oriented, smooth 4-manifolds with the same Euler characteristic and signature. Is there a torus link L in X with trivial normal bundle such that some choice of torus surgery along L transforms X into a manifold diffeomorphic to Y? What if we only require that the result be homeomorphic to Y?*

REMARKS.

(1) These questions were discussed in [Ste06, BS13, FS11].

(2) Let T be a torus in a 4-manifold W with trivial normal bundle. We say that a 4-manifold W' is obtained from W by torus surgery along T if $W' = W\backslash\nu(T) \cup (T^2 \times D^2)$ for some choice of gluing map $\partial\nu(T) \to \partial(T^2 \times D^2)$.

(3) It is a theorem of Iwase [Iwa90] that there is a 4-manifold Z and links of tori $L_X \subset X, L_Y \subset Y$ so that every component of L_X, L_Y has trivial normal bundle and there exists some choice of torus surgeries on

L_X, L_Y transforming X, Y into manifolds equivalent to Z. Here, if X, Y are smooth, then L_X, L_Y may be taken to be smooth, and "equivalent" means "diffeomorphic." Otherwise, surfaces are locally flat, and "equivalent" means "homeomorphic."

(4) In Iwase's paper, he actually proves that X can be transformed into $\#^a\mathbb{C}\mathrm{P}^2\#^b\overline{\mathbb{C}\mathrm{P}}^2\#^c S^1 \times S^3$ by surgery on a torus link in X for sufficiently large a with $b = a - \sigma(X)$, $c = (a + b + 2 - \chi(X))/2$. That is, we may take Z in the above discussion to be a connected sum of copies of $\mathbb{C}\mathrm{P}^2$s, $\overline{\mathbb{C}\mathrm{P}}^2$s, and $S^1 \times S^3$s. Iwase's argument holds in both categories.

A generalization of this result, extended over to the nonorientable 4–manifolds, is announced in a recent preprint of Baykur and Morgan [BM25], which states that any closed smooth 4–manifold is obtained by a surgery along a link of tori in a Z that is a connected sum of copies of $S^2 \times \mathbb{RP}^2$, $\mathbb{RP}^4$, $\mathbb{C}\mathrm{P}^2$s, $\overline{\mathbb{C}\mathrm{P}}^2$s, and $S^1 \times S^3$s.

(5) The problem asks whether, instead of X and Y being related by a *sequence* of two surgeries on torus links, all necessary torus surgeries transforming X into Y can be performed simultaneously.

(6) The problem seeks the 4-dimensional analog of the fact that any two closed, oriented 3-manifolds M, N are related by Dehn surgery along a link, rather than a sequence of Dehn surgeries. In dimension three, these two facts are clearly equivalent by dimensionality, but since surfaces generically intersect in 4-manifolds the situation is different.

(7) Fintushel–Stern [FS11] asked the following version of this question in the case the 4–manifolds are simply connected.

Question ([FS11, §9]). *Can a simply-connected closed, smooth 4-manifold always be obtained from torus surgery on a link of tori in a connected sum of* $\mathbb{C}\mathrm{P}^2, S^2 \times S^2$ *and* $K3$ *summands, taken with either orientations?*

See [BS13] for an approach to this question via 5-dimensional round handles. Round 2-handle attachments correspond to certain torus surgeries, and the work of [BS13] falls short at the same point as discussed above; the authors build cobordisms made out of only round 2-handles, but it is not clear if there is always a cobordism where these round 2-handles can be attached independently.

(8) Fintushel–Stern noted that many interesting simply connected 4-manifolds arise from a single null-homologous torus surgery, e.g. for $n = 2, \ldots, 7, 9$ there are infinitely many exotic $\mathbb{C}\mathrm{P}^2\#^n\overline{\mathbb{C}\mathrm{P}}^2$ that each arises from a single torus surgery on a null-homologous torus in $\mathbb{C}\mathrm{P}^2\#^n\overline{\mathbb{C}\mathrm{P}}^2$ [FS11, Theorem 6].

Question (Fintushel and Stern). *If* X *and* Y *are homeomorphic, simply-connected, smooth 4-manifolds, is it possible to obtain* Y *from surgery on a single torus in* X*? If so, can we arrange for the torus to be null-homologous?*

Proposed for K3 and scribed by: S. Kim, M. Miller

Problem 4.58.

(a) *Which Seifert fibered homology spheres $\Sigma(a_1, \dots, a_n)$ bound acyclic manifolds? Are there any examples with four or more singular fibers that bound acyclic manifolds?*

(b) *Which Seifert fibered homology spheres bound contractible manifolds? Is there an example that bounds an acyclic manifold but not a contractible manifold?*

(c) *Are there any Seifert fibered homology spheres that arise as cork boundaries?*

Remarks.

(1) The second part of Problem (a) (regarding the number of fibers) is in [**Kir97**, Problem 4.123] in a different guise, with a different motivation.

(2) Many Seifert fibered homology spheres bound acyclic (and, in fact, contractible) manifolds (see for example [**AK79b, CH81, Fic84, Şav20**]) but a complete characterization is unknown. All known such examples have three singular fibers. Amongst spheres $\Sigma(p, q, r)$ with three singular fibers, there is no closed characterization of which bound acyclic manifolds in terms of p, q, and r.

(3) It is a longstanding conjecture that no Seifert fibered homology sphere with four or more singular fibers bounds an acyclic manifold; see [**FS87b, Kol08**]. This is related to the Montgomery-Yang conjecture, which states that every pseudofree action of S^1 on S^5 has at most 3 non-free orbits. One can also ask the related question of which Seifert fibered homology spheres embed in $\mathbb{R}^4$. In the setting of symplectic topology, it is known that no Seifert fibered homology sphere (of any number of fibers) occurs as a hypersurface of contact type in $(\mathbb{R}^4, \omega_{std})$ [**MT22**]. A recent preprint [**AC24**] builds on this work and announces that no (standardly-oriented) Seifert fibered homology sphere bounds a Stein rational ball.

(4) One can also ask about the difference between bounding an acyclic manifold and bounding a contractible manifold. In general, these notions differ: Taubes' periodic ends theorem implies that $\Sigma(2, 3, 5) \# -\Sigma(2, 3, 5)$ bounds no contractible manifold [**Tau87**], whereas this trivially bounds an acyclic manifold. Many other examples can be obtained through instanton Floer theory. However, no such example consisting of an *individual* Seifert fibered homology sphere is known.

(5) A similar question is whether or not any Seifert fibered homology sphere Y forms a cork boundary. Here, recall that Y is a cork boundary if there exists a contractible manifold W with boundary Y, together with a self-diffeomorphism of Y that does not extend over W (as a diffeomorphism). It is known that the standard cyclic group actions on any Brieskorn sphere $\Sigma(p, q, r)$ do not extend (smoothly) as group actions to any contractible manifold with boundary $\Sigma(p, q, r)$ [**AH16, AH21**]. However, these do extend as diffeomorphisms. Current Floer-theoretic techniques devoted to establishing corks [**AKS20, DHM23**] are known to fail for Seifert fibered homology spheres. Note that several authors take W to be Stein in the definition of a cork; the notion defined here is sometimes called a loose cork.

Scribed by: I. Dai, J. Van Horn-Morris

PROBLEM 4.59. *Are lens spaces topologically homology cobordant if and only if they are homeomorphic?*

REMARKS.

(1) Livingston conjectures that the statement is correct.
(2) If $L(a_1, b_1)$ and $L(a_2, b_2)$ are homology cobordant, then $a_1 = a_2$. The problem can be restated: if $L(n, b_1)$ and $L(n, b_2)$ are homology cobordant, then $L(n, b_1)$ and $L(n, b_2)$ are homeomorphic. Gilmer and Livingston [**GL83**] proved this in the case that n is a prime power, using Atiyah–Singer signature invariants $\rho_\alpha(L)$ associated to characters $\alpha : \pi_1(L) \to U(1)$ of prime-power order. The simplest unknown case is the pair they identified, $L(231, 53)$ and $L(231, 86)$. It remains unknown whether these lens spaces are homology cobordant.
(3) In the smooth category, the conjecture was proved for n even by Fintushel–Stern [**FS87a**], with later generalizations by Matić [**Mat88**] and Ruberman [**Rub88**]. The conjecture in the smooth setting can also be proved using Heegaard Floer theory; see [**DW15**].
(4) For higher dimensional lens spaces, Cappell and Ruberman [**CR88**] showed that the ρ_α-invariants for α of prime-power order give the homology cobordism classification. This uses homology surgery theory [**CS74, Vog82**] which is known [**Akb79**] to fail in dimension 4, even topologically.

    It is conceivable to try to construct a topological homology cobordism W between non-diffeomorphic $L(n, b_1)$ and $L(n, b_2)$ with n composite using ordinary surgery theory. A first step might be to find an appropriate homotopical model for such a cobordism. The Gilmer–Livingston argument implies that the n-fold cyclic $\widetilde{W}$ would have to have nontrivial b_1, so that $\pi_1(W)$ would have to be large in this sense; see [**AGL18**] for more information on this.

Proposed for K3 and scribed by: C. Livingston

PROBLEM 4.60.

(a) *Let X be an open, spin, smooth 4-manifold. Does X have a proper smooth embedding in $\mathbb{R}^6$?*
(b) *By choosing a proper exhaustion function on W, (a) would follow from an affirmative answer to the following. Let $(W; M, N)$ be a compact, smooth spin cobordism with a smooth embedding f of M in S^5. Is there a smooth embedding*

$$F\colon (W; M, N) \to (S^5 \times I; S^5 \times \{0\}, S^5 \times \{1\})$$

whose restriction to M coincides with f?
(c) *Let $(W; M, N)$ be a compact, smooth spin cobordism with smooth embeddings f of M in S^5 and g of N in S^5 such that $\sigma(N, S^5) - \sigma(M, S^5) =$*

$\sigma(W)$. Is there a smooth embedding

$$F\colon (W; M, N) \to (S^5 \times I; S^5 \times \{0\}, S^5 \times \{1\})$$

whose restriction to M coincides with f and whose restriction to N coincides with g?

REMARKS.

(1) The spin condition is necessary in both parts of the problem.
(2) One motivation for part (a) is to understand the *embedding dimension* for a Stein surface (of real dimension four). By definition, this is the minimal d for which X has a proper holomorphic embedding in $\mathbb{C}^d$. General results about embedding dimension due to Eliashberg and Gromov [**EG92**] imply that a Stein 4-manifold has a proper holomorphic embedding into $\mathbb{C}^4$. Since a 4-manifold properly embedded in $\mathbb{C}^3 = \mathbb{R}^6$ is spin, this is the best possible result for non-spin Stein 4-manifolds, but it is conceivable that one could get embeddings in $\mathbb{C}^3$ for spin Stein 4-manifolds. Part (a) is a topological version of that question.
(3) Part (b) asks for a relative version of the result, announced by Cappell-Shaneson in [**CS79**] and proved by Ruberman in [**Rub82**], that a closed spin 4-manifold embeds in $\mathbb{R}^6$ if and only if its signature is 0.
(4) For part (c), note that an embedding of a 3-manifold M in S^5 has a well-defined signature $\sigma(M, S^5)$, given by the signature of any 4-manifold that M bounds in S^5. The equality $\sigma(N, S^5) - \sigma(M, S^5) = \sigma(W)$ is necessary for W to be a cobordism as in (b). It follows, in the setting of part (b), that one cannot specify the embedding of both M and N in advance. It is not clear if there are further obstructions, so part (c) represents a sharpening of part (b).

Proposed for K3 and scribed by: D. Ruberman

4.7. Smooth 4-manifold invariants

Here we present a few problems on diffeomorphism invariants of 4-manifolds, coming from gauge theory and Khovanov homology.

PROBLEM 4.61. *What do different 4-manifold gauge theories see?*

REMARKS.

(1) It is conjectured that the Donaldson, Seiberg–Witten, and Heegaard Floer invariants of closed 4-manifolds coincide (after organizing the Donaldson invariant as a generating function [**KM95**]). Yet each of these theories taken more broadly seem to see different geometric and topological properties. This problem asks how to understand some of these properties proved in one theory via one of the other theories.

(2) Is it possible to prove the existence of uncountable many exotic structures on $\mathbb{R}^4$ using Seiberg–Witten or Heegaard Floer theory? One might try to do this by adapting Taubes' periodic end gauge theory [**Tau87**] to the Seiberg–Witten setting. This can be done but to date requires rather strong hypotheses such as positive scalar curvature on the periodic end. Another possibility is to use limit invariants of ends such as the Heegaard Floer end invariant introduced by Gadgil [**Gad10**].
(3) Is there a Seiberg–Witten proof of the Donaldson–Sullivan [**DS89**] results that there are 4-manifolds without quasiconformal (and hence Lipschitz) structure, and that 4-manifolds can admit more than one such structures? A seemingly fundamental difficulty here is whether Lipschitz or quasiconformal manifolds have something like a Dirac operator; see the discussion in [**Sul87, Sul99**] and also Problem 4.131.
(4) Donaldson and Seiberg-Witten theory have parameterized versions that can be used to study invariants of diffeomorphisms and families of 4-manifolds as well as families of symplectic structures [**Rub98, BK22, Kon21, Kro97**]. Are there family versions of Heegaard Floer theory that would be useful for such applications?
(5) Seiberg-Witten invariants can be extended to give the Bauer–Furuta invariant [**BF04**] living in (equivariant) stable homotopy groups. Are there similar stable homotopy theoretic invariants coming from Donaldson or Heegaard Floer theory?
(6) Seiberg–Witten theory can be used to show that certain 4-manifolds admit no Riemannian metric of positive scalar curvature (PSC) [**Wit94**] and to distinguish path components in the space of PSC metrics [**Rub01**]. Find a way to do this using Donaldson or Heegaard Floer theory.
(7) All three theories [**Frø02, Frø96, OS03a**] give statements about the definite intersection forms of 4-manifolds with given boundary. Are these statements equivalent?
(8) Hambleton and Lee [**HL95**] used an equivariant version of Donaldson's original argument for his definite manifolds theorem to study smooth group actions on a simply connected (positive) definite 4-manifold. Among other results, they showed that a homologically trivial cyclic group action has the same fixed-point data and tangential isotropy representations as an equivariant connected sum of linear actions on $\mathbb{C}P^2$. Are there Seiberg–Witten or Heegaard Floer proofs of their results?

Proposed for K3 and scribed by: D. Ruberman

Problem 4.62. *Let X be a smooth, closed, connected, oriented 4-manifold with $b_2^+(X) > 1$.*

(a) *Does X have Donaldson simple type?*
(b) *Does X have Seiberg–Witten simple type?*

Remarks.

(1) The Simple Type Conjecture holds that the answer is affirmative. Part (a) is in [**Kir97**, Problem 4.131], and was raised by Kronheimer and Mrowka in [**KM95**] for simply connected 4–manifolds.

(2) A 4-manifold is said to have Donaldson simple type if the Donaldson polynomials q_k for principal $SU(2)$-bundles with $c_2 = k$ satisfy

$$q_{k+1}(\nu, \Sigma_1, \ldots, \Sigma_d) = 4q_k(\Sigma_1, \ldots, \Sigma_d)$$

where $\nu = \mu(1)$ ($\mu\colon H_0(X;\mathbb{Z}) \to H_4(M_{k+1})$), $\Sigma_i \in H_2(X;\mathbb{Z})$, and $2d = \dim M_k$.

(3) Manifolds which have Donaldson simple type [KM95] include:
- complete intersections,
- elliptic surfaces,
- any manifold with a Gompf nucleus,
- manifolds with a smoothly embedded surface F satisfying $2(\text{genus}(F)) - 2 = F \cdot F > 0$.

(4) The Kronheimer–Mrowka structure theorem from [KM95] says that, for manifolds of simple type, the Donaldson invariants are determined by finitely many basic classes $K_1, \ldots, K_s$ and rational numbers $\beta_1, \ldots, \beta_s$.
When $b_2^+ = 1$, some manifolds, e.g. $\mathbb{C}\mathrm{P}^2$, $S^2 \times S^2$, $\mathbb{C}\mathrm{P}^2 \# \overline{\mathbb{C}\mathrm{P}}^2$, do not have Donaldson simple type.

(5) A Seiberg–Witten *basic class* $\mathfrak{s}$ is a spin^c structure $\mathfrak{s}$ with nonzero Seiberg-Witten invariant $\mathrm{SW}_X(\mathfrak{s})$. A 4-manifold has *Seiberg-Witten simple type* if the virtual dimension of the Seiberg-Witten moduli space

$$d_X(\mathfrak{s}) = \frac{1}{4}(c_1(\mathfrak{s})^2 - 2\chi(\mathfrak{s}) - 3\sigma(X))$$

is zero for every basic class $\mathfrak{s}$.
Note that, when $d_X(\mathfrak{s}) = 0$, the Seiberg–Witten invariants are defined by a signed count of monopoles. When $d_X(\mathfrak{s}) > 0$, they are defined by evaluating a higher degree cohomology class on the moduli space of monopoles; the conjecture says that, in such cases, the evaluation is always zero.

(6) We list some important progress.
- Taubes' equivalence between the Seiberg–Witten invariants and the Gromov-Taubes invariants implies that all symplectic 4-manifolds are Seiberg–Witten simple type [Tau96].
- Kato-Nakamura-Yasui proved the conjecture for the mod 2 Seiberg-Witten invariants under a mild condition on the homology ring [KNY22].
- Baraglia [Bar23a] proved that the mod 2 Seiberg–Witten simple type conjecture holds for spin structures without any extra assumptions.

Just as in the Donaldson case, there are manifolds with $b_2^+ = 1$ that are not of simple type, because of the wall-crossing formula.

(7) Witten's conjecture [Wit94] states that if X has Seiberg–Witten simple type, then it also has Donaldson simple type, and there is a precise relation between its Donaldson and Seiberg–Witten invariants. The conjecture was proved in many cases by Feehan and Leness; see [FL15], [FL18].

Scribed by: C. Manolescu

Problem 4.63. *How many independent basic classes can a simply connected smooth 4-manifold X have, as measured by $\mathrm{br}(X)$, the rank of the span of the basic classes?*

- *(a) Is there an upper bound for $\mathrm{br}(X)$ in terms of topological invariants of X?*
- *(b) In particular, is $\mathrm{br}(X) \leqslant b_2^+(X)$ for all simply connected X with $b_2^+(X)$ odd?*
- *(c) Is there a smooth indefinite, simply connected 4-manifold X with $b_2(X) \geqslant 3$ for which the image of $\mathrm{ev}_*\colon \mathrm{Diff}(X) \to \mathrm{Aut}(Q_X)$ is finite, or even trivial?*

Remarks.

(1) Here we define

$$\mathrm{br}(X) = \mathrm{Rank}\,(\mathrm{Span}\{c_1(\mathfrak{s}) \mid \mathfrak{s} \text{ is a Seiberg-Witten basic class on } X\})$$

if X has any basic classes, and 0 otherwise.

(2) Knot surgery [**FS98**] on n disjoint and homologically independent tori in a manifold with nontrivial Seiberg-Witten invariant would create a manifold X for which $\mathrm{br}(X) = n$. For example, starting with an elliptic surface, the construction in [**GM93**] produces examples of manifolds X for which $\mathrm{br}(X) = b_2^+(X)$.

(3) Questions (a) and (b) are relevant to the study of the map $\mathrm{ev}_* : \mathrm{Diff}(X) \to \mathrm{Aut}(Q_X)$ giving the action of a diffeomorphism on the intersection form. Since any diffeomorphism must permute the basic classes up to sign, the presence of many basic classes can restrict the size of the image of ev_* .

(4) The restriction to indefinite manifolds and $b_2 > 2$ is to rule out intersection forms with finite automorphism groups. If $\mathrm{br}(X) = b_2(X)$, then $\mathrm{ev}_*(\mathrm{Diff})$ is contained in a finite permutation group and hence is finite.

Proposed for K3 by: D. Auckly
Scribed by: D. Ruberman

Problem 4.64. *Find an irreducible, closed, smooth 4-manifold with nontrivial Bauer–Furuta invariant but with trivial Seiberg–Witten invariant.*

Remarks. The Bauer–Furuta invariant Ψ [**BF04**] is a stable cohomotopy refinement of the Seiberg–Witten invariant [**Wit94**], building on Furuta's proof of the 10/8-theorem [**Fur01**]. There are examples for which the Bauer–Furuta invariant is strictly stronger than the Seiberg–Witten invariant; for instance, Ψ can be used to distinguish between certain connected sums of homotopy K3 surfaces (which have vanishing Seiberg–Witten invariant). Such examples rely on a gluing formula due to Bauer [**Bau04**]; no irreducible examples are known.

Proposed for K3 by: M. Stoffregen
Scribed by: I. Dai

PROBLEM 4.65. *Suppose X is a smooth 4-manifold with the homology of $S^1 \times S^3$ whose infinite cyclic cover $\widetilde{X}$ has $H^1(\widetilde{X}) = 0$. Furuta and Ohta* [**FO93**] *define an invariant $\lambda_{FO}(X)$ as 1/4 of the signed count of irreducible flat* SU(2) *connections on X; this requires an orientation of X and a specified generator of $H^1(X)$.*

(a) *Does the following hold? The invariant $\lambda_{FO}(X)$ is an integer, and $\lambda_{FO}(X) \equiv \rho(Y, \mathfrak{s})$, where $\rho(Y, \mathfrak{s})$ is the Rokhlin invariant of an oriented spin 3 manifold Y that is Poincaré dual to the generator of $H^1(X)$.*
(b) *Mrowka–Ruberman–Saveliev* [**MRS11**] *give an approach, defining an invariant $\lambda_{SW}(X)$ by counting solutions to the Seiberg-Witten equations and adding an index-theoretic correction term. Is*

$$\lambda_{FO}(X) = -\lambda_{SW}(X)?$$

REMARKS.

(1) Part (a) was conjectured by Furuta–Ohta.
(2) A solution to (a) would imply that the Wall group $L_5(\mathbb{Z}[\mathbb{Z}])$ does not act on the smooth structure set of $S^1 \times S^3$; compare the discussion in Problem 4.22. By construction, $\lambda_{SW}(X)$ is an integer, and it is shown in [**MRS11**] that it reduces mod 2 to $\rho(Y, \mathfrak{s})$. So a positive answer to (b) implies a positive answer to (a).
(3) The invariant $\lambda_{FO}(X)$ is defined in greater generality; one could require only that X is a homology $S^1 \times S^3$ whose twisted cohomology $H^1(X; \mathbb{C}_\alpha)$ vanishes for any homomorphism $\alpha : \pi_1(X) \to \mathrm{U}(1)$. In this setting, neither part of (a) holds, but (b) is still plausible. The paper [**LRS21**] shows that (b) holds for mapping tori of all orientation preserving diffeomorphisms of homology spheres generating a semifree finite cyclic group action. For involutions, the result also follows from [**LRS23b**].

Scribed by: D. Ruberman, N. Saveliev

PROBLEM 4.66. *Can the skein lasagna module detect exotic smooth structures on closed 4-manifolds?*

REMARKS.

(1) The skein lasagna module is an extension of Khovanov homology. It is an invariant of 4-manifolds with (possibly empty) boundary and a framed link in their boundary. It was defined by Morrison, Walker and Wedrich in [**MWW22**]. Ren and Willis [**RW24**] gave examples of exotic compact 4-manifolds with boundary that are detected by the skein lasagna module. For closed 4-manifolds, the computations so far are limited; see [**MN22**], [**MWW23**], [**RW24**]. The invariant is nonvanishing for S^4, $S^1 \times S^3$ and $\overline{\mathbb{CP}}^2$. On the other hand, it vanishes for manifolds that contain a smoothly embedded sphere with positive self-intersection (cf. Theorem 1.3 in [**RW24**]); e.g. for $\mathbb{CP}^2$ or $-K3$.
(2) The invariant is multiplicative under connected sums, and it takes the value 0 on $S^2 \times S^2$. Thus, it has a chance of detecting exotic smooth

structures on simply connected, closed 4-manifolds, even though such structures become standard after sufficiently many stabilizations.

Proposed for K3 and scribed by: C. Manolescu

4.8. Symmetries of 4-manifolds

Symmetries of 4-manifolds refers primarily to the topological groups of self-homeomorphisms $\mathrm{Homeo}(X)$ or self-diffeomorphisms $\mathrm{Diff}(X)$ of a fixed 4-manifold X; it can also refer to group actions on the manifold. The smooth (resp. topological) *mapping class group of* X is the group of isotopy classes of these diffeomorphisms (resp. homeomorphisms).

We begin with problems asking for exotic diffeomorphisms, namely diffeomorphisms that are topologically but not smoothly isotopic to the identity. After that, the next few problems consider higher families of symmetries, for instance the homotopy groups of $\mathrm{Diff}(X)$ or $\mathrm{Homeo}(X)$. Then come a series of problems on stabilizations. After a few more problems, on non-smoothability, the Nielsen problem, uniform perfectness, and 5-dimensional mapping tori, we end the section with a collection of problems on group actions on 4-manifolds.

We collect a few conventions here. For connected X, we write $\mathring{X} := X \backslash \mathring{D}^4$ for a punctured X obtained by removing an open 4-ball from the interior of X. In the following definitions, we use morphism to refer to either homeomorphisms or diffeomorphisms, depending on the category. The groups $\mathrm{Diff}^+(X)$ and $\mathrm{Homeo}^+(X)$ denote the respective subgroups of orientation-preserving morphisms. The notation $\mathrm{Diff}_\partial(X)$ and $\mathrm{Homeo}_\partial(X)$ denotes morphisms of X that fix the boundary ∂X pointwise. A pseudo-isotopy is a morphism $F\colon X \times I \to X \times I$ that restricts to the identity on $(X \times 0) \cup (\partial X \times I)$. We say that the morphism $F|_{X\times 1}$ is pseudo-isotopic to the identity.

PROBLEM 4.67.

(a) Compute $\pi_0(\mathrm{Diff}^+(S^4))$*. Do we have* $\pi_0(\mathrm{Diff}^+(S^4)) = \{1\}$*?*

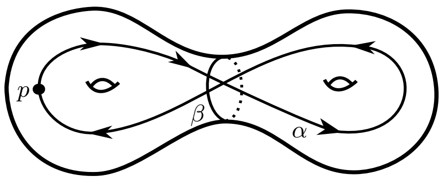
(b) In particular, does some implantation of the barbell map provide a nontrivial element in $\pi_0(\mathrm{Diff}^+(S^4))$*?*

REMARKS.

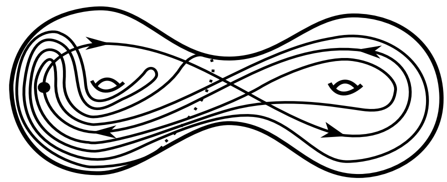
(1) The *4-dimensional generalized Smale conjecture* asked whether the inclusion $SO(5) \hookrightarrow \mathrm{Diff}^+(S^4)$ is a homotopy equivalence (see Problems 4.34 and 4.126 in [**Kir97**]). Watanabe [**Wat19**] disproved this conjecture, by proving that $\pi_k(\mathrm{Diff}_\partial(D^4))$ is nontrivial for many k, including $k = 1, 4, 8$.

A fundamental issue remaining to understand is $\pi_0(\mathrm{Diff}^+(S^4))$, the orientation-preserving mapping class group of S^4, and it is presently unknown whether or not this group is trivial.

Some sources of possibly nontrivial diffeomorphisms of S^4 are suggested in [**Gay25**], [**BG19**] and [**GGH$^+$23**]. The Pin(2)-equivariant family Bauer-Furuta invariant could potentially detect such nontrivial diffeomorphisms; see [**LM25**].



(a) Before pushing



(b) After pushing

FIGURE 2. Point push map on a surface

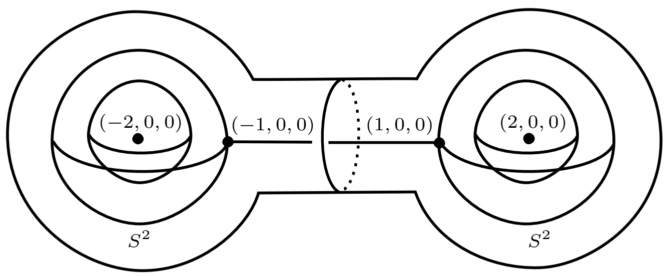


FIGURE 3. Barbell

(2) The *barbell map* is defined by Budney and Gabai in [**BG19**] (see also [**BG25**] for the 'X-resolution') using isotopy extension, via a generalization of Birman's 'push map' [**Bir69**]. We give an exposition here.

Birman's map can be described by pushing a point p (see Figure 2(a)) along a path α in a closed surface S and back to p. This isotopy of the surface can be assumed to fix a disk D centered at p at the end of the isotopy. Removing the interior of D, we get a diffeomorphism of the punctured surface that is the identity on ∂D and also the identity outside a nice neighborhood of the arc α. Figure 2(b) shows what happens to the loop β under this diffeomorphism.

In general, when pushing a 0-dimensional point p along a 1-dimensional loop α in a 2-dimensional surface, the point crosses a loop (say β) and bulldozes it over D.

Consider a pair of 2-spheres in $\mathbb{R}^3$ centered at $(\pm 2, 0, 0)$ of radius one, which are joined by the arc in the x-axis $[-1, 1]$. We can use the z-axis to define the equator $(x^2 + y^2 = 1)$, latitudes, and the two poles. Thicken this slightly in $\mathbb{R}^3$, to what might be called a (hollow) barbell. Now cross with $[-3, 3]$ to get a four-dimensional analogue called B. We will find an interesting diffeomorphism β of B that is the identity on ∂B.

Notice the two line segments $(\pm 2, 0, 0) \times [-3, 3]$. These can be thickened so that they fill in the "hollows", the pair of $S^2 \times [-1, 1]$s, so that we have a solid 4-dimensional barbell, which is obviously B^4. Figure 3 may help.

Now focus just on the left line segment and move a portion of it, namely $(-2, 0, 0) \times [-1, 1]$, around the 2-sphere on the right side, sort of like lassoing a horse's head. We will do this with a 1-parameter family of

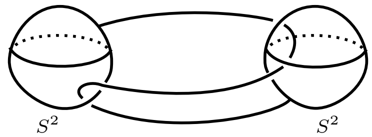


FIGURE 4. Embedding of barbell

Birman push maps, parameterized by $t \in [-3, 3]$. The push maps occur on a 2-dimensional surface given by fixing t and z.

The push map for $t = 0$ will push the point $p = (-2, 0, 0) \times 0$ over to and then around the equator ($z = 0$) of the 2-sphere at $t = 0$ and back to p; for $t \in (-1, 1)$, p goes over to and around the latitude at $z = t$ and back to p; for $t = \pm 1$, p goes over to a pole and then back to p; for $t \in (-2, -1) \cup (1, 2)$, p goes partway to a pole and then back to p; finally, for $t \in (-3, -2) \cup (2, 3)$, p does not move.

This isotopy of the left line segment extends to an isotopy of B^4 that is the identity on ∂B^4 and on the two line segments. But if this isotopy fixes the two line segments, then it can also be made to fix the above thickenings of the segments. Therefore, the end of this isotopy is a diffeomorphism β taking the 4-dimensional hollow barbell B back to itself, and fixing its boundary.

Note that the lasso can go over the horse's head with two possible orientations. Also there is a rotation ρ switching the two balls in the barbell. It can be checked that $\rho\beta\rho^{-1} = \beta^{-1}$.

The 4-dimensional barbell, B, can be embedded, by f, in a 4-manifold X in many ways, and these are called implantations and the induced barbell map can be called $f_*\beta$.

(3) A specific example of a barbell diffeomorphism to consider is as follows.

QUESTION. *Does the following embedding f induce a nontrivial diffeomorphism of S^4, where the embedding is given by mapping the two 2-spheres to separate 2-spheres in S^4, and the bar goes from the left sphere over to link the right sphere and then back again to link the first and finally attaching to the right sphere? See Figure 4.*

A positive answer has been announced by Gabai–Gay–Hartman [**GGH25**].

(4) We can also ask the following structural question.

QUESTION. *Is $\mathrm{Diff}^+(S^4)$ (finitely) generated up to isotopy by a composition of $f_*\beta$s?*

Scribed by: H. Konno, R. Kirby

PROBLEM 4.68. *Does every closed smooth 4-manifold admit an exotic diffeomorphism? How about the following special cases?*

(a) *Is there a definite smooth closed 4-manifold that admits an exotic diffeomorphism?*

(b) Does $S^2 \times S^2$ or $K3$ admit an exotic diffeomorphism?

REMARKS.

(1) A self-diffeomorphism $f: X \to X$ of a smooth manifold X is called *exotic* if it is topologically but not smoothly isotopic to the identity. The first examples of exotic diffeomorphisms of 4-manifolds were given by Ruberman [**Rub98**]. Most known examples of closed 4-manifolds confirmed to admit exotic diffeomorphisms are of the form $\#^m\mathbb{CP}^2\#^n\overline{\mathbb{CP}}^2$ for $m, n > 0$ [**Rub98**], $\#^m K3\#^n S^2 \times S^2$ for $m, n > 0$ [**BK20**] or $m = 0$ [**AR25**], and $K3\#K3$ [**KM20**]. There exist irreducible 4-manifolds that admit exotic diffeomorphisms [**BK24a**], but the proof in [**BK24a**] does not apply to S^4, $S^2 \times S^2$, $\mathbb{CP}^2$, or $K3$.

(2) In the literature, the smallest (in term of second Betti number) closed 4-manifold that is known to admit an exotic diffeomorphism is $\#^2\mathbb{CP}^2\#^{10}\overline{\mathbb{CP}}^2$, announced in [**Qiu24**]. It is natural to ask how small a closed 4-manifold with an exotic diffeomorphism can be. In particular, whether such a diffeomorphism exists on S^4 is the π_0 case of the Smale conjecture; see Problem 4.67.

For 4-manifolds with boundary, there exists an example of a contractible (hence, definite) 4-manifold that has an exotic diffeomorphism (relative to the boundary) [**KMT23a, KMPW24, KPT26, KLMME24**].

(3) For a simply connected closed smooth 4-manifold X, every exotic diffeomorphism of X is smoothly isotopic to the identity after sufficiently many stabilizations by $S^2 \times S^2$. This fact follows by combining work of Kreck [**Kre79**] and either Quinn [**Qui86**] (cf. [**GGH+23**]) or Gabai [**Gab22**]. It is an interesting question to determine how many stabilizations are needed to kill the exotic property of a given diffeomorphism. Many known examples of exotic diffeomorphisms, such as those in [**Rub98, BK20**], are smoothly isotopic to the identity after only one stabilization [**AKMR15**]. On the other hand, Lin [**Lin23**] proved that the exotic diffeomorphism of $K3\#K3$ from [**KM20**] stays exotic after one stabilization. There is no known upper bound for how many stabilizations are needed to trivialize this diffeomorphism. See also Problem 4.79.

Proposed for K3 and scribed by: H. Konno

PROBLEM 4.69. *Does there exist a diffeomorphism of a closed 3-manifold $f: M \to M$ such that f is topologically but not smoothly pseudo-isotopic to the identity?*

REMARKS.

(1) Friedman–Witt [**FW86**] constructed diffeomorphisms f that are homotopic but not isotopic to the identity. These are also topologically pseudo-isotopic to the identity [**KS96**].

QUESTION. *Are the Friedman–Witt diffeomorphisms smoothly pseudo-isotopic to the identity?*

(2) The following observation yields a potentially useful reformulation. Consider the embedding

$$i_{1/2}\colon M \to M \times \{1/2\} \hookrightarrow M \times [0,1]$$

where the first map is the canonical identification. Then f is smoothly (topologically) pseudo-isotopic to the identity if and only if $i_{1/2} \circ f$ and $i_{1/2}$ are smoothly (topologically) isotopic as embeddings.

The proof of this observation, which we give next, was provided separately by Hatcher and Igusa. It applies in all dimensions.

If f were pseudo-isotopic to the identity via $F\colon M \times I \to M \times I$, then $i_{1/2} \circ f$ and $i_{1/2}$ would isotopic as embeddings. To see this, note that by translation it suffices to show that i_0 and $i_1 \circ f$ are isotopic. But $g_t\colon M \to M$ defined by $g_t(x) = F(x,t)$ gives such an isotopy.

Conversely, if $i_{1/2}$ and $i_{1/2} \circ f$ are isotopic as embeddings, then apply isotopy extension rel. $M \times \{0,1\}$ to the isotopy, to obtain a diffeomorphism of $M \times [0,1/2]$ that restricts to the identity on $M \times \{0\}$ and to f on $M \times \{1/2\}$ (and similarly in $M \times [1/2,1]$). Thus after rescaling we obtain a pseudo-isotopy from f to the identity.

Proposed for K3 and scribed by: I. Agol

Problem 4.70. *Do there exist $k \geqslant 0$ and a smooth closed 4-manifold X such that the map $\pi_k(\mathrm{Diff}(X)) \to \pi_k(\mathrm{Homeo}(X))$ induced by the inclusion $\mathrm{Diff}(X) \hookrightarrow \mathrm{Homeo}(X)$ is an isomorphism?*

Remarks.

(1) Lin–Xie [**LX23**] proved that, for every orientable compact smooth 4-manifold X, at least one of the following holds:
- $\pi_1(\mathrm{Diff}(X)) \to \pi_1(\mathrm{Homeo}(X))$ is not injective.
- $\pi_2(\mathrm{Diff}(X)) \to \pi_2(\mathrm{Homeo}(X))$ is not surjective.

They prove also that, if $\partial X \neq \varnothing$ or the signature of X is non-zero, then there are many degrees k for which $\pi_k(\mathrm{Diff}(X)) \to \pi_k(\mathrm{Homeo}(X))$ is not an isomorphism. However, it is still possible that for some 4-manifold X and some degree k, the map $\pi_k(\mathrm{Diff}(X)) \to \pi_k(\mathrm{Homeo}(X))$ is an isomorphism.

The above result of Lin–Xie is based on Watanabe's work [**Wat19**] for $X = S^4$. Watanabe proved that nontrivial elements in the kernel of $\pi_k(\mathrm{Diff}(S^4)) \to \pi_k(\mathrm{Homeo}(S^4))$ exist whenever $\mathcal{A}_{k+1} \neq 0$. Here $\mathcal{A}_{k+1}$ is the degree $(k+1)$-part of a specific graph cohomology.

(2) Many other negative results are obtained by (mainly family) gauge theory, such as [**AR25**, **FM88**, **Don90**, **MS97**, **Rub98**, **BK20**, **KKN21b**, **BK22**, **Bar21**, **BK23**, **KN23**, **KM20**, **Lin23**, **KT22b**, **IKMT25**, **KMT23a**, **GL25**].

(3) As a positive result, a classical theorem by Wall [**Wal64a**] shows that there are many 4-manifolds X for which the map $\pi_0(\mathrm{Diff}(X)) \to \pi_0(\mathrm{Homeo}(X))$ is surjective. However, there also exist examples where this map is not surjective; see [**FM88**, **Don87b**].

Proposed for K3 and scribed by: H. Konno

PROBLEM 4.71.

(a) Do there exist $k \geqslant 0$ and a smooth closed orientable 4-manifold X such that $\pi_k(\operatorname{Diff}(X))$ is finitely generated?
(b) Do there exist $k > 0$ and a smooth closed orientable 4-manifold X such that $H_k(B\operatorname{Diff}(X);\mathbb{Z})$ is finitely generated?

REMARKS.

(1) For each $k > 0$, there exist simply-connected closed smooth 4-manifolds X where $H_k(B\operatorname{Diff}(X);\mathbb{Z})$ are not finitely generated [**Kon24b**]. Also, Auckly–Ruberman [**AR25**] proved that, for each $k > 0$, there exist simply-connected closed smooth 4-manifolds X where $\pi_k(\operatorname{Diff}(X))$ is not finitely generated. In dimension $\neq 4$, there are several finiteness results. See [**Kup19b, BKK24**]. See also Problem 4.72 for the analogous question in the topological category.
(2) If X is oriented, we can ask an analogous question for $\operatorname{Diff}^+(X)$, the orientation-preserving diffeomorphism group, in place of $\operatorname{Diff}(X)$. (Note that $\operatorname{Diff}(X) = \operatorname{Diff}^+(X)$ if X has non-zero signature.) For homotopy groups $\pi_k(\operatorname{Diff}(X))$ and $\pi_k(\operatorname{Diff}^+(X))$, finite generation of $\pi_k(\operatorname{Diff}(X))$ and that of $\pi_k(\operatorname{Diff}^+(X))$ are equivalent. However, for $H_k(B\operatorname{Diff}(X);\mathbb{Z})$ and $H_k(B\operatorname{Diff}^+(X);\mathbb{Z})$, the questions may not be equivalent. For example, when $\operatorname{Diff}(X) \neq \operatorname{Diff}^+(X)$, and if we take rational coefficients, the covering map $\mathbb{Z}/2\mathbb{Z} \to B\operatorname{Diff}^+(X) \to B\operatorname{Diff}(X)$ induces an isomorphism
$$H_k(B\operatorname{Diff}^+(X);\mathbb{Q})^{\mathbb{Z}/2} \cong H_k(B\operatorname{Diff}(X);\mathbb{Q}),$$
where the superscript $\mathbb{Z}/2$ indicates the monodromy invariant part. Thus finite generation of $H_k(B\operatorname{Diff}^+(X);\mathbb{Q})$ implies that of $H_k(B\operatorname{Diff}(X);\mathbb{Q})$, but the converse may not be true in general.
(3) As sets, the homotopy and homology groups of diffeomorphism groups of compact manifolds are always countable.
(4) The diffeomorphism groups of non-compact 4-manifolds can have finitely generated homotopy and homology groups, e.g. $\operatorname{Diff}(\mathbb{R}^4)$, in the weak C^∞-topology, is homotopy equivalent to $O(4)$. Nevertheless, there also exist exotic $\mathbb{R}^4$'s with infinitely generated mapping class groups [**Gom18**].
(5) The problem is open even for $X = S^4$ or D^4.

Proposed for K3 and scribed by: H. Konno, M. Powell

PROBLEM 4.72. *Let X be a closed orientable topological 4-manifold with finite $\pi_1(X)$.*

(a) Is $\pi_k(\operatorname{Homeo}(X))$ finitely generated for every $k \geqslant 0$?
(b) Is $H_k(B\operatorname{Homeo}(X);\mathbb{Z})$ finitely generated for every $k \geqslant 0$?

Remarks.

(1) For $\pi_1(X) = 1$, it follows from a result by Perron [Per86] and Quinn [Qui86] (cf. [GGH+23]) that $\pi_0(\mathrm{Homeo}(X))$ is finitely generated, which implies that $H_1(B\,\mathrm{Homeo}(X);\mathbb{Z})$ is also finitely generated. There is no known finiteness result on $\pi_k(\mathrm{Homeo}(X))$ for $k \geqslant 1$ and $H_k(B\,\mathrm{Homeo}(X);\mathbb{Z})$ for $k \geqslant 2$. See Problem 4.71 for the analogous question in the smooth category. The following question is closely related.

Question. *Does* Top(4) *have finitely generated homotopy groups in each degree?*

Indeed, Top(4) has finitely-generated homotopy groups if and only if $\mathrm{Homeo}(S^4)$ has finitely-generated homotopy groups, because we have a fibration

$$\mathrm{Top}(4) \to \mathrm{Homeo}(S^4) \to S^4,$$

and the homotopy groups of S^4 are finitely generated. The question on the homotopy groups of Top(4) is itself closely related to Problem 4.73 on the Morlet correspondence in dimension 4.

(2) Here is another closely related question, on topological embedding spaces.

Let Σ be a compact surface, and consider a compact 4-manifold X. Fix a locally flat embedding $\iota\colon \partial\Sigma \hookrightarrow \partial X$. Let $\mathrm{Emb}^t_\partial(\Sigma, X)$ be the space of locally flat embeddings of Σ extending ι. This is defined as the geometric realization of a semi-simplicial set, where the p-simplices are locally flat embeddings $\Sigma \times \Delta^p \to X \times \Delta^p$ over the projection to Δ^p, and extending $\iota \times \mathrm{Id}\colon \partial\Sigma \times \Delta^p \to X \times \Delta^p$. Let $f_0\colon \Sigma \hookrightarrow X$ be a 0-simplex.

Question. *Suppose that $\pi_1(X \backslash f_0(\Sigma))$ is finite, and fix $k > 0$. Is $\pi_k(\mathrm{Emb}^t_\partial(\Sigma, X), f_0)$ finitely generated?*

Randal-Williams [RW] has announced that the answer to the analogous question is no, in a case where the fundamental group of the surface complement is infinite. Using [BG19, BG25], Randal-Williams deduced that $\pi_4(\mathrm{Emb}^t(S^2, S^4), U)$ is infinitely generated, where U is the trivial 2-knot.

The homotopy groups spaces of embeddings are closely related to the homotopy groups of the homeomorphism groups of both the ambient space and of the exterior of the basepoint embedding. The space of thickenings of a fixed embedding to a closed tubular neighborhood plays an important rôle as well. As demonstrated by Randal-Williams' note, information about any of these characters often leads to information about the others.

Proposed for K3 and scribed by: H. Konno, M. Powell

Problem 4.73. *Does the Morlet correspondence*

$$\mathrm{BDiff}_\partial(D^n) \simeq \Omega^n_0(\mathrm{Top}(n)/O(n)) \tag{10}$$

hold for $n = 4$?

Remarks.

(1) Here $\mathrm{BDiff}_\partial(D^n)$ denotes the classifying space of the diffeomorphism group of D^n relative to its boundary. We write $\mathrm{Top}(n)$ for the group of homeomorphisms on $\mathbb{R}^n$ that fix the origin, and $\mathrm{Top}(n)/O(n)$ for the homotopy fiber of $\mathrm{BO}(n) \to \mathrm{BTop}(n)$. Finally let $\Omega_0^n(\mathrm{Top}(n)/O(n))$ denote the unit component of the loop space $\Omega^n(\mathrm{Top}(n)/O(n))$.
(2) In dimension $n \neq 4$, the weak equivalence (10) follows from smoothing theory [**BL74, KS77**]. It is known that smoothing theory fails in dimension 4. For example, the manifold $E_8 \# E_8$ has a formal smooth structure (i.e. a vector bundle structure on its tangent microbundle) but has no smooth structure. However, (10) may still hold for $n = 4$.
(3) Watanabe [**Wat19**] disproved the 4-dimensional Smale conjecture by showing that $\pi_k(\mathrm{BDiff}_\partial(D^4)) \otimes \mathbb{Q} \neq 0$ for many values of k including $2, 5, 9$. It is known that the group $\pi_{k+4}(\mathrm{Top}(n)/O(n)) \otimes \mathbb{Q}$ is also nonvanishing for these values of k [**LX23**].
(4) Gauge theory can distinguish non-diffeomorphic smooth structures on a closed 4-manifold that are isomorphic as formal smooth structures. So gauge theory could potentially be used to disprove (10).
(5) The question is closely related to Problem 4.67. If there is a diffeomorphism of D^4 not isotopic to the identity, and this is detected using gauge theory, then this could show that the Morlet correspondence does not hold.

Proposed for K3 and scribed by: J. Lin

PROBLEM 4.74. *Does there exist a closed, smooth 4-manifold X and a diffeomorphism $f\colon X \xrightarrow{\cong} X$ such that f is smoothly pseudo-isotopic to the identity, but f is not stably smoothly isotopic to the identity?*

REMARKS.
(1) We can stabilize a diffeomorphism by making a choice of isotopy to one that is the identity on a 4-ball, connect summing with $\#^k S^2 \times S^2$ using that 4-ball, for some k, and then extending by the identity on the new $\#^k S^2 \times S^2$. If some stabilization of f (for some choice of isotopy and for some k) is smoothly isotopic to the identity, then we say that f is smoothly stably isotopic to Id.
(2) Gabai [**Gab22**] proved that there is a smooth pseudo-isotopy F with vanishing Hatcher-Wagoner pseudo-isotopy obstruction $\Sigma(F) \in \mathrm{Wh}_2(\pi_1(X))$ if and only if f is smoothly stably isotopic to Id. The question is whether there exists an f such that $\Sigma(F)$ is nontrivial for *all* pseudo-isotopies F from f to Id. Or perhaps, for any pair (f, F), there is always a choice of F with $\Sigma(F) = 0$ restricting to the same f. Singh [**Sin25**] proved that the Hatcher–Wagoner obstruction Σ can be stably realized, which could be useful if one could control the diffeomorphism produced by his realization procedure.

Proposed for K3 and scribed by: M. Powell

Problem 4.75. *Let X be a connected smooth 4-manifold with nonempty boundary, with finite $\pi_1(X)$, and let $k \geqslant 0$. Let $\operatorname{Diff}_\partial(X)$ denote the group of diffeomorphisms of X that are the identity near ∂X. Is the image of the natural map*

$$s_*\colon \pi_k(\operatorname{Diff}_\partial(X)) \to \operatorname*{colim}_{N\to\infty} \pi_k(\operatorname{Diff}_\partial(X\#_N S^2 \times S^2)) \tag{11}$$

finitely generated? What about the analogous problem in the topological category?

Remarks.

(1) Fixing a model of the interior connected sum $X\#S^2 \times S^2$ defined by

$$X\#S^2 \times S^2 = X \cup ((\partial X \times [0,1])\#S^2 \times S^2),$$

we have a well-defined stabilization map

$$s\colon \operatorname{Diff}_\partial(X) \to \operatorname{Diff}_\partial(X\#S^2 \times S^2),$$

extending diffeomorphisms by the identity map on $(\partial X \times [0,1])\#S^2 \times S^2$.

(2) In the topological category, Problem 4.72 asks the analogous question without stabilization.

(3) We take a closer look at the problem for $k = 0$, $\pi_1(X) = 1$, and connected boundary. A diffeomorphism $f\colon X \to X$ determines a Poincaré variation [Sae06]. The group of Poincaré variations is denoted $\mathcal{V}(H_2(X), \lambda_X)$, where λ_X is the intersection form of X. Saeki [Sae06, Theorem 3.7] proved that the stable mapping class group of X is isomorphic to the group of stable Poincaré variations $S\mathcal{V}(H_2(X), \lambda_X)$. Let

$$V := \operatorname{Im}(\theta\colon \pi_0 \operatorname{Diff}_\partial(X) \to \mathcal{V}(H_2(X), \lambda_X)).$$

The image of s_* is thus isomorphic to the image of V under the algebraic stabilization map $\mathcal{V}(H_2(X), \lambda_X) \to S\mathcal{V}(H_2(X), \lambda_X)$. This latter map is injective, so in fact $\operatorname{Im}(s_*) \cong V$. We need to decide whether V is finitely generated.

By [Sae06, Section 4], the group $\mathcal{V}(H_2(X), \lambda_X)$ sits in an exact sequence

$$0 \to \wedge^2 H^1(\partial X; \mathbb{Z}) \to \mathcal{V}(H_2(X), \lambda_X) \to \operatorname{Aut}_\partial(H_2(X; \mathbb{Z}), \lambda_X).$$

The first group is finitely generated, and the image of the second map is a finite index subgroup of $\operatorname{Aut}_\partial(H_2(X; \mathbb{Z}), \lambda_X)$, the automorphisms of the intersection form of X whose algebraic boundary is trivial. The latter group is arithmetic, so is finitely generated. It follows that $\mathcal{V}(H_2(X), \lambda_X)$ is finitely generated, since finite index subgroups and extensions of finitely generated groups are again finitely generated. Is V finitely generated?

In the special case that $\partial X = S^3$

$$\mathcal{V}(H_2(X), \lambda_X) \cong \operatorname{Aut}(H_2(X; \mathbb{Z}), \lambda_X).$$

By capping off with a 4-ball we can apply Wall's theorem [Wal64a] to see that θ is surjective when X is of the form $X = M\#S^2 \times S^2$, where M is indefinite or $b_2(M) < 9$. Thus in these cases the image of s_*, for $k = 0$, is known to be finitely generated. In other cases, such as for $X = K3\setminus \mathring{D}^4$, V is finite index in $\mathcal{V}(H_2(X), \lambda_X) \cong \operatorname{Aut}(H_2(X; \mathbb{Z}), \lambda_X)$, so is finitely generated. However there are also examples, such as for X the punctured Dolgachev surface, where V is infinite index [FM88] in $\operatorname{Aut}(H_2(X; \mathbb{Z}), \lambda_X)$. In such cases, is V finitely generated?

(4) Without stabilization, it is known that $\pi_k(\mathrm{Diff}(X))$ need not be finitely generated, even for $\pi_1(X) = 1$. This was proven by Ruberman [Rub99b] for $k = 0$, with later proofs by Baraglia [Bar23c] and Konno [Kon24b]. For $k = 1$ it was proven by Baraglia[Bar23b] and Lin [Lin22]. For $k \geqslant 1$ this is announced in recent work of Auckly–Ruberman [AR25]. Different 4-manifolds are used in each work. All of these results are proven using gauge theory for families.

(5) For 4-manifolds with infinite fundamental group, Budney–Gabai [BG19] and Watanabe [Wat23] proved (without using gauge theory) that some 4-manifolds with infinite fundamental group (e.g. $S^1 \times S^3$ in [BG19], and hyperbolic manifolds of dimension at least 4 in [BG25]) have infinitely generated mapping class groups. However the diffeomorphisms they construct are pseudo-isotopic to the identity, and hence are stably isotopic to the identity by Gabai's theorem [Gab22]. It would be interesting to know whether there are counterexamples when $\pi_1(X)$ is infinite.

(6) Since $\pi_k(\mathrm{Diff}_\partial(X)) \cong \pi_{k+1}(B\,\mathrm{Diff}_\partial(X))$, the problem can be described in terms of $B\,\mathrm{Diff}_\partial(X)$. If one considers the analogous question for the homology groups of $B\,\mathrm{Diff}_\partial(X)$, the answer is often positive. This is because it follows from work by Galatius and Randal-Williams [GRW17] that

$$\operatorname*{colim}_{N\to\infty} H_k(B\,\mathrm{Diff}_\partial(X \#^N S^2 \times S^2)) \tag{12}$$

is finitely generated for all k and many X, such as for simply-connected X. In particular, any subgroup of (12) such as the image of the stabilization map is also finitely generated. However, this is not necessarily strong evidence to hope for a positive solution to the problem for homotopy groups, since there are many spaces with finitely generated homology groups but infinitely generated homotopy groups (for example, $H_2(S^1 \vee S^2) = \mathbb{Z}$ but $\pi_2(S^1 \vee S^2) = \mathbb{Z}^\infty$).

(7) In even, higher dimensions, the answer to the analogous problem is positive, and indeed this holds without taking the image in a colimit. For a compact smooth manifold M of dimension $2n \geqslant 6$ with finite fundamental group, $\pi_k(\mathrm{Diff}_\partial(M))$ is finitely generated, due to Bustamante–Krannich–Kupers [BKK24, Theorem 6.1].

Proposed for K3 and scribed by: H. Konno, M. Powell

Problem 4.76. *Let X be a closed, oriented, simply connected, smooth 4-manifold and fix $k > 0$. Is there $N \geqslant 0$ such that, for every $n \geqslant N$, the natural map*

$$\pi_k(\mathrm{Diff}^+(X \#^n S^2 \times S^2)) \to \pi_k(\mathrm{Homeo}^+(X \#^n S^2 \times S^2))$$

is surjective?

Remarks.

(1) If $k = 0$, the answer to the analogous question is affirmative. First, for $N = 2$, it follows from Wall's theorem [Wal64a] that the natural map

$$\pi_0(\mathrm{Diff}^+(X\#^n S^2 \times S^2)) \to \mathrm{Aut}(H_2(X\#^n S^2 \times S^2;\mathbb{Z}), \lambda_{X\#^n S^2\times S^2})$$

is surjective for every $n \geqslant N$. Here $\mathrm{Aut}(H_2(X;\mathbb{Z}), \lambda_X)$ denotes the automorphism group of the intersection form. On the other hand, work of Freedman [Fre82], Kreck, [Kre79], Perron [Per86], and Quinn [Qui86] (plus [GGH+23]), implies that the natural map

$$\pi_0(\mathrm{Homeo}^+(X\#^n S^2 \times S^2)) \to \mathrm{Aut}(H_2(X\#^n S^2 \times S^2;\mathbb{Z}), \lambda_{X\#^n S^2\times S^2})$$

is an isomorphism. Thus, considering the obvious commuting triangle, we have

$$\pi_0(\mathrm{Diff}^+(X\#^n S^2 \times S^2)) \to \pi_0(\mathrm{Homeo}^+(X\#^n S^2 \times S^2))$$

is surjective.

(2) For the analogous problem obtained by replacing "surjective" with "injective", the answer is negative for $k = 0$. Indeed, for any simply-connected closed smooth 4-manifold X, one can find a strictly increasing divergent sequence $0 < N_1 < N_2 < \cdots \to \infty$ such that

$$\pi_0(\mathrm{Diff}(X\#^{N_i} S^2 \times S^2)) \to \pi_0(\mathrm{Homeo}(X\#^{N_i} S^2 \times S^2))$$

is not injective for every $i \geqslant 0$ (*cf.* [KL23c, Theorem 1.5]).

Proposed for K3 by: H. Konno, M. Powell
Scribed by: H. Konno

Problem 4.77. *For which $k \geqslant 0$ and closed smooth 4-manifold X does the equality*

$$\begin{aligned} &\ker(i_*\colon \pi_k(\mathrm{Diff}_\partial(\mathring{X})) \to \pi_k(\mathrm{Homeo}_\partial(\mathring{X}))) \\ &= \ker(s_*\colon \pi_k(\mathrm{Diff}_\partial(\mathring{X})) \to \operatorname*{colim}_{n\to\infty} \pi_k(\mathrm{Diff}_\partial(\mathring{X}\#^n S^2 \times S^2))) \end{aligned} \tag{13}$$

hold?

Remarks.

(1) Here $i\colon \mathrm{Diff}_\partial(\mathring{X}) \to \mathrm{Homeo}_\partial(\mathring{X})$ is the inclusion, and $s\colon \mathrm{Diff}_\partial(\mathring{X}) \to \mathrm{Diff}_\partial(\mathring{X}\#^n S^2 \times S^2)$ is the stabilization map from Problem 4.75.

(2) For $k = 0$ and $\pi_1(X) = 1$, the equality (13) is known to hold: by a result by Quinn [Qui86] and Perron [Per86], the kernels in (13) coincide with the group of mapping classes of diffeomorphisms that act trivially on homology.

(3) See Problem 4.45 for the analogous question on smooth and topological embedding spaces of surfaces in 4-manifolds.

(4) Gabai [Gab22] proved that diffeomorphisms that are pseudo-isotopic, via a pseudo-isotopy with trivial Wh_2 obstruction, are smoothly stably isotopic. If we knew a topological analogue of this result, that could help to find counterexamples for $k = 0$, and be an interesting development in its own right.

QUESTION. *Suppose that a self-homeomorphism $f\colon X \to X$ of a compact 4-manifold X is topologically pseudo-isotopic to the identity via a pseudo-isotopy F with vanishing primary topological Hatcher–Wagoner invariant, $\Sigma(F) = 0 \in \mathrm{Wh}_2(\pi_1(X))$. Is f topologically stably isotopic to* Id_X*?*

See [GN25] for a definition of the primary topological Hatcher–Wagoner invariant.

Proposed for K3 and scribed by: H. Konno

PROBLEM 4.78. *Let X be a simply connected closed smooth 4-manifold. Does $B\,\mathrm{Diff}(X)$ satisfy homological stability over $\mathbb{Q}$?*

REMARKS.

(1) As before, let
$$s\colon \mathrm{Diff}_\partial(\mathring{X}) \to \mathrm{Diff}_\partial(\mathring{X}\# S^2 \times S^2)$$
be the stabilization map from Problem 4.75. The question asks whether, for each $k \geqslant 0$, the induced maps
$$s_*\colon H_k(B\,\mathrm{Diff}_\partial(\mathring{X}\#^N S^2 \times S^2);\mathbb{Q}) \to H_k(B\,\mathrm{Diff}_\partial(\mathring{X}\#^{N+1} S^2 \times S^2);\mathbb{Q})$$
are isomorphic for all $n \gg 0$ large enough relative to k.

(2) For a simply connected compact manifold of dim $= 2n \neq 4$, an analogous stability holds over any (untwisted) coefficients due to work by Harer [Har85] for dim $= 2$ Galatius and Randal-Williams [GRW18] for dim $\geqslant 6$. The colimit
$$\operatorname*{colim}_{n\to\infty} H_k(B\,\mathrm{Diff}_\partial(\mathring{X}\#^N S^n \times S^n))$$
is called the stable homology, which has been extensively studied, especially with $\mathbb{Q}$ coefficients: indeed, the stable homology over $\mathbb{Q}$ coefficient has been determined [MW07, GRW18].

(3) With $\mathbb{Z}$ coefficients, an analogous stability in dimension 4 was shown to fail by Konno and Lin [KL23c]. However, the unstable homology classes detected in [KL23c] are 2-torsion, so the result in [KL23c] does not imply rational instability.

Proposed for K3 and scribed by: H. Konno

PROBLEM 4.79. *Given $k > 0$, is there a closed, simply connected, smooth 4-manifold X and a nonzero homotopy class $\alpha \in \pi_k(\mathrm{Diff}_\partial(\mathring{X}))$ such that*
$$\alpha \in \ker(i_*\colon \pi_k(\mathrm{Diff}_\partial(\mathring{X})) \to \pi_k(\mathrm{Homeo}_\partial(\mathring{X}))) \tag{14}$$
and
$$\alpha \notin \ker(s_*\colon \pi_k(\mathrm{Diff}_\partial(\mathring{X})) \to \pi_k(\mathrm{Diff}_\partial(\mathring{X}\# S^2 \times S^2)))? \tag{15}$$

Remarks.

(1) Here we use the notation i, and s of Problem 4.75 and the previous two problems.
(2) The answer to an analogous statement for $k = 0$ is known to be affirmative: Kronheimer–Mrowka [KM20] proved that the Dehn twist on $X = K3\#K3$ along S^3 gives non-zero class α that lies in (14), and Lin [Lin23] proved that this α satisfies (15).

Proposed for K3 and scribed by: H. Konno

Problem 4.80. *Is there $n > 2$ and a smooth closed simply connected 4-manifold X for which there is an element of order n in the subgroup $\ker(\pi_0 \operatorname{Diff}(X) \to \pi_0 \operatorname{Homeo}(X))$? More generally, is there a subgroup of order n in $\ker(\pi_0 \operatorname{Diff}(X) \to \pi_0 \operatorname{Homeo}(X))$?*

Remarks.

(1) An element of $\ker(\pi_0 \operatorname{Diff}(X) \to \pi_0 \operatorname{Homeo}(X))$ is called an *exotic diffeomorphism*. Examples of exotic diffeomorphisms by Ruberman [Rub98] and Baraglia–Konno [BK20] are of infinite order in $\ker(\pi_0 \operatorname{Diff}(X) \to \pi_0 \operatorname{Homeo}(X))$ A Dehn twist on $K3\#K3$ along the connected sum S^3, detected by Kronheimer–Mrowka [KM20], is of order 2. These exotic diffeomorphisms are detected by $\mathbb{Z}$ or $\mathbb{Z}/2$-valued invariants defined using parameterized ASD Yang–Mills or Seiberg-Witten theory. It would seem that to detect an exotic diffeomorphism of order n one would need $\mathbb{Z}/n$-valued invariants of this type.

    Note that, for a simply connected, closed, oriented 4-manifold, the orientation-preserving topological mapping class group $\pi_0(\operatorname{Homeo}^+(X)))$ is isomorphic to $\operatorname{Aut}(H_2(X;\mathbb{Z}), \lambda_X)$ by work of Freedman [Fre82], Kreck, [Kre79], Perron [Per86], and Quinn [Qui86].
(2) One could consider the following special cases.

    Question.
    (i) Is there a smooth, closed, simply connected 4-manifold X with a subgroup isomorphic to $\mathbb{Z}/2 \times \mathbb{Z}/2$ in $\ker(\pi_0 \operatorname{Diff}(X) \to \pi_0 \operatorname{Homeo}(X))$?
    (ii) In particular, for $X = K3\#K3\#K3$, do the Dehn twists on the two connected sum copies of S^3 in X generate such a subgroup?
    (iii) More generally, is there a copy of $(\mathbb{Z}/2)^{n-1}$ in $\ker(\pi_0 \operatorname{Diff}(X) \to \pi_0 \operatorname{Homeo}(X))$ when X is a connected sum of n copies of the $K3$ surface?

Proposed for K3 and scribed by: H. Konno, D. Ruberman

Problem 4.81. *Is there a smooth, closed, simply connected 4-manifold X for which the group $\ker(\pi_0(\operatorname{Diff}(X)) \to \pi_0(\operatorname{Homeo}(X)))$ is finitely generated?*

Remarks.

(1) Let TDiff(X) be the group of diffeomorphisms that act trivially on $H_*(X;\mathbb{Z})$. Quinn [Qui86] and Perron [Per86] proved that, for $\pi_1(X) = \{1\}$, the group $\ker(\pi_0(\mathrm{Diff}(X)) \to \pi_0(\mathrm{Homeo}(X)))$ is isomorphic to $\pi_0(\mathrm{TDiff}(X))$, called the Torelli group.
(2) Ruberman [Rub99b] proved that there exist simply connected closed 4-manifolds X for which $\pi_0(\mathrm{TDiff}(X))$ is not finitely generated.
(3) In dimension 2, for the closed oriented surface Σ_g of genus $g \geqslant 2$, Johnson [Joh83] proved that the Torelli group is finitely generated for $g > 2$, and McCullough–Miller [MM86] proved that the Torelli group is not finitely generated for $g = 2$.

Scribed by: H. Konno

Problem 4.82. *Let φ be a self-diffeomorphism of a closed, simply-connected, smooth 4-manifold X. Suppose that for every smooth surface Σ in X, the surfaces Σ and $\varphi(\Sigma)$ are smoothly isotopic.*

(a) Is φ necessarily smoothly isotopic to the identity map?
(b) Is φ smoothly isotopic to a diffeomorphism that is supported on a $B^4 \subseteq X$?

Remarks.

(1) In dimension three, a self-homeomorphism f of an irreducible 3-manifold M^3 that preserves free homotopy classes of loops is isotopic to the identity map [ABD+20]. The analogous result holds for simply connected 4-manifolds in the topological category by Perron [Per86] and independently by work of Quinn [Qui86] combined with that of Gabai–Gay–Hartman–Krushkal–Powell [GGH+23]. (These authors show that in the topological category, the weaker hypothesis that φ induces the identity map on $H_2(X;\mathbb{Z})$ implies that φ is topologically isotopic to the identity map.)

This question essentially asks whether these theorems have an analogue in dimension four in the smooth category. The stronger hypothesis is necessary, as there are many examples of exotic self-diffeomorphisms of simply connected 4-manifolds, e.g. the Dehn twist $\phi\colon K3\#K3 \to K3\#K3$ is topologically but not smoothly isotopic to the identity map [KM20]. In this case, the induced map from ϕ clearly preserves $H_2(K3\#K3;\mathbb{Z})$, but it is not clear whether $\phi(\Sigma)$ is smoothly isotopic to Σ for every surface Σ inside $K3\#K3$.
(2) The first question, in the special case $X = S^4$, would imply that every orientation-preserving diffeomorphism of S^4 is isotopic to the identity, answering Problem 4.67. To see that every diffeomorphism $\varphi\colon S^4 \to S^4$ satisfies the hypothesis of the problem, isotope φ to fix some B^4, and then isotope Σ into that B^4. The second question decouples this problem from Problem 4.67.
(3) Another variation on the problem allows the stronger hypothesis that for every $g \geqslant 0$, every smooth embedding $h\colon \Sigma_g \hookrightarrow X$ is smoothly isotopic to the embedding $\varphi \circ h$.

(4) Assuming instead that φ becomes isotopic to the identity after connected sum with $S^2 \times S^2$ (which in particular implies that φ induces the identity on $H_2(X;\mathbb{Z})$), then Krushkal-Mukherjee-Powell-Warren [KMPW24] showed that φ can be isotoped so as to be supported on a contractible submanifold. Another point of view on the question asks whether the assumption that Σ and $\varphi(\Sigma)$ are smoothly isotopic for every Σ, enables us to show that the contractible supporting manifold can be assumed to be a 4-ball.

Proposed for K3 and scribed by: J. Chaidez, M. Miller

PROBLEM 4.83. *For which 4-manifolds does there exist a smooth structure such that there exists a non-smoothable homeomorphism with respect to that smooth structure?*

REMARKS.

(1) We say that a homeomorphism is *smoothable* if it is rel. boundary isotopic to a diffeomorphism. If this does not hold, we say that the homeomorphism is *non-smoothable*.

(2) This question is interesting for closed 4-manifolds, compact 4-manifolds with boundary, and for open 4-manifolds.

(3) Recent progress for compact simply connected 4-manifolds with boundary was made by Galvin–Ladu [GL25] and Konno–Taniguchi [KT22b]. Here is an open question for such 4-manifolds.

QUESTION (i). *Let X be a simply connected, smooth, spin 4-manifold with $\partial X = Y_1 \sqcup Y_2$ having two connected components that do not admit generalized Dehn twists. Consider a boundary-fixing homeomorphism that has trivial Poincaré variation but that acts nontrivially on the relative spin structures of X (there are two such relative spin structures). Is it isotopic to a diffeomorphism?*

See [Sae06, OP25] for the definition of a Poincaré variation. If the Y_i are hyperbolic, then they do not admit generalized Dehn twists. The answer is also yes in the case that there is a separating embedding of a 3-manifold Z, with Y_1 and Y_2 in different connected components of $X \backslash Z$, where Z admits a generalized Dehn twist.

(4) One can wonder whether there is a relationship with the existence of exotic smooth structures.

QUESTION. *Does there exist a 4-manifold that admits a non-smoothable homeomorphism, but no exotic smooth structure? Or vice versa, a 4-manifold admitting exotic smooth structures for which every self-homeomorphism is smoothable?*

See e.g. the work of Donaldson [Don90], Friedman–Morgan [FM88], and Baraglia [Bar21].

Proposed for K3 and scribed by: M. Powell

PROBLEM 4.84. *Is there a closed oriented smooth 4-manifold X for which every finite subgroup G of the mapping class group $\pi_0(\mathrm{Diff}^+(X))$ can be realized by a finite group action on X?*

REMARKS.

(1) This question relates to the study of the algebraic properties of diffeomorphism groups as opposed to their homotopy type.

If there is a group-theoretic section $G \to \mathrm{Diff}^+(X)$ of the quotient map $\mathrm{Diff}^+(X) \to \pi_0(\mathrm{Diff}^+(X))$ over G, we say that G is *realized* by a finite group action. The question whether a given G is realized is known as the Nielsen realization problem.

(2) For orientable surfaces X, Kerckhoff [**Ker83**] proved that every finite subgroup of $\pi_0(\mathrm{Diff}^+(X))$ is realized. On the other hand, in dimension 4, there are several known examples of 4-manifolds X for which there are non-realizable finite subgroups of $\pi_0(\mathrm{Diff}^+(X))$ [**RS77, BK23, FL24a, Kon24a, KMT23b, AB25, Bar23c**].

(3) One may also ask the analogous questions for the extended mapping classes group of X, which also contains the orientation-reversing diffeomorphisms, and for nonorientable X.

(4) The existence of *asymmetric manifolds*, i.e. manifolds that do not admit any effective action of a finite group, has been studied in higher dimensions. See e.g. [**Pup07, CR72**]. Related to the Nielsen realization problem, one may ask whether there is a smooth asymmetric 4-manifold where the mapping class group has a nontrivial finite subgroup.

(5) Here is a variant of the Nielsen realization problem, and for some specific X, several non-realizability results are known. Let $\mathrm{Aut}(H_2(X;\mathbb{Z}))$ denote the automorphism group of the intersection form and set

$$I(X) := \mathrm{Im}(\mathrm{Diff}^+(X) \to \mathrm{Aut}(H_2(X;\mathbb{Z}))).$$

QUESTION. *Which X admits a (finite) group G and a homomorphism $\varphi\colon G \to I(X)$ that cannot be realized by a smooth action on X?*

Here we say that φ is *realized* by a smooth action if there a homomorphism $\tilde{\varphi}\colon G \to \mathrm{Diff}^+(X)$ that descends to φ.

Scribed by: H. Konno

PROBLEM 4.85. *Is there a closed orientable smooth 4-manifold X for which the identity component $\mathrm{Diff}_0(X)$ of the diffeomorphism group is not uniformly perfect?*

REMARKS.

(1) By Mather and Thurston [**Mat71, Mat74, Thu74a**], the group $\mathrm{Diff}_0(X)$ is perfect for every closed orientable manifold X.

(2) For a given group G, the *commutator length* of an element $g \in [G, G]$ is defined to be the minimal number of factors in expressions of g as products of commutators. The commutator length has been studied for Lie groups, groups of automorphisms of (topological, smooth, or symplectic) manifolds, and mapping class groups in contexts of dynamics and bounded cohomology. It is known to be difficult to compute the commutator length of a given element in general, which leads us to consider the following qualitative property for a perfect group (a group that is equal to its commutator subgroup). Given a perfect group G, we say that G is *uniformly perfect* if the commutator lengths of elements of G are uniformly bounded.

(3) For $\dim X \neq 2, 4$, $\mathrm{Diff}_0(X)$ is known to be uniformly perfect due to work by Burago–Ivanov–Polterovich [**BIP08**] and Tsuboi [**Tsu08, Tsu12**]. Their works also show that $\mathrm{Diff}_0(X)$ is uniformly perfect for $X = S^2$ and S^4. On the other hand, Bowden–Hensel–Webb [**BHW22**] proved that, for $\dim X = 2$, $\mathrm{Diff}_0(X)$ is not uniformly perfect if the genus of X is positive. Nothing is known in dimension 4 except for $X = S^4$.

Proposed for K3 and scribed by: H. Konno

Problem 4.86. *Is it the case that for every closed, smoothable topological* 4*-manifold X, there exists a locally linear finite group action on X, such that for every smooth structure on X, the action is non-smoothable?*

Remarks.

(1) Given a topological manifold X and a smooth structure $\mathcal{O}$ on X, we say that a locally linear topological action φ of a group on X is *non-smoothable* with respect to $\mathcal{O}$ if φ is not conjugate to a smooth action on X.

(2) A variant of the problem is the following.

Question. *If we fix a smooth structure $\mathcal{O}$ on X, is there a locally linear finite group action on X that is non-smoothable with respect to $\mathcal{O}$?*

(3) There are many examples of locally linear finite group actions on closed 4-manifolds that are non-smoothable with respect to any smooth structure, such as [**KL88**, **KL93**, **HL95**, **Bry98**, **HT04**, **Nak09**, **Bar19**, **Bar21**, **Kat22**].

Scribed by: H. Konno

Problem 4.87. *Is there an exotic action of $\mathbb{Z}/n$ on S^4 with* 0*-dimensional fixed point set?* 1*-dimensional?* 2*-dimensional?*

Remarks.

(1) In the case that the fixed set is empty, any nontrivial symmetry is orientation-reversing, so $n = 2$. In this case Cappell and Shaneson [CS76] constructed an exotic smooth $\mathbb{R}P^4$, whose double cover is S^4 [AK79a, Gom91b]. Another construction was given by Fintushel and Stern [FS81]. It is not known whether these examples are diffeomorphic, or more generally whether there is more than one exotic $\mathbb{R}P^4$. (In the topological setting there are exactly two homeomorphism classes of 4-manifolds that are homotopy equivalent to $\mathbb{R}P^4$ according to a calculation from the surgery exact sequence [Wal99, Chapter 14]). These two manifolds are distinguished by their Kirby-Siebenmann invariant. As a point of interest, we refer the reader to Ruberman's explicit construction of the non-smoothable homotopy $\mathbb{R}P^4$ [Rub84, Section 2].)

(2) In the case that the fixed set is 0-dimensional or 1-dimensional, very little is known in the smooth setting. In the case that the fixed set is 2-dimensional, there exist actions with knotted fixed point set [Gif66, Gor74, Sum75], but the tools for establishing this are topological. The case that the fixed point set has dimension 3 has been extensively studied (see e.g. [Maz61]); examples arise from gluing two copies of a cork along the boundary.

Scribed by: K. Hendricks

PROBLEM 4.88. *Let $\tau\colon S^4 \to S^4$ be a free (hence orientation-reversing) involution. Is there an embedded $S^2 \subseteq S^4$ that is invariant under τ? This is equivalent to the existence of an embedded $\mathbb{R}P^2$ in $S^4/\tau \simeq \mathbb{R}P^4$, carrying the nontrivial class in $H_2(\mathbb{R}P^4; \mathbb{Z}/2)$. This condition on the homology class will be assumed for the rest of the problem.*

REMARKS.

(1) This is an instance of the codimension two splitting problem [CS74] and may be asked in the smooth or topological category (where one would be looking for an $\mathbb{R}P^2$ with a normal bundle). Another classic phrasing is that the problem is asking if the involution desuspends twice.

(2) In the topological category, there is only one 4-manifold homotopy equivalent but not homeomorphic to $\mathbb{R}P^4$, which is sometimes denoted $*\mathbb{R}P^4$.

QUESTION (i). *Does $*\mathbb{R}P^4$ have an embedded $\mathbb{R}P^2$ with a normal bundle?*

(3) There are two known constructions of exotic $\mathbb{R}P^4$s in the smooth category, due to Cappell–Shaneson [CS76] and Fintushel–Stern [FS81]. By construction, the Cappell–Shaneson $\mathbb{R}P^4$s all contain an embedded $\mathbb{R}P^2$.

QUESTION (ii). *Do the Fintushel–Stern $\mathbb{R}P^4$s contain smoothly embedded $\mathbb{R}P^2$s?*

A negative answer would show that the Fintushel–Stern $\mathbb{R}P^4$s are not diffeomorphic to those obtained by the Cappell–Shaneson construction; see also Problem 4.87.

Proposed for K3 and scribed by: D. Ruberman

Problem 4.89. *Classify smooth, effective circle actions on simply connected 4-manifolds with boundary.*

(a) *Classify simply connected 4-manifolds with boundary that admit circle actions.*

(b) *For each such 4-manifold, classify the circle actions up to conjugation by diffeomorphisms.*

(c) *Given a 4-manifold X with a circle action on its boundary, when does the action extend over X? When does it extend uniquely?*

Remarks.

(1) Information about the smooth classification of the closed 4-manifolds admitting a circle action was obtained by Fintushel and Pao [**FP77**] via Pao's replacement trick [**Pao77**]. (These papers assumed the truth of the 3-dimensional Poincaré conjecture, an assumption that is now known to be satisfied.) Part (a) asks for similar results in the bounded case.

(2) The classification of circle actions on *closed* simply connected 4-manifolds was carried out in the 1970s by Fintushel [**Fin77, Fin78**] in terms of orbit data. Part (b) is asking for an analogous result in the bounded case. Note that in the bounded case, one necessarily has a Seifert-fibered space on the boundary, and so the orbit data is somewhat more complicated than in the closed case.

(3) There are obstructions to the extension problem in part (c) coming from gauge theory. Konno–Mallick–Taniguchi [**KMT23a**] studied, for M equal to one of the Milnor fibers $M(2,3,7)$ or $M(2,3,11)$, the loop in $\pi_1(\mathrm{Diff}(\partial M))$ corresponding to the circle action coming from the Seifert-fibred structure of the boundary. This gives rise to a *boundary generalized Dehn twist*, a diffeomorphism of M supported in a collar neighborhood of ∂M. This diffeomorphism is isotopic to the identity rel. boundary if and only if the element of $\pi_1(\mathrm{Diff}(\partial M))$ extends to a loop in $\pi_1(\mathrm{Diff}(M))$. Konno–Mallick–Taniguchi showed that there is no such extension, and hence there is no corresponding circle action. Montague [**Mon23**] drew the same conclusion for the Gompf nuclei $N(2n)$ and simple plumbing $P(2n)$ with boundary $-\Sigma(2,3,12n-5)$, by proving non-extension results for $\mathbb{Z}/p$ actions embedded in the circle action on the boundary. All of these results continue to hold when M is replaced by $M\#S^2\times S^2$. Further results along these lines have been announced in [**KLMME24, KPT26**].

Question. *Does there exist a compact 4-manifold X, and a circle action on ∂X, such that the latter extends to a loop of diffeomorphisms in $\pi_1(\mathrm{Diff}(X))$ but not to a circle action on X?*

(4) If ∂X admits a unique circle action, then an answer to (c) would follow from an answer to (a). But there could exist distinct circle actions on ∂X such that one extends over X and one does not.

(5) A complementary obstruction to extending would be via a classical theorem of Atiyah–Hirzebruch [**AH70**] that restricts the characteristic classes

of closed *spin* manifolds of dimension $4k$ admitting a circle action; they show in this case that the $\widehat{A}$ genus of the manifold must vanish. In dimension 4, this means that the signature vanishes.

QUESTION. *Find an analogue of the Atiyah–Hirzebruch result for* 4*-manifolds* (*or more generally* $4k$*-manifolds*) *with nonempty boundary.*

It is easy to see that the signature does not necessarily vanish for spin manifolds with circle actions if the boundary is nonempty. For instance, one could take the disk bundle over S^2 with even, nonzero, Euler class. It has signature ± 1 but supports an S^1 action. From such examples, it seems plausible that there could be some sort of boundary correction to the Atiyah–Hirzebruch argument, presumably involving an S^1-equivariant η-invariant [**Don78**].

Proposed for K3 and scribed by: D. Ruberman

4.9. Symplectic and complex structures

This section focuses on problems concerning symplectic and complex structures on 4–manifolds, including geography and botany questions, the realization and diversity of symplectic representatives in fixed homology classes, and the existence of distinct Stein fillings and Weinstein structures. A *symplectic structure* on a smooth, oriented 4–manifold X is a closed, nondegenerate 2-form $\omega \in \Omega^2(X)$; that is, $d\omega = 0$ and $\omega^2 > 0$ everywhere, compatible with the orientation on X. An important homotopy invariant of a symplectic 4–manifold (X, ω) is its *canonical class* $K = -c_1(X, \omega)$, the first Chern class of any almost complex structure J compatible with the symplectic form ω, which is unique up to homotopy. When equipped with the associated Kähler form, a *compact complex algebraic surface* — that is, a complex smooth projective surface—becomes a symplectic 4-manifold where ω is compatible with an integrable complex structure J.

A *Weinstein domain* is a quadruple (W, ω, ϕ, J) where:

(i) W is a compact, smooth 4-manifold with boundary;
(ii) λ is a 1-form on W such that the symplectic form ω equals $d\lambda$;
(iii) the associated Liouville vector field V defined by $\iota_V d\lambda = \lambda$ is gradient-like for a Morse function $\phi : W \to \mathbb{R}$ with ∂W as a regular level set;
(iv) J is an almost complex structure compatible with $d\lambda$.

A *(compact) Stein surface* is a complex 2-dimensional manifold S with boundary, admitting a proper, strictly plurisubharmonic function $\phi\colon W \to \mathbb{R}$ such that S is exhausted by compact sublevel sets. (We often drop the adjective 'compact' not to avoid irritating algebraic geometers — for whom Stein manifolds are by definition non-compact — but because it will be clear from the context that we are restricting attention to Stein subdomains with contact-type boundary.) A Stein surface S admits the structure of a Weinstein domain, with $\lambda = -d^c\phi$ and symplectic form $d\lambda = -dd^c\phi$, and the complex structure on it is Kähler.

Problem 4.90. *Do the Chern numbers c_1^2 and c_2 of every closed, symplectic 4–manifold X that is not a ruled surface satisfy the following?*

(a) $c_1^2 \leqslant 3\,c_2$.
(b) $c_1^2 = 3\,c_2 > 0$ *if and only if* $X = \mathbb{C}\mathrm{P}^2$ *or* X *is a complex ball quotient.*
(c) $c_2 \geqslant 0$.

Remarks.

(1) These invariants depend only on the underlying homotopy type of the 4–manifold X, satisfying the identities $c_1^2 = 2\chi + 3\sigma$ and $c_2 = \chi$ where χ and σ are the Euler characteristic and the signature of X.
(2) All hold true for Kähler surfaces [**BPVdV84**]; part (a) is the well-known *Bogomolov–Miyaoka–Yau inequality* (BMY) and part (b) follows from Yau's celebrated solution of the Calabi conjecture [**Yau78**].
(3) The inequality (c) was conjectured to hold by Gompf. If the non-ruled symplectic 4–manifold X is *minimal*, then $c_1^2 \geqslant 0$ by [**Tau94, Tau95, Tau96, LL95**]; so part (c) is implied by part (a) in this case.
(4) These geographic constraints would have strong consequences. For instance, if true, part (b) would imply that there is no symplectic 4–manifold X homeomorphic but not diffeomorphic to $\mathbb{C}\mathrm{P}^2$; part (a) or (c) would imply the same for any ruled surface over Σ_h, with $h \geqslant 2$.
(5) If there exist symplectic 4–manifolds with $c_1^2 = 3c_2$ that are not diffeomorphic to any Kähler surface, a further line of inquiry for part (b) would be whether there is a symplectic analog of Yau's theorem for them; e.g., are they always $K(G,1)$s? See also Problem 4.91.
(6) One approach to an affirmative solution for part (a) is via branched coverings. Auroux proved that every symplectic 4-manifold admits a simple branched covering to $\mathbb{C}\mathrm{P}^2$, where the branch locus in $\mathbb{C}\mathrm{P}^2$ is an immersed symplectic surface with nodes and simple cusps [**Aur00**]. One can compute the Chern numbers in terms of the degree of the branch locus, the number of cusps, the genus of the branch curve, and the number of sheets of the cover (see [**Aur06b**, p.266]). Thus, one can translate the existence of an example on or above the BMY line into the existence of a braided singular surface in $\mathbb{C}\mathrm{P}^2$ with a coloring with certain constraints.
(7) One can ask part (a) more generally as follows (see also Problem 4.16):

Question. *Does any closed 4–manifold X with $b_2^+ > 1$ and with nontrivial Seiberg–Witten invariants satisfy the BMY inequality?*

Recall that by the work of Taubes [**Tau94, Tau95**], if such a 4–manifold X admits a symplectic structure, then it has a Seiberg-Witten basic class.

Feehan and Leness have announced a program to answer this broader question affirmatively [**Fee22, FL23, FL24b**]. Their approach is to adapt Hitchin's Morse theory analysis of the moduli space of Higgs monopoles on a rank-2 Hermitian vector bundle over a Riemann surface [**Hit87**] to the moduli space of non-Abelian monopoles (A, Φ) on a rank-2 Hermitian vector bundle E.

Scribed by: I. Baykur, L. Starkston

PROBLEM 4.91. *Present a topological construction of symplectic fake projective planes. Does there exist a symplectic fake projective plane that is not a complex ball quotient?*

REMARKS.

(1) A *fake projective plane* (FPP) is a closed 4–manifold X with the same rational cohomology ring as the complex projective plane $\mathbb{C}\mathrm{P}^2$ but not diffeomorphic to it.

By Yau's solution of the Calabi Conjecture [**Yau78**], every *complex* FPP is a torsion-free quotient of the complex unit ball by a discrete co-compact subgroup of $PU(1,2)$. The first example of a complex FPP was given by Mumford [**Mum79**] (hence the alternate name *Mumford surface* for a complex FPP) using p-adic uniformization, with further examples later given by Ishida and Kato [**IK98**] and Keum [**Keu06**]. Notably, Prasad and Yeung [**PY07**], with the help of computer-assisted calculations by Cartwright and Steeger, established that there are exactly 50 diffeomorphism types for complex FPPs [**PY07, PY10, CS10**].

In fact, these 50 Kähler surfaces are the only known examples of symplectic FPPs to date and their constructions involve arithmetic geometry. (Keum's work [**Keu06, Keu11**] may be largely reinterpreted using topological arguments but also relies on arithmetic geometric results [**Ish88**].) The problem asks if there are "softer" constructions via symplectic topology.

(2) A reconstruction of even the known complex FPPs using topological methods may lead to the further discovery of non-Kähler, symplectic examples on the Bogomolov–Miyaoka–Yau line. See Problem 4.90.

(3) Every complex FPP is smoothly irreducible [**BSS24**, Proposition 6.1]; this fact extends to any other potential symplectic FPP under mild assumptions on its fundamental group.

Indeed, Fintushel and Stern raised the more general question of whether there exist smoothly irreducible FPPs besides the complex ones and what could be said about their π_1. (For reducible examples, one can simply take a connected sum of any rational homology 4-sphere with $\mathbb{C}\mathrm{P}^2$.) Irreducible FPPs with various fundamental groups were recently constructed in [**BSS24**]; for instance, any finite abelian group with a $\mathbb{Z}_2$ factor is claimed to be realized as the π_1 of an irreducible FPP. None of these FPPs admit symplectic structures, and in fact, it can be shown that the approach in [**BSS24**] falls short of producing *symplectic* examples.

(4) The existence of an (irreducible) FPP with trivial π_1, which amounts to an exotic smooth structure on $\mathbb{C}\mathrm{P}^2$, is a particularly important open question. See Problem 4.2.

Let G be a finitely presented group and Q be an integral intersection form. Baldridge and Kirk conjectured that the diffeomorphism type of a closed symplectic 4–manifold that minimizes the Euler characteristic among all with $\pi_1 = G$ and intersection form Q is unique [**BK07**, Conjecture 23]. A *symplectic* FPP with trivial π_1 would be a counter-example to a special case of this conjecture, known as the *Symplectic Poincaré Conjecture*.

Proposed for K3 by: R. Fintushel, R. Stern
Scribed by: I. Baykur

PROBLEM 4.92. *Is every symplectic Calabi-Yau surface diffeomorphic to either the K3 surface, the Enriques surface or a T^2-bundle over T^2?*

REMARKS.

(1) Here a *symplectic Calabi-Yau surface* (SCY) is defined as a closed symplectic 4-manifold with torsion first Chern class.
(2) To date, the only smooth 4–manifolds known to support a symplectic structure with torsion c_1 are the ones listed in the problem. The problem asks whether this list indeed provides a complete classification of the diffeomorphism types of SCYs.
(3) T-J Li [**Li06a**, **Li06b**] and Bauer [**Bau08**] established that any SCY has the rational homology type of one of these standard 4-manifolds. (This extends the earlier results of Morgan and Szabó [**MS97**] when $b_1 = 0$ and Ruberman and Strle [**RS00**] when $b_1 = 4$.)
    Any SCY with $b_2^+ > 1$ has only one Seiberg-Witten basic class, namely the (trivial) canonical class, and its SW invariant is one. When $b_2^+ = 1$, the SW invariant of an SCY is determined by the wall crossing formula, and only depends on the cohomology ring structure.
(4) The list has been verified to be complete in some specific cases. Any SCY that smoothly fibers over a circle, a surface, or a 3–manifold is standard [**BF15**, **Bay14**, **FV13**, **LN14**, **Ni17b**]. The same holds when the SCY admits certain finite symplectic group actions [**Che20a**, **Che20b**].
    Moreover, many well-known construction techniques—such as Luttinger surgery, generalized fiber sums, knot surgery, and the simplest rational blow-downs—have been shown not to yield new SCYs from standard symplectic 4-manifolds; see [**Li19**] for a detailed survey and references.
(5) One approach towards an affirmative answer is to first detect the existence of a (possibly singular) T^2–fibration. In the case of a homology K3, if further assuming that it has a winding family of symplectic forms (a symplectic generalization of a hyperkähler family), then a parameterized version of Taubes' SW $\Rightarrow$ GW theorem, including a parametrized wall-crossing formula, produces a 2–dimensional family of embedded symplectic tori in the winding family [**Li10**, Section 7.4].
(6) One may probe the existence of new SCYs via symplectic Lefschetz pencils and multisections by analyzing certain positive factorizations in mapping class groups. New constructions of SCYs realizing all possible rational homology types are given in [**BH16b**, **Bay22**, **BHM23**] adapting this approach. Only a handful of these SCYs are confirmed to be standard so far [**HH18a**, **Ful23**].

Due to the shortcomings of gauge-theoretical invariants in distinguishing SCYs (up to diffeomorphism) within the same homotopy type, the possibility of detecting a new SCY hinges on identifying a new SCY group. An intriguing, not-yet-ruled-out possibility is $\pi_1 = \mathbb{Z}^2$ [**FV13**, **Bay22**].

Proposed for K3 and scribed by: T.J. Li, I. Baykur

PROBLEM 4.93. *Is every symplectic form on the standard K3 surface symplectomorphic to a Kähler form?*

REMARKS.

(1) This question has several notable generalizations. First, since every symplectic form on the K3 surface is cohomologous to a Kähler form [**Li08**, 4.10], this is a special case of the following broader question [**Don06**]:

QUESTION (Donaldson). *Let (X, ω_0) be a closed Kähler surface. Is any other symplectic form ω on X with $[\omega] = [\omega_0]$ and $c_1(X, \omega) = c_1(X, \omega_0)$ symplectomorphic to ω_0?*

As every closed hyperkähler surface is diffeomorphic to either the K3 surface or T^4, the problem also constitutes a special case of the following conjecture.

CONJECTURE. *Let X be a closed hyperkähler surface and let $a \in H^2(X; \mathbb{R})$ be such that $a^2 > 0$. Then the space of symplectic forms on X representing the class a is connected.*

Finally, the problem is a significant instance of the more general Problem 4.96.

(2) Donaldson proposed using the almost Kähler Calabi-Yau equation given in [**Don06**] to get a family of cohomologous symplectic forms connecting the given symplectic form ω to the Kähler form ω_0. A partial a priori estimate for the almost Kähler Calabi-Yau equation was obtained in [**TWY08**].

(3) There are also approaches to this problem via Donaldson's geometric flow [**Don00**, **KS19**] and hypersymplectic flow [**FY19**].

Scribed by: T.J. Li

PROBLEM 4.94. *Are homotopy equivalent Horikawa surfaces in different deformation classes diffeomorphic as 4–manifolds? Are they symplectomorphic?*

REMARKS.

(1) A *Horikawa surface* is a minimal (non-singular) complex projective surface of general type whose Chern numbers satisfy $5c_1^2 = c_2 - 36$ (i.e. it lies on the *Noether line*), or equivalently, $c_1^2 = 2p_g - 4$, where $p_g \geqslant 3$ is the geometric genus. They are all simply connected.

These surfaces were studied extensively by Horikawa in [**Hor76b**, **Hor76a**, **Hor78**, **Hor79**], who classified them up to deformation. For each $r \geqslant 2$, there are two deformation classes of surfaces with $c_1^2 = 8r - 8$ and $c_2 = 40r - 4$. The smooth 4–manifolds $H(r)$ and $H'(r)$ in the two respective deformation classes are distinguished by their intersection form when r is even; whereas, when r is odd, they are homotopy equivalent. They have the same Donaldson polynomials and the same Seiberg-Witten invariants. Some 50 years after Horikawa, it remains unknown whether any pair of the latter surfaces are diffeomorphic.

(2) The question on the number of diffeomorphism classes was originally raised by Horikawa in [**Hor76b**] and it is in [**Kir97**, Problem 4.101(A)].

If two compact complex manifolds X and X' are deformation equivalent, then there exists a diffeomorphism $f\colon X \to X'$ such that

$$f^*(c_1(X, \omega)) = c_1(X', \omega').$$

The bold speculation of the time, namely that the converse holds in complex dimension two (the refined *"DEF = DIFF Conjecture"*), was disproved by Manetti in [**Man01**], and there are simply connected counterexamples as well [**CW07**].

A minimal complex surface of general type has a canonical symplectic structure, unique up to symplectomorphism, which is invariant under smooth deformation [**Cat09**]. Catanese showed that Manetti surfaces are indeed symplectomorphic [**Cat09**], so they also constitute counterexamples to the elusive *"DEF = SYMP Conjecture"*.

Nonetheless, it is still open whether DEF equivalence is more strict than DIFF or SYMP equivalence for the Horikawa surfaces.

(3) Let $\mathbb{F}_{2r}$ denote the Hirzebruch surface with fiber f and sections Δ_0 and Δ_∞ with self-intersections $\Delta_0^2 = 2r$ and $\Delta_\infty^2 = -2r$. The Horikawa surface $H(r)$ is the double cover of $\mathbb{F}_0$ branched over a smoothing of $6\Delta_0 + 4rf$ and $H'(r)$ is the double cover of $\mathbb{F}_{2r}$ branched over a disconnected branch locus that is a smoothing of $5\Delta_0 + \Delta_\infty$. Based on this, one can describe $H(r)$ and $H'(r)$ as smooth 4–manifolds using Kirby diagrams for branched covers, and as symplectic 4–manifolds via monodromy factorizations for compatible Lefschetz fibrations/pencils [**Ful98**, **Aur06a**]. In [**Aur06a**], Auroux compared the canonical symplectic Lefschetz pencils of the same genus on $H(3)$ and $H'(3)$ and observed that they are related through fibered Luttinger surgeries. As suggested by the status of the problem, there has been otherwise no success in relating such diagrams via Kirby calculus or the pencils via monodromy manipulations to prove DIFF or SYMP equivalence.

(4) Another possible strategy to obtain a diffeomorphism between $H(r)$ and $H'(r)$ is based on the observation of Manetti, that a common degeneration with Wahl singularities would prove that they are diffeomorphic [**Man01**][Theorem 1.5]. Recently in [**MNU24**], through an extensive study of complex surfaces with Wahl singularities, the authors invalidated this approach. On the other hand, a different common degeneration was recently found in [**RR24**], without any interpretation of moving from one deformation component to the other as diffeomorphism.

Scribed by: I. Baykur, G. Urzúa

PROBLEM 4.95.

(a) Is there a closed hyperbolic oriented 4-manifold that admits a symplectic structure?

(b) Do the Seiberg–Witten invariants vanish on every closed hyperbolic 4-manifold?

REMARKS.

(1) In a preprint [**Sto25**], Stover announces the construction of a hyperbolic orbifold structure on $\mathbb{C}P^2$. This resolves an orbifold version of this problem.

(2) By work of Taubes [**Tau94**], a positive answer to (a) would imply a negative answer to (b); see [**Rei06**, Proposition 4.5]. By Donaldson [**Don99**] and Gompf–Stipsicz [**GS99**, Theorem 10.2.18], (a) is equivalent to asking whether there is a closed hyperbolic oriented 4-manifold that admits a smooth Lefschetz pencil. Compare this to Problem 2.23, which asks whether there exists a surface bundle over a surface whose total space is a hyperbolic 4-manifold.

(3) In [**LeB02**, Conjecture 1.1], LeBrun conjectures that the answer to (b) is, 'yes,' and hence the answer to (a) is 'no.' LeBrun provides evidence towards the moduli spaces being empty (for suitable perturbations) and proposes that one should consider the generalization of Seiberg-Witten invariants involving the evaluation of cohomology classes in $\Lambda^* H^1(X;\mathbb{Z})\otimes\mathbb{Z}[U]$ on higher dimensional moduli spaces. The first examples of manifolds for which all these invariants vanish were provided in [**AL20**], followed by more concrete examples in [**BFS24**]. It is worth noting that there are no known examples of closed hyperbolic 4-manifolds with $b_2^+ \leqslant 1$, cf. Problem 4.19.

Proposed for K3 and scribed by: F. Lin, B. Martelli

PROBLEM 4.96. *Does there exist a pair of symplectic* 4*–manifolds* (X_1, ω_1) *and* (X_2, ω_2)*, where there is a diffeomorphism* $f\colon X_1 \to X_2$ *such that* $f^*(c_1(X_2, \omega_2)) = c_1(X_1, \omega_1)$ *and* $f^*([\omega_2]) = [\omega_1] \in H^2(X_1, \mathbb{R})$*, but* (X_1, ω_1) *is not symplectomorphic to* (X_2, ω_2)*?*

REMARKS.

(1) It may be difficult to detect such a difference with current invariants due to Taubes' result relating Gromov–Witten invariants and Seiberg–Witten invariants [**Tau96**].

(2) There is also a relative version of this problem for fillings.

QUESTION. *Does there exist a pair of Stein manifolds filling the same contact* 3*-manifold that are diffeomorphic where the diffeomorphism takes* *the first Chern class of one to that of the other, but such that the Stein manifolds are not symplectomorphic?*

One could also ask whether there is such a pair of Stein manifolds that are not Weinstein homotopic. See also Problem 5.17.

Scribed by: L. Starkston, J. Van Horn-Morris

PROBLEM 4.97. *Let $\lambda := c_1^2/c_2$ be the Chern slope of a closed, almost complex 4–manifold X. What is the supremum of λ as X ranges over the following families?*

(a) Symplectic Σ_g–bundles over Σ_h, with $g, h \geqslant 2$.
(b) Holomorphic Σ_g–bundles over Σ_h, with $g, h \geqslant 2$.
(c) Symplectic Lefschetz fibrations over S^2.
(d) Holomorphic Lefschetz fibrations over S^2.

(*Here, the Lefschetz fibrations are assumed to have critical points.*)

REMARKS.

(1) Recall that the Chern numbers of X are determined by its homotopy type. Any Σ_g–bundle over Σ_h with $g \geqslant 2$ or a Lefschetz fibration (with nonempty critical set) over S^2 can be made symplectic, but not always holomorphic. See e.g. [**Joh86, Bay12b**]. In fact, the sole obstruction to making a surface bundle over a surface holomorphic is whether the smooth 4–manifold X admits a complex structure [**Hil00, Kot99**].

(2) The first line of research in this direction, pioneered by Atiyah, Kodaira, and Hirzebruch in the late 1960s, and Endo in the late 1990s, is to understand for which pairs of $g \geqslant 3$ and $h \geqslant 2$ one can have a Σ_g–bundle over Σ_h with $\sigma > 0$. (The signature vanishes when $g \leqslant 2$ or $h \leqslant 1$ [**Mey73**].) This geography problem was nearly resolved in the recent work of Baykur and Korkmaz [**BK24b**], who showed that for all but 19 possible pairs (g, h), there are symplectic Σ_g–bundles over Σ_h with $\sigma > 0$. However, the remaining few cases are quite significant for symplectic geography; see below.

(3) The Bogomolov–Miyaoka–Yau inequality for complex surfaces is encoded as $\lambda \leqslant 3$, providing an upper bound on the slopes of X in (b) and (d). Since no complex ball quotient can smoothly fiber over a surface, it is moreover known that any X in (b) satisfies $\lambda < 3$.

Two of the unsettled cases in [**BK24b**] have direct implications on symplectic geography: any example with $(g, h) = (3, 2)$ and $\sigma > 0$ is a symplectic X of general type violating the BMY inequality, whereas one with $(g, h) = (4, 2)$ and the smallest possible positive signature would give a symplectic X on the BMY line and cannot possibly be complex. See [**BK24b**, Question 2] and Problem 4.90. Do such surface bundles over surfaces exist?

An unpublished preprint by Hamenstädt [**Ham20**] announces that $\lambda \leqslant 3$ more generally for any X in (a).

(4) Presumably, these questions will all have different answers. Nonetheless, the largest currently known slope for (a) and (b) coincide: these are the

examples by Catanese and Rollenske with $\lambda = 8/3$ [CR09]. There exist holomorphic Σ_g–bundles over Σ_h with positive signatures, for $h = 2$ (and large g), as shown by Bryan and Donagi in [BD02], and with $g = 3$ (and unspecified, presumably very large h), as shown by Kazuhiro Konno (in works only available in Japanese). The situation for (c) and (d) is more mysterious; for instance, (symplectic) Lefschetz fibrations over S^2 with $\sigma > 0$ were only recently discovered in [BH24c].

There are fewer constraints on the slopes of Lefschetz fibrations over higher genera surfaces; e.g. the Cartwright-Steger surface on the BMY line admits a holomorphic Lefschetz fibration over T^2 [KY21], realizing the maximal possible slope $\lambda = 3$ in this case.

Proposed for K3 and scribed by: I. Baykur

PROBLEM 4.98. *Does every closed symplectic 4-manifold admit inequivalent Lefschetz pencils with the same fiber genus g, for sufficiently large g? How about infinitely many?*

REMARKS.

(1) Here Lefschetz pencils are assumed to have base points, whereas Lefschetz fibrations do not. Equivalence is defined by a diffeomorphism of the symplectic 4–manifold commuting with any pair of pencil/fibration maps.
(2) Any symplectic 4–manifold can be equipped with Lefschetz pencils with arbitrarily high fiber genera by increasing the degree in Donaldson's construction. However, the number of base points also increases, meaning the Lefschetz fibrations derived by blowing up the base points would be on birational yet different 4–manifolds.

    Except for rational and ruled surfaces, the number of base points in a pencil is bounded above by $2g - 2$, where g is the genus of the fiber. Therefore, a positive answer to the second question would imply the existence of infinitely many genus–g Lefschetz pencils with the same number of base points. By blowing up the base points of such examples, one can obtain inequivalent Lefschetz fibrations over S^2 with the same fiber genus, but the converse is not always feasible.
(3) By [Bay16], any symplectic 4–manifold that is not a rational or ruled surface, possibly after blow-ups, admits arbitrarily many non-isomorphic Lefschetz pencils with the same genus, the same number of base points, and matching topological types of singular fibers. A slightly weaker result also holds for rational and ruled surfaces [Bay19]. These constructions require blow ups, addressing the problem only up to birational equivalence.

    Other examples of inequivalent Lefschetz pencils and fibrations on a few specific 4–manifolds are given in [PY09, PY17, BH16b, BHM23, Ham17]. The monodromy invariants and arguments used in these works do not distinguish more than finitely many classes; for a positive resolution of the problem, one needs finer monodromy invariants.

Proposed for K3 by: I. Baykur
Scribed by: I. Baykur, N. Salter, L. Starkston

PROBLEM 4.99. *Let X be a closed symplectic 4-manifold. Let $T \subset X$ be a symplectic submanifold that is diffeomorphic to a 2-dimensional torus such that $[T]^2 = 0$. Let X_K be a manifold obtained by Fintushel–Stern knot surgery on T using a knot $K \subset S^3$. If X_K has a symplectic structure, must K be a fibered knot?*

REMARKS.

(1) Fintushel and Stern [**FS98**] showed that X_K has a symplectic structure if K is a fibered knot, and raised the question in the problem.
(2) One can ask the question for the particular case when X admits a Lefschetz fibration whose regular fibers are tori and T is a regular fiber. This particular case was explicitly stated in [**Ni17a**]. An interesting example is the case when X is the K3 surface, then $SW(X)$ is essentially the Alexander polynomial Δ_K of K [**FS98**], so we know Δ_K should be monic if X_K has a symplectic structure [**Tau94, Tau95**]. No other constraint is known.
(3) The answer to this problem is "Yes" when $X = T^2 \times S^2$ and $T = T^2 \times \{\text{point}\}$ by Friedl–Vidussi [**FV11**], and when X is a torus bundle over a closed surface with homologically essential fibers and T is a fiber by Ni [**Ni17a**].

Scribed by: Y. Ni

PROBLEM 4.100. *Given a closed, connected, symplectic 4-manifold (X, ω) and $c \in H_2(X, \mathbb{Z})$ represented by an embedded, connected, oriented, smooth surface S such that*

(i) $\langle [\omega], c \rangle > 0$, *and*
(ii) $\langle c_1(X, \omega), c \rangle = \chi(S) + S^2$,

is c represented by an embedded, connected, symplectic surface?

REMARKS.

(1) This is a special case of the question of which homology classes in a symplectic manifold are represented by symplectic surfaces. By the adjunction formula, the homology class of every embedded symplectic surface satisfies both (i) and (ii). The *Symplectic Thom Conjecture* [**OS00**] states that symplectic surfaces minimize genus in their homology classes. In their proof of this conjecture, Ozsváth and Szabó showed that every class satisfies the adjunction inequality, $-\chi(S) \geqslant S^2 + |\langle c_1(X, \omega), c \rangle|$, provided $b_2^+(X) > 1$. (The result extends to $b_2^+(X) = 1$ unless X is a rational or ruled surface.) The question is whether this is a sufficient condition to conclude the existence of a symplectic representative. See also [**DLW18**].
(2) By Donaldson [**Don99**], if $[\omega] \in H_2(X, \mathbb{Z})$, then all large enough multiples of $[\omega]$ are represented by symplectic submanifolds. (When $b_2^+(X) = 1$, one can say a bit more; see e.g. [**Li08**, Proposition 3.18].) When $b_2^+(X) > 1$,

Taubes [**Tau96**] showed this is also true for the Poincaré dual of the canonical class $c_1(X, \omega)$; also see [**DS03**].

Scribed by: I. Baykur, T.J. Li, J. Van Horn-Morris

PROBLEM 4.101. *Is every smooth symplectic surface in* $(\mathbb{CP}^2, \omega_{\text{FS}})$ *symplectically isotopic to a complex curve? Equivalently, is there a unique symplectic isotopy class of smoothly embedded symplectic surfaces of degree* d *in* $(\mathbb{CP}^2, \omega_{\text{FS}})$*?*

REMARKS.

(1) Here ω_{FS} denotes the standard Fubini–Study symplectic form on $\mathbb{CP}^2$. A symplectic surface in $\mathbb{CP}^2$ is said to have degree d if it represents the homology class $dh \in H_2(\mathbb{CP}^2; \mathbb{Z})$ where h is the generator of $H_2(\mathbb{CP}^2; \mathbb{Z}) \cong \mathbb{Z}$ represented by a complex projective line. Note that symplectic surfaces always represent a *positive* multiple of h because they have positive symplectic area.

(2) A complex projective algebraic plane curve of degree d is the zero set of a homogeneous polynomial of degree d in $\mathbb{CP}^2$. Such a curve represents $dh \in H_2(\mathbb{CP}^2; \mathbb{Z})$. If the zero set is regularly cut out, the complex algebraic curve is smooth. Any two smooth complex algebraic curves of the same degree are isotopic, i.e. connected through a family of smooth complex algebraic curves. This can be seen by observing that the set of singular curves of degree d (corresponding to homogeneous polynomials that do not regularly cut out 0) is a complex subvariety of positive complex codimension (cut out by the discriminant) in the space of all degree-d curves (a high dimensional complex projective space parametrized by the coefficients of the monomials in a degree-d homogeneous polynomial). Thus the smooth curves form a path connected space, as it is the complement of a subset of real codimension at least 2. Thus, the two questions in the problem statement are equivalent and are known as the "symplectic isotopy problem."

(3) Key progress on the symplectic isotopy problem relies on the fact that every symplectic surface can be realized as a J-holomorphic curve for some almost complex J compatible with the symplectic form. The development of pseudoholomorphic curves originates from Gromov's seminal paper [**Gro85**]. It is proven in Gromov's work that the answer to the symplectic isotopy question is yes in degrees 1 and 2. Extensive further work using Gromov's strategy of pseudoholomorphic curves shows that the answer is yes for degrees less than or equal to 17 [**She00, Sik03, ST05**]. After degree 17, we have been unable to rule out the possibility that a family of J_t-holomorphic curves may degenerate to a singular curve with unreduced components, and it is unknown whether such degenerations can produce an unavoidable codimension-1 "wall" in the moduli space of pseudoholomorphic curves.

(4) Another approach to this problem is via quasipositive factorizations in the braid group. Given a smooth symplectic surface in $\mathbb{CP}^2$ realized as a J-holomorphic curve, one can generate a J-holomorphic linear pencil

on $\mathbb{CP}^2$, $\pi : \mathbb{CP}^2\backslash\{p\} \to \mathbb{CP}^1$. Restricting the pencil to the symplectic surface gives a simple branched covering from the surface to $\mathbb{CP}^1$, assuming the pencil point p is chosen sufficiently generically. The branch points correspond to places where the fibers of the pencil are tangent to the symplectic surface. Looking at the preimages under π of loops in $\mathbb{CP}^1$ gives a braid monodromy presentation that fully encodes the symplectic surface [MT88, MT91]. Through this, the symplectic isotopy problem can be related to the following problem in the braid group.

QUESTION. *Let Δ^2 denote the full twist on d strands in the braid group B_d, and let σ_i denote the standard generators of the braid group (a half twist of two adjacent strands). If $\rho_1 \dots \rho_k = \Delta^2$ and each ρ_j is a conjugate of some σ_i, then is there a sequence of Hurwitz moves and global conjugations one can perform on $\rho_1 \dots \rho_k$ to turn it into $(\sigma_1 \dots \sigma_{d-1})^d$?*

For more background on this perspective and the definition of Hurwitz moves, see this survey by Auroux [Aur06b].

(5) Another strategy to attack the symplectic isotopy problem is proposed in [Sta20], which uses deformations of the smooth symplectic surface to a singular surface (a symplectic line arrangement) to show that the symplectic isotopy problem is equivalent to the existence of certain Lagrangian disks with boundary on the symplectic surface. Finding or obstructing the existence of such Lagrangians could potentially be approached using Floer theoretic/Fukaya categorical techniques.

(6) Note that there are smooth surfaces in $\mathbb{CP}^2$ that are not smoothly isotopic to any complex curve [Fin02, Kim06]. However, one can mix the symplectic and smooth categories for the following weakening of the symplectic isotopy problem, which is currently equally open.

QUESTION. *Are any two symplectic surfaces in $\mathbb{CP}^2$ of the same degree smoothly isotopic?*

A strategy for answering this weaker version of the problem using transverse bridge trisections was proposed by Lambert-Cole [LC23].

(7) There are examples of symplectic surfaces that are homologous but not symplectically isotopic in other closed symplectic 4-manifolds [FS99b]. However, the question appears to also be open more generally for rational and ruled surfaces, and their blow-ups (symplectic manifolds of Kodaira dimension $-\infty$).

In ruled surfaces $S^2 \times S^2$, $\mathbb{CP}^2\#^n\overline{\mathbb{CP}}^2$, or an S^2 bundle over a higher-genus surface, in a fixed homology class, is there a unique symplectic isotopy class of symplectic surfaces?

Scribed by: L. Starkston

PROBLEM 4.102. *Is every symplectic rational cuspidal curve in $(\mathbb{CP}^2, \omega_{\mathrm{FS}})$ equisingularly symplectically isotopic to a complex curve? More generally, which types of singular symplectic surfaces in $(\mathbb{CP}^2, \omega_{\mathrm{FS}})$ (where type is specified by the genus* *of each irreducible component and the topological types of the singularities), admit symplectic representatives that are not equisingularly symplectically isotopic to complex curves?*

REMARKS.

(1) This is a singular version of the symplectic isotopy problem (Problem 4.101). Note that "curve" here refers to a real 2-dimensional surface in analogy with the terminology for complex/pseudoholomorphic curves. A rational cuspidal curve is a singular curve homeomorphic to S^2.

(2) We say that two singular symplectic surfaces are *equisingularly symplectically isotopic* if there is a family of symplectic surfaces connecting them such that the topological type of each singularity remains constant through the isotopy. (The topological type is the isotopy class of the link of the singularity.)

(3) Although one could define symplectic surfaces whose singularities are a cone on any transverse knot, for there to be any chance that the symplectic curve is equisingularly isotopic to a complex curve, its singularities must be modeled on the singularities appearing in complex plane curves. By a result of McDuff [**McD92**] (reproved by Micallef–White [**MW95**] through different methods), these are precisely the singularity types which can be realized in J-holomorphic curves for some almost complex structure J compatible with the standard symplectic form on $\mathbb{C}\mathrm{P}^2$. Thus these types of singular symplectic surfaces can be studied with pseudoholomorphic techniques.

(4) There are a number of works which approach this problem for certain classes of singular surfaces and are able to prove in some cases that the symplectic surfaces with certain specified singularity types are symplectically isotopic to complex curves. For example Baurraud established symplectic isotopy results for nodal symplectic spheres [**Bar99**] and sufficiently generic line arrangements [**Bar00**]. Shevchishin proved results for nodal symplectic surfaces of sufficiently low genus [**She04**], conjecturing this holds more generally with a positivity assumption on the Chern number. Ohta–Ono proved uniqueness up to symplectic isotopy of a cuspidal cubic [**OO05**]. More generally, unicuspidal single Puiseux pair families and low degree examples of symplectic rational cuspidal curves were shown to always be equisingularly symplectically isotopic to complex curves in [**GS22b, GK23**], motivating the plausibility of a positive answer to the first question in the problem statement.

(5) There are a number of ad hoc examples of singular symplectic surfaces in $\mathbb{C}\mathrm{P}^2$ that are *not* equisingularly isotopic to any complex curve. Possibly the first such examples appeared in Moishezon's work [**Moi94**]. These examples were high degree and contained a very large number of nodes (positive transverse double points) and simple cusps (modeled on $z_1^3 = z_2^2$). An important tool in constructing and detecting such examples is braid monodromy [**MT88, KK03**]. An easier and lower-degree example is the "fake Pappus" line arrangement which cannot be realized by complex projective lines, but which can be realized symplectically (see [**RS19**]). For an irreducible example of degree 8 with locally reducible singularities see [**GS22b**, Section 8]. However, we do not currently have any examples

where the surface is irreducible and the singularities are locally irreducible, a.k.a., "cuspidal".

(6) One could also ask whether every pair of equisingular singular symplectic surfaces in $\mathbb{CP}^2$ are equisingularly symplectically isotopic to each other. In contrast to the smooth symplectic isotopy problem, this is not equivalent to asking whether every singular symplectic surface is equisingularly symplectically isotopic to a complex curve. In fact, it is well known that there are examples of equisingular complex curves in $\mathbb{CP}^2$ that are not equisingularly symplectically isotopic. Additionally there are examples of singular symplectic surfaces such that there does not exist any complex curve with the same singularities as mentioned above.

   Equisingular complex algebraic curves that are not equisingularly isotopic are often known as "Zariski pairs" (due to the first example being a pair of sextics discovered by Zariski [**Zar29**]) and are of great interest in the study of complex algebraic plane curves. (Note that Zariski pair sometimes is used to refer to examples that are not related by a weaker equivalence than equisingular isotopy, such as the fundamental groups of the complements being non-isomorphic; this implies they are not symplectically isotopic.) See [**ABCT08**] for an in-depth survey on Zariski pairs. For singular symplectic curves, it is interesting to ask when there are further "Zariski pairs" that can be realized symplectically than can be realized complexly.

Proposed for K3 by: M. Golla, L. Starkston
Scribed by: L. Starkston

Problem 4.103.

(a) What polynomials can occur as the Alexander polynomials of complex plane algebraic curves?

(b) More generally, what are the conditions that must be satisfied by a finitely presented group G, so that there exist a plane algebraic curve having G as the fundamental group of its complement?

Remarks.

(1) Let C be an algebraic curve in $\mathbb{C}^2$. The Alexander polynomial of C relative to a surjection $\pi_1(\mathbb{C}^2 \backslash C) \to \mathbb{Z}$ is the characteristic polynomial the automorphism of the homology of the associated infinite cyclic cover $H_1(\widetilde{\mathbb{C}^2 \backslash C}, \mathbb{C})$ induced by a generator of the group of deck transformations of the cover.

(2) Since a constraint on the Alexander polynomial is also a constraint on the fundamental group, part (a) is a very special case of part (b). One of many other questions underlying the second part is: which *finite* groups can occur as the fundamental groups of the complements to irreducible algebraic curves in $\mathbb{CP}^2$?

(3) A root of the Alexander polynomial of C must be a root of the Alexander polynomial of the link of at least one of the singularities of C and also

a root of the Alexander polynomial of the link at infinity, i.e., the intersection of C with a 3-sphere in $\mathbb{C}^2$ of a sufficiently large radius [**Lib83, Lib21**]. In particular the Alexander polynomial of an algebraic curve is cyclotomic but degrees of the factors that can occur are unknown. For example, for an irreducible plane curve having only ordinary cusps and nodes as singularities (i.e., locally homeomorphic to $x^2 + y^3 = 0$ and $x^2 + y^2 = 0$ respectively) the Alexander polynomial has the form $(t^2 - t + 1)^r$ and the largest known value for r is 4; see [**CAL14**]. Is the set of possible degrees of the Alexander polynomials of curves C of arbitrary degrees but with fixed local types of singularities bounded?

(4) For a recent survey of this circle of questions and mentioned properties of the Alexander polynomials, see [**Lib82**]. Further developments discussed there include characteristic varieties providing a multivariable generalization of Alexander polynomials, the Alexander polynomials of the complements to ample singular divisors on projective simply connected surfaces, and the role of the Alexander polynomials in the study of Zariski pairs (see Problem 4.102).

(5) It would be interesting to understand a symplectic analog of the above problems. The Alexander polynomials of the fundamental groups of the complements to pseudoholomorphic curves with isolated singularities can be similarly defined, but their characterization, and the question "are the classes of realizable polynomials different in the symplectic or algebraic context?" are open. In particular, it is unknown if the classes of the fundamental groups of the complements to plane algebraic and pseudoholomorphic curves are different. Recently, a symplectic analog of the divisibility theorem mentioned in (3) was announced in the symplectic context; see [**AG24**].

Proposed for K3 and scribed by: A. Libgober

PROBLEM 4.104. *Does there exist a transverse link $L \subset (S^3, \xi_{std})$ bounding a pair of complex curves in $B^4 \subset \mathbb{C}^2$ that are isotopic through embedded smooth surfaces but not through complex curves? Smoothly isotopic but not symplectically? Can there be infinitely many of them?*

REMARKS.

(1) By Rudolph [**Rud83**] and Boileau-Orevkov [**BO01**], L should be a quasipositive link and the complex curves can be realized as quasipositive surfaces in B^4. The boundary braids of the latter do not necessarily need to be the same quasipositive braid representatives of K, but after Markov stabilization one can assume they are.

In fact, there are transverse knots in (S^3, ξ_{std}) with quasipositive braid representatives that bound infinitely many quasipositive surfaces that are not smoothly isotopic [**BVHM18**], but these are distinguished by the topology of their complements. There are also pairs that are topologically isotopic but not smoothly isotopic, announced in [**Hay21a**].

(2) This can be regarded as a relative version of the symplectic isotopy problem; see Problem 4.101. The case of $K = T(d, d)$, the (d, d) torus link, is

equivalent to the symplectic isotopy problem. This is because the symplectic isotopy classes of a non-singular surface of degree d are in bijection with the equisingular symplectic isotopy classes of the union of the non-singular degree d curve with a generically intersecting line [**GS22b**, Proposition 5.1]. The complement of a neighborhood of the line is symplectomorphic to B^4 and the degree d symplectic surface will intersect the boundary S^3 in a transverse (d, d) torus link. There may be some generalizations of this to other algebraic links besides the (d, d) torus link, by allowing the degree d surface to intersect the line tangentially or by studying degree d surfaces with singularities.

(3) There are infinitely many examples of a Legendrian link $L \subset (S^3, \xi_{std})$ bounding infinitely many (exact) Lagrangians in $B^4 \subset \mathbb{C}^2$ that are smoothly isotopic but not Hamiltonian isotopic [**CG22**].

Proposed for K3 by: I. Baykur, J. Van Horn-Morris
Scribed by: I. Baykur, L. Starkston, J. Van Horn-Morris

PROBLEM 4.105. *Does there exist a planar contact 3-manifold that has infinitely many distinct Stein fillings?*

REMARKS.

(1) A contact 3-manifold is *planar* if it is supported by a planar open book decomposition.
(2) There are many interesting classes of planar contact 3-manifolds, for example lens spaces and links of rational surface singularities with reduced fundamental cycle. By Wendl [**Wen10**], and Wendl and Niederkrüger [**NW11**], any minimal weak symplectic filling of a planar contact manifold is in fact Stein, fills any given planar open book, and hence is determined by a positive Dehn twist factorization of the monodromy of that open book. Additionally, by Plamenevskaya [**Pla12**] and separately Wand [**Wan12**], there are only finitely many "homological types" of positive factorizations of the monodromy of a planar open book. (See [**BVHM18**] for an explicit statement.) Lisca [**Lis08**] classified the diffeomorphism types of Stein fillings of the standard contact structure on lens spaces, showing that there are only finitely many.
(3) The adjective "distinct" could be interpreted in many ways and most are interesting. One strong "no" result would be to show there exist only finitely many Stein fillings up to diffeomorphism. Stronger would be to prove this up to symplectomorphism and deformation. Lisi and Wendl conjecture that there are only finitely many deformation classes of minimal symplectic fillings [**LW21**].
(4) Such examples are known when the page genus is 1 and higher [**OS04a**] [**BVHM18**]. Moreover, for genus 2 and higher the Stein fillings can have arbitrarily large b_2. This is thus related to the *support genus* of the contact manifold; see Problem 3.45.

Scribed by: J. Van Horn-Morris

PROBLEM 4.106. *Is the exact symplectomorphism type of T^*X^4 sensitive to the smooth structure on a 4-manifold X, or does it depend only on the simple-homotopy or homeomorphism type of X?*

REMARKS.

(1) Recall that the cotangent bundle T^*X has a canonical 1-form λ_{can}; the canonical symplectic form on T^*X is $d\lambda_{can}$. A diffeomorphism $\phi\colon T^*X \to T^*Y$ is an *exact symplectomorphism* if $\phi^*\lambda_{can} = \lambda_{can} + df$ for some function $f\colon X \to \mathbb{R}$.

(2) The problem is a version of Arnol'd's *Nearby Lagrangian Conjecture*, which states that any exact Lagrangian submanifold of a cotangent bundle is Hamiltonian isotopic to the 0-section. In particular, a positive solution to the Nearby Lagrangian Conjecture would imply that the exact symplectomorphism type of T^*X determines the diffeomorphism type of X.

(3) Evidently, the symplectomorphism type of T^*X determines the homotopy type of T^*X and hence of X. In fact, deep results in Floer theory show that it determines the simple homotopy type of X [**AK18**].

(4) In higher dimensions, the symplectic structure on the cotangent bundle can detect (some) exotic smooth structures [**Abo12, EKS16**]. In dimension 3, a relative version of this construction—considering the unit conormal bundle to a knot—gives a strong invariant of knots [**Ng05, ENS18**], though of course the homotopy type of a knot complement is itself a strong knot invariant. It would be interesting to know whether this conormal construction detects exotic embeddings of surfaces in 4-manifolds.

Scribed by: R. Lipshitz

PROBLEM 4.107. *Problems about contact hypersurfaces:*

(a) *Let (Y, ξ) be a contact manifold and $(\mathbb{R} \times Y, \omega)$ its symplectization. Let $f : Y \to \mathbb{R} \times Y$ be a smooth embedding such that f induces an isomorphism on homology. When can we isotope f so that its image is a contact type hypersurface?*

(b) *Special case: Is every embedded 3-sphere $Y \subset (\mathbb{R}^4, \omega_{std})$ smoothly isotopic to a hypersurface of contact type?*

(c) *Is there any contact rational homology sphere other than (S^3, ξ_{std}) which embeds as a contact type hypersurface in $(\mathbb{R}^4, \omega_{std})$)?*

REMARKS.

(1) There are some geometric obstructions to a hypersurface Y being isotopic to a contact type hypersurface in the ambient symplectic 4–manifold (M, ω); e.g. [**Sul76**] provides a necessary condition formulated in terms of a characteristic foliation of ω on Y and [**Cie98**] shows that the existence of certain concentric annuli is an obstruction.

(2) Question (b) has a bearing on the Schoenflies conjecture. If it is possible to isotope a smooth embedding of S^3 to make it a contact hypersurface in $(\mathbb{R}^4, \xi_{std})$ then it bounds a symplectic filling that is necessarily the 4-ball by Gromov [Gro85]. A strategy towards trying to isotope a smoothly embedded S^3 to a contact type hypersurface has been proposed by Lambert-Cole [LC21].

(3) It was conjectured by Gompf [Gom13] that no Brieskorn sphere with either orientation admits a pseudoconvex embedding in $\mathbb{C}^2$. This has a bearing on question (c). Mark and Tosun proved half of this conjecture in [MT22] that no positively oriented Brieskorn sphere (with any number of singular fibers) embeds as a contact type hypersurface of $(\mathbb{R}^4, \omega_{std})$.

Proposed for K3 by: J. Chaidez, B. Tosun
Scribed by: J. Chaidez, L. Starkston, J. Van Horn-Morris

PROBLEM 4.108. *Let W_+ and W_- be two 4-dimensional Liouville domains with a contactomorphism $\Phi : \partial W_- \simeq \partial W_+$. This determines an $\mathbb{R}$-invariant contact structure ξ on $\mathbb{R} \times X$ where X is the gluing*

$$X = W_+ \cup_\Phi W_-$$

Moreover, $0 \times X \subset \mathbb{R} \times X$ is a convex hypersurface in the sense of Giroux.

(a) Is there a 4–dimensional version of Giroux's criterion in the case where W_+ and W_- are Weinstein? That is, a necessary and sufficient topological criterion on (Φ, W_+, W_-) for ξ to be overtwisted.

(b) Does every 4-manifold X admit a decomposition $W_+ \cup_\Gamma W_-$ so that the corresponding contact structure ξ is tight (i.e. not overtwisted)?

(c) Is there a 4-dimensional Liouville domain W and a contactomorphism $\Phi : \partial W \to \partial W$ that extends to a diffeomorphism Ψ of W, but so that the contact structure corresponding to (Φ, W, W) is overtwisted?

REMARKS.

(1) A hypersurface $\Sigma \subset Y$ of a contact manifold (Y, ξ) is *convex* if there is a contact vector-field V that is transverse to Σ. A contact manifold is *overtwisted* if there is an embedded codimension one disk D_{ot} [BEM15] with characteristic foliation determining a standard overtwisted contact structure in a neighborhood.

(2) Overtwistedness is essentially characterized by an h-principle, and the overtwisted-tight dichotomy is emblematic of the general "flexible-rigid" dichotomy in symplectic topology. The theory of convex surfaces in contact 3-manifolds was pioneered by Giroux [Gir02, GM03].

(3) The foundational work on convex surfaces was extended to higher dimensions by several works of Honda-Huang [HH18b, HH19], Breen–Honda–Huang [BHH23] and Eliashberg–Pancholi [EP23]. When W_+ and W_- are Weinstein, the data (Φ, W_+, W_-) can be described using a variant of the Weinstein-Kirby diagrams due to Gompf (c.f. Breen-Christian [BC24c]). In particular, many of the outstanding open problems in higher

dimensional convex surface theory may be particularly interesting to study in dimension four.

(4) On part (a): given a splitting $\Sigma = W_+ \cup_\Phi W_-$ of a closed, oriented, connected surface, *Giroux's criterion* states that the resulting contact structure on $\mathbb{R} \times \Sigma$ is overtwisted if and only if either $\Sigma \simeq S^2$ and Γ has more than one component or $\Sigma \not\simeq S^2$ and Γ contains a contractible curve. A higher dimensional, algebraic analogue of this criterion has been obtained by Avdek [**Avd23**], but a true topological analogue is still unknown.

(5) On Part (b): a celebrated result of Etnyre–Honda [**EH01b**] gives an example of a closed contact 3-manifold with no tight contact structures. Part (b) of this problem may be viewed as a 4-manifold analogue of that result.

(6) On Part (c): this problem is motivated by the goal of finding diffeomorphisms fixed at the boundary that act nontrivially on the space of symplectic structures. Indeed, the extension Ψ of Φ in part (c) would constitute such a map, since the double $W \cup_{Id} W$ is always tight by Avdek [**Avd23**]. Moreover, such a map may be usable as a sort of symplectic cork twist - by embedding W into a closed symplectic manifold X, removing W and regluing it by Ψ to acquire a new symplectic manifold X', one may be able to find diffeomorphic, non-symplectomorphic closed 4-manifolds (see Problem 4.96).

Scribed by: J. Chaidez

PROBLEM 4.109. *If $\Sigma \subseteq (X, \omega)$ is a symplectic surface in a closed symplectic 4-manifold with $[\Sigma] = PD(k[\omega])$, $k \in \mathbb{Z}$, does $(X \backslash \Sigma, \omega)$ support a Weinstein structure?*

REMARKS.

(1) In particular, one could ask this for surfaces in $(\mathbb{C}\mathrm{P}^2, \omega_{\mathrm{std}})$. A negative answer to the question in this case would yield a counterexample to the symplectic isotopy problem (Problem 4.101).

(2) Donaldson [**Don96**] and Giroux [**Gir02, Gir17**] prove that for suitably large k there is some symplectic surface for which this is true. However, there are not currently known bounds on k, or known examples where $k = 1$ is not enough. In principle, large upper bounds could be obtained by carefully tracking the proof in Donaldson's construction. Such bounds are likely much larger than needed, but any explicit bound in terms of the symplectic manifold would represent progress on this question. Obtaining more effective bounds through new techniques would be particularly interesting.

Scribed by: L. Starkston, J. Van Horn-Morris

PROBLEM 4.110. *Does there exist a 2-handlebody W that admits an exact symplectic structure with convex contact boundary that does not admit a Weinstein structure filling the same contact boundary?*

REMARKS.

(1) So far, all known examples of exact symplectic manifolds that we can obstruct from having a Weinstein structure are known to have handle structures requiring 3-handles [**McD91**, **Bow12**].
(2) We have limited tools for obstructing an exact filling from being Weinstein when it has the topology of a 2-handlebody. Bowden's argument in [**Bow12**] utilized a theorem of Eliashberg which says that a Weinstein filling of a connected sum is necessarily a boundary sum of Weinstein fillings of the summands [**Eli90**]. A potential strategy to generalize this obstruction to other examples may be to use Menke's "mixed tori" [**CM19**], which gives a different decomposition criterion for Weinstein fillings.

Scribed by: L. Starkston, J. Van Horn-Morris

4.10. Trisections of 4-manifolds

A *trisection* of a 4–manifold X, introduced by Gay and Kirby in [**GK16**], is a decomposition $X = Z_1 \cup Z_2 \cup Z_3$ such that, for each $i \in \mathbb{Z}_3$,

(1) Z_i is a 4–dimensional 1–handlebody;
(2) $H_i = Z_i \cap Z_{i-1}$ is a 3–dimensional handlebody; and
(3) $\Sigma = Z_1 \cap Z_2 \cap Z_3$ is a closed, genus g surface.

We say that such a trisection has *genus* g. If $Z_i = \natural^{k_i}(S^1 \times B^3)$, we say the trisection has *complexity* $(g; k_1, k_2, k_3)$, or (g, k) when $k_i = k$ for all k.

PROBLEM 4.111. *Is trisection genus additive? In other words, must it be the case that $g(X \# X') = g(X) + g(X')$.*

REMARKS.

(1) This is Conjecture 1.6 of [**LCM22**].
(2) The *trisection genus* of a 4–manifold X is

$$g(X) = \min\{g \,|\, X \text{ admits a genus } g \text{ trisection}\}.$$

(3) If $\mathfrak{T}$ and $\mathfrak{T}'$ are trisections for X and X' of genus g and g', respectively, then $\mathfrak{T} \# \mathfrak{T}'$ is a trisection of genus $g + g'$ for $X \# X'$. It follows that trisection genus satisfies:

$$g(X \# X') \leqslant g(X) + g(X').$$

Equality holds when the lower-bound on $g(X)$ and $g(X')$ coming from standard algebraic topology is sharp; see Problem 4.117.

(4) A positive resolution to this problem would have sweeping consequences. If trisection genus is additive, then no manifold from the following set has an exotic copy:

$$\mathcal{M} = \{S^4, \mathbb{C}\mathrm{P}^2, S^1 \times S^3, 2\mathbb{C}\mathrm{P}^2, \mathbb{C}\mathrm{P}^2\#\overline{\mathbb{C}\mathrm{P}}^2, S^2 \times S^2\}.$$

This follows from the classification of trisections up to genus two [**GK16, MZ17b**] and the observation that if trisection genus is additive, homeomorphic smooth four-manifolds have the same trisection genus; see Proposition 1.7 of [**LCM22**], where modest evidence for this phenomenon is given; see also Remark 5 of Problem 4.113.

In particular, if trisection genus is additive, then any exotic copy of $\mathbb{C}\mathrm{P}^2\#2\overline{\mathbb{C}\mathrm{P}}^2$ admits a genus three trisection. Such exotic copies are known to exist [**AP10, FS11**]. This motivates the important problem of classifying genus three trisections; see Problem 4.113.

(5) There is an analogous problem for orientable surface-links in S^4 that is related to the *Meridional Rank Conjecture* for orientable surface-links [**JP25**]; see Problem 1.18. For more details and precise definitions, see [**MZ17a**] and [**AAD+23**, Question 6.1]. Given an orientable surface-link $\mathcal{K} \subset S^4$, let $p(\mathcal{K})$ denote its *patch number*, the minimum value p such that $\mathcal{K}$ admits a $(b; p, p', p'')$–bridge trisection.

QUESTION. *Is patch number* (-1)*–additive for orientable surface-links? In other words, must it be the case that*

$$p(\mathcal{K}_1\#\mathcal{K}_2) = p(\mathcal{K}_1) + p(\mathcal{K}_2) - 1$$

for orientable surface-links $\mathcal{K}_1$ *and* $\mathcal{K}_2$*?*

Proposed for K3 and scribed by: J. Meier

PROBLEM 4.112. *Is every trisection of the 4-sphere with positive genus a stabilization of the genus zero trisection?*

REMARKS.

(1) This is Conjecture 3.11 of [**MSZ16**]. The analogous result regarding Heegaard splittings of the 3–sphere is *Waldhausen's Theorem* [**Wal68a**]. Connections between this problem, the *Generalized Property R Conjecture* (Problem 1.10), and the *Andrews–Curtis Conjecture* (Problem 5.10) are detailed in [**MZ18**] and elaborated in [**MZ22**]. In particular, if every positive-genus trisection of S^4 is stabilized, then well-known potential counter-examples (dating back to [**AK85**]) to the two conjectures mentioned above are not, in fact, counter-examples.

(2) The following related question may be more tractable and of independent interest [**MZ17a**]. It is a four-dimensional analog of the classical result of Otal that says the unknot has a unique bridge splitting for each bridge number [**Ota82**].

QUESTION. *Is every* b*–bridge trisection of the unknotted 2–sphere in* S^4 *with* $b > 1$ *a perturbation of the 1–bridge trisection?*

Proposed for K3 and scribed by: J. Meier

PROBLEM 4.113. *Which closed, oriented, smooth* 4*–manifolds admit genus–*3 *trisections? Which ones admit genus–*3 *simplified trisections? How about genus–*4*?*

REMARKS.

(1) There is a complete classification of trisections of genus $g \leqslant 2$ and only a handful of standard 4–manifolds admit them: S^4 for $g = 0$; $\mathbb{CP}^2$, $\overline{\mathbb{CP}}^2$, or $S^1 \times S^3$ for $g = 1$; and $S^2 \times S^2$, or connected sums of $\mathbb{CP}^2$, $\mathbb{CP}^2$ and $S^1 \times S^3$ with two summands, for $g = 2$ by [**GK16, MZ17b**].

(2) While the *simplified trisections* (see [**BS23b**] for a precise definition) constitute a subclass of Gay-Kirby trisections, the following question [**BS18c**, Question 2] is still open:

QUESTION. *Is there a closed* 4*–manifold that admits a trisection, but not a simplified trisection of the same genus?*

The classification of genus $g \leqslant 2$ simplified trisections coincide with that of standard trisections [**BS18c, Hay20**]. So, the genus $g = 3$ is the lowest genus where a discrepancy may appear.

(3) Some partial results are known: A trisection with complexity $(3; k_1, k_2, k_3)$ and $k_i \geqslant 2$ for some $i \in \mathbb{Z}_3$ is reducible [**MSZ16**]. Recall that a trisection that is the connected sum of smaller trisections is called *reducible*; otherwise, it is *irreducible*.

Infinitely many, pairwise homotopy inequivalent 4–manifolds admit genus–3 trisections; e.g. every spun lens space admits a (simplified) genus–3 trisection [**Mei18, BS18c**]. Similarly, there are genus–4 (simplified) trisections of 3–manifold bundles over S^1 with lens space or $S^1 \times S^2$ fibers [**Mei18, BS18c**].

(4) The classification of low genera trisections is intimately related to that of *simplified broken Lefschetz fibrations* [**BS23b, BS18c**]. The classification for the latter is complete for $g \leqslant 1$ and spans a larger class of 4–manifolds [**BK15a, Bay12a, Hay11, Hay14**].

(5) In the case of (broken) Lefschetz fibrations, the following exotic phenomenon already appears when we hit $g = 2$: there are pairs of homeomorphic but not diffeomorphic 4–manifolds which both admit genus–2 Lefschetz fibrations; for instance, let $X_i := E(1)_{K_i}$, $i = 1, 2$, be the knot surgered rational elliptic surface for K_1 a trefoil knot and K_2 the figure eight knot [**FS04, Bay09**]. There are analogous results for (simplified) trisections when $g = 20$ [**BS23b**] and many other, larger genera, starting at $g = 23$, where one of the exotic pairs is an algebraic surface [**ST18, LCM22**]. It is reasonable to expect this exotic phenomenon to manifest for much smaller g. (This is related to the question on the additivity of triseciton genus under connected sum; see Problem 4.111.)

QUESTION. *What is the smallest* g *for which there is an exotic pair of closed, oriented* 4*–manifolds admitting genus–*g *trisections?*

(6) While the problem is formulated only for orientable 4–manifolds, the classification of small-genera (simplified) trisections and (simplified) broken Lefschetz fibrations on *nonorientable* 4–manifolds is also within reach, and suggests analogous classification schemes; see [**MN24**, **ST22**, **BM25**].

Proposed for K3 and scribed by: I. Baykur, J. Meier

Problem 4.114. *For a given Heegaard splitting of a closed orientable* 3*–manifold, classify self-indexing Morse functions that give the given Heegaard splitting, up to* C^∞ *right-left* (*or right*) *equivalence. Likewise, for a given* (*simplified*) *trisection of a closed orientable* 4*–manifold, classify generic maps that give the given trisection, up to right-left* (*or right*) *equivalence.*

Remarks.

(1) Let $f_i\colon M_i \to N_i$ be C^∞ maps between smooth manifolds, $i = 0, 1$. We say that they are C^∞ *right-left equivalent* if there exist diffeomorphisms $\varphi\colon M_0 \to M_1$ and $\psi\colon N_0 \to N_1$ such that $f_1 = \psi \circ f_0 \circ \varphi^{-1}$. If $N_0 = N_1$ and ψ can be taken to be the identity, we say that f_0 and f_1 are C^∞ *right equivalent*.

Figure 5. Two commutative diagrams. Left: the maps f_0, f_1 are right-left equivalent. Right: the maps f_0, f_1 are right-equivalent.

It is known that to a self-indexing Morse function on a closed orientable 3–manifold is canonically associated a Heegaard splitting. Likewise, to a certain generic map, called *Morse 2–function*, on a closed orientable 4–manifold is canonically associated a trisection [**GK16**] or a simplified trisection [**BS23b**, **BS18c**].

(2) The 4–dimensional problem can be rephrased as follows. Let M be a closed orientable 4–manifold and let $f\colon M \to \mathbb{R}^2$ a C^∞ be a stable map that corresponds to a (simplified) trisection. If two such maps are C^∞ right-left equivalent, then the resulting trisections are naturally equivalent. Does the converse hold? Namely, if the resulting (simplified) trisections are equivalent, then are the original C^∞ stable maps C^∞ right-left equivalent? If not, how different are they?

Some related results are obtained in [**Hay20**] and [**Asa23**].

(3) It is known that Morse functions on closed surfaces can be classified by means of Kronrod-Reeb graphs up to C^∞ right-left (or right) equivalence (for example, see [**Mak05**]).

Proposed for K3 and scribed by: O. Saeki

PROBLEM 4.115.

(a) Find two diffeomorphic but non-isotopic trisections of the same 4–manifold.
(b) Find two non-diffeomorphic balanced trisections of the same genus on a closed simply connected 4–manifold.

REMARKS.

(1) Part of the difficulty of Problem (a) is that it is connected to the problem of understanding the smooth mapping class groups of 4–manifolds. In particular, if a 4–manifold X has trivial smooth mapping class group, so that all (orientation preserving) diffeomorphisms are isotopic, then all diffeomorphic trisections are obviously isotopic. On the other hand, 4–manifolds with nontrivial homology can have diffeomorphisms that act nontrivially on homology, and one might imagine that one could apply such a diffeomorphism to a given trisection to get a diffeomorphic but non-isotopic trisection. However this diffeomorphism might be isotopic to a diffeomorphism that is not the identity but which fixes the initial trisection (i.e. fixes the sectors setwise), in which case the new trisection would in fact be isotopic to the initial one.

(2) Problem (b) seems to be easier than Problem (a), with the first vague approximation being to come up with examples of trisections that look at first glance like they might be diffeomorphic but which in fact are not. Examples of non-diffeomorphic same-genus trisections of a 4-manifold were first constructed by Islambouli [**Isl21**]. However, Islambouli's techniques rely on the 4-manifold having nontrivial fundamental group G, as the trisections are distinguished by the Nielsen classes of associated presentations of G. This mirrors an argument in the 3–dimensional Heegaard splitting setting [**Eng70**]

(3) In principle, invariants of trisections up to diffeomorphism should be easier to find than invariants up to isotopy. Meier and Lambert-Cole construct various genus–22 trisections of K3 that may or may not be diffeomorphic and/or isotopic by viewing K3 as a branched cover in different ways [**LCM22**]. See Problem 4.112 (and examples of Meier and Zupan [**MZ18**]) for a related problem regarding the uniqueness of trisections of S^4.

Proposed for K3 by: D. Gay
Scribed by: D. Gay, J. Meier

PROBLEM 4.116. *Is there an algorithm to compute 'distance' in the cut complex of a trisection? Is the L-invariant computable?*

REMARKS.

(1) A Heegaard splitting for a 3-manifold M is defined by two "cut systems" of curves on a Heegaard surface Σ. Various notions of 'distance' (e.g.

the *Hempel distance*) for Heegaard splittings have been extensively studied and yielded valuable geometric information about M. A trisected smooth closed 4-manifold X is defined by three "cut systems" of curves on a surface Σ. One can define various analogous notions of 3-manifold Heegaard distance for a trisected X by measuring the length of a shortest loop in the cut complex (or pants complex) that includes a representative of each of the three specified cut systems. Given a specific trisection, the first question asks whether any such distance is computable. The L*-invariant* [**KT22a**] uses this idea in a limit to define a 4-manifold invariant.

(2) When L is zero and X is a homology sphere, X is diffeomorphic to the 4-sphere. Hence a positive answer to the second question is related to recognizability of the 4-sphere.
(3) In a preprint, Asano–Naoe–Ogawa [**ANO24**] gave a lower bound on the L-invariant of a 4-manifold X in terms of the first Betti number of X.

Proposed for K3 and scribed by: A. Thompson

Problem 4.117. *Let X be a closed, orientable, smooth 4-manifold, with $g(X)$ the trisection genus of X. Does*

$$g(X) = \chi(X) - 2 + 3\mathrm{rk}(\pi_1(X))?$$

Remarks.

(1) Chu and Tillmann proved that $\chi(X) - 2 + 3\mathrm{rk}(\pi_1(X))$ is a lower bound for $g(X)$ [**CT19**]. It is reasonable to expect there to exist manifolds for which this inequality is strict, but at present, Chu and Tillmann's result remains the only known lower bound on trisection genus. Following the classification of trisections up to genus two [**MZ17b**], if the problem has an affirmative answer, then there do not exist exotic copies of any X such that $g(X) \leqslant 2$, including S^4, $S^1 \times S^3$, $\mathbb{C}\mathrm{P}^2$, and $S^2 \times S^2$. (See also Problem 4.111.)
(2) Like many problems in trisections, there is an analogous problem related to Heegaard splittings of 3-manifolds, the rank-genus conjecture. The now-disproved rank-genus conjecture asks whether $g(Y) = \mathrm{rk}(\pi_1(Y))$ for a closed, orientable 3-manifold Y. Originally posed by Waldhausen [**Wal78**], this conjecture was also called the *Generalized Poincaré Conjecture*, since it implies the 3-dimensional Poincaré conjecture [**Hak70**]. The first examples of a 3-manifold Y for which $g(Y) > \mathrm{rk}(\pi_1(Y))$ were exhibited by Boileau and Zieschang [**BZ84**], while the first hyperbolic counterexamples to the conjecture were produced by Li [**Li13**].
(3) A related problem for bridge trisections of a knotted surface $\mathcal{K} \subset S^4$ involves the *meridional rank* of the group $\pi_1(S^4 \backslash \mathcal{K})$, denoted $\mathrm{mrk}(\pi_1(S^4 \backslash \mathcal{K}))$, the smallest number of meridians needed to generate the group, and the bridge number $b(\mathcal{K})$, the minimal b such that $\mathcal{K}$ admits a b-bridge trisection.

QUESTION. *Let $\mathcal{K}$ be an orientable knotted surface in S^4. Does*

$$b(\mathcal{K}) = -\chi(\mathcal{K}) + 3\mathrm{mrk}(\pi_1(S^4 - \mathcal{K}))?$$

As in the case of trisections, the question is suggested by a straightforward (and the only currently known) lower bound. To derive the bound, note that if $\mathcal{K}$ admits a $(b; c_1, c_2, c_3)$-bridge trisection, then $\chi(\mathcal{K}) = c_1 + c_2 + c_3 - b$ and $\mathrm{mrk}(\pi_1(S^4 \backslash \mathcal{K})) \leqslant c_i$ for all i [MZ17a, Corollary 5.3]). Thus,

$$b = -\chi(\mathcal{K}) + c_1 + c_2 + c_3 \geqslant -\chi(\mathcal{K}) + 3\mathrm{mrk}(\pi_1(S^4 - \mathcal{K})).$$

This problem can be compared to the meridional rank conjecture ([Kir97, Problem 1.11] and Problem 1.18), an unsolved problem that asks whether the bridge number of a knot K in S^3 is equal to the minimal number of meridional generators required to generate its knot group. See also Problem 4.111.

(4) Meier and Zupan showed that if $\mathcal{K}$ is the spin of a classical knot K that satisfies the meridional rank conjecture in dimension three, then $\mathcal{K}$ satisfies the equality in the problem above [MZ17a]. Note that a positive answer to the problem would imply that any 2-sphere or torus in S^4 with infinite cyclic fundamental group has bridge number one or three, respectively, and as such is smoothly unknotted by the classification of b-bridge trisections with $b \leqslant 3$ [MZ17a]. Miyazawa's recent construction of an exotic $\mathbb{R}\mathrm{P}^2$, announced in [Miy23], yields a negative answer to the problem in the case of nonorientable surfaces.

Proposed for K3 and scribed by: A. Zupan

4.11. Other structures on 4-manifolds

The problems in this final subsection ask about various structures on 4-manifolds not hitherto considered, including low-complexity handle decompositions, CW-structures, achiral pencils, open books, branched covers, foliations, hyperbolic structures, PSC metrics and Lipschitz structures.

PROBLEM 4.118. *Does every simply connected, closed, smooth 4-manifold admit a handle decomposition without any 1-handles? Without 1-handles and 3-handles?*

REMARKS.

(1) A manifold is called *geometrically simply connected* if it admits a handle decomposition without 1-handles. Any simply connected closed manifold of any dimension other than four is geometrically simply connected; this follows from the celebrated works of Smale in higher dimensions [Sma62b] and of Perelman in dimension three [Per02, Per03b, Per03a].

(2) This is [Kir97, Problem 4.18]. Some candidates for counter-examples, such as the Dolgachev surface $E(1)_{2,3}$, have since been shown to admit handle decompositions without any 1- and 3-handles [Yas08, Akb12].

(3) If one allows the 4-manifold to have boundary, the answer is negative. Indeed, there are many contractible 4-manifolds that require 1-handles, by the following argument due to Casson: If a compact, contractible 4–manifold X can be built without 1-handles, then, turning the handlebody upside down, we can also build X from ∂X by adding the same number of 1- and 2–handles and a 4-handle. So $\pi_1(\partial X)$ can be killed by adding the same number of generators and relators. However, a finitely presented group with a nontrivial representation to a compact connected Lie group cannot be trivialized by adding the same number of generators and relators by [**GR62**] and there are contractible X where $\pi_1(\partial X)$ is such a group. In fact, geometrization now implies all nontrivial 3-manifold groups admit nontrivial finite quotients by [**Hem87**], so any contractible 4-manifold with boundary other than S^3 requires 1-handles, such as the Mazur manifold with boundary $\Sigma(2, 3, 13)$ [**AK79b**].

(4) If a geometrically simply connected closed 4-manifold X has $b_2^+(X) > 1$ and $b_2^-(X) = 0$, then all the stable cohomotopy Seiberg-Witten invariants of X vanish [**Yas19**]; so, e.g. it cannot admit a symplectic structure—see also [**HL19**].

In the same paper, Yasui shows that if X is geometrically simply connected, then every $\alpha \in H_2(X; \mathbb{Z})$ has a neighborhood W diffeomorphic to a 2-handle attached to a 4-ball, where α is the image of the generator of $H_2(W; \mathbb{Z}) \cong \mathbb{Z}$ under the inclusion induced homomorphism. Any simply connected X with some $\alpha \in H_2(X; \mathbb{Z})$ that does not admit such a neighborhood would be a counter-example.

(5) Admitting a handle decomposition without 1- and 3-handles has strong implications. For instance, if this is true for every homotopy S^4 or homotopy $\mathbb{C}P^2$, then there are no exotic copies of these 4-manifolds. The conclusion for homotopy $\mathbb{C}P^2$s follows from [**GL89**].

Scribed by: I. Baykur, M. Powell

Problem 4.119. *Is every topological 4-manifold homeomorphic to a CW complex?*

Remarks.

(1) It follows from the work of Kirby and Siebenmann [**KS77**] that every topological manifold M has the homotopy type of a CW complex, and moreover there is a canonical simple homotopy type of CW complexes homotopy equivalent to M.

(2) Every smooth manifold is triangulable and hence homeomorphic to a CW complex. See [**Cai35**], [**Whi40**]. In particular, non-compact 4-manifolds are smoothable [**Qui82**]; therefore, the question has a positive answer for those.

(3) Every topological manifold of dimension $n \neq 4$ has a handlebody structure, and hence is homeomorphic to a CW complex. See [**Moi77a**] for $n = 3$, [**KS77**, p.104] for $n \geqslant 6$ and [**Qui86**] for $n = 5$.

(4) Any 4-manifold with a handlebody structure is smooth. Indeed, when a handle is attached to a smooth 4-manifold, the attaching map is in dimension three, where everything is smoothable.
(5) Topological, non-smoothable 4-manifolds (such as the E_8 manifold) are known to not be homeomorphic to simplicial complexes. See [**AM90**] and [**Man13**, Remark 4.2].

Scribed by: C. Manolescu

PROBLEM 4.120. *Which closed, smooth 4–manifolds admit achiral Lefschetz pencils? Does every simply connected 4–manifold have one?*

REMARKS.

(1) Achiral Lefschetz pencils are generalizations of Lefschetz pencils, where the local models for nodal singularities and base points are allowed to reverse orientations. So, they are also defined on nonorientable 4–manifolds. Here we allow achiral pencils to possibly have no critical points and/or no base points.
(2) There are only a couple of known obstructions to the existence of an achiral Lefschetz pencil on a given closed, oriented 4–manifold X; see [**GS99**, Theorem 8.4.13] and [**Sco03**, Theorem 4.15]. These obstructions rule out definite 4–manifolds with $b_2 + 1 < b_1$, such as $\#^m (S^1 \times S^3)$, for $m \geqslant 2$. A curious question is: *Are there homotopy equivalent smooth 4–manifolds X and X', where X admits an achiral pencil but X' does not?*
(3) Any closed, orientable X admits an achiral Lefschetz fibration (without base points) after surgery along a curve, and specifically, $X\#(S^2 \times S^2)$ always admits one when X is simply connected [**EF06**].
(4) Just like Lefschetz pencils can be equipped with certain symplectic forms making all the fibers symplectic, achiral Lefschetz pencils can be equipped with certain *folded-symplectic* forms. Furthermore, a variation of achiral Lefschetz pencils, defined in the complement of a 1–manifold, support *folded-Kähler* forms on all closed, oriented, smooth 4–manifolds. See [**Bay06, Hit16**].

Proposed for K3 and scribed by: I. Baykur

PROBLEM 4.121. *Which closed, smooth 4–manifolds admit open book decompositions? In particular, does every closed, simply connected 4–manifold with signature zero admit one?*

REMARKS.

(1) An *open book decomposition* of an n-dimensional manifold X is given by a smooth fibration $f\colon X\backslash L \to S^1$, where the *binding* $L \subset X$ is an

$(n-2)$–dimensional embedded submanifold with a trivial normal bundle, and there is a neighborhood $N(L) \cong L \times D^2$ such that f conforms to the local model $f(x,(r,\theta)) = \theta$ for $r \neq 0$, where $x \in L$ and $(r,\theta) \in D^2$ are the polar coordinates.

(2) If a closed, oriented n–dimensional manifold X admits an open book, its signature vanishes. An extension of this necessary condition is the vanishing of the *asymmetric signature*; see [**Ran98**] for a definition and related discussion. Vanishing signature is also a sufficient condition for
   - any odd $n \geqslant 3$, by the works of Alexander, Lawson and Quinn [**Law78, Qui79**];
   - any even $n \geqslant 6$, when X is simply connected, by Winkelnkemper [**Win73**] for $n \geqslant 8$, and for $n = 6$ by Quinn [**Qui79**].

(3) Kastenholz [**Kas25**] claims that the simplicial volume vanishes for 4-manifolds that admit an open book, and gives examples of non-simply-connected 4-manifolds with vanishing asymmetric signature that do not admit open book decompositions.

(4) Open books on 4–manifolds are related to Engel structures. If a closed, oriented 4–manifold X admits an open book with a binding that is a link of tori and a monodromy that preserves a framing on the fiber, then X admits an Engel structure [**CPV18**].

(5) It would be interesting to see if there are *smooth* obstructions to admitting open books in dimension four. If X_1 and X_2 admit open book decompositions, then so does $X_1 \# X_2$. There are natural open books on any Σ_g–bundle over S^2, where the binding is a pair of fibers. Combining these general constructions, one gets an open book on every $\#^m(S^2 \times S^2)$ and $\#^n(\mathbb{C}\mathrm{P}^2 \# \overline{\mathbb{C}\mathrm{P}}^2)$. Do their exotic copies always admit open books?

Scribed by: I. Baykur

PROBLEM 4.122. *Is there a universal branching surface $S \subseteq S^4$ such that every closed, orientable 4-manifold W admits a branched covering $W \to S^4$ with branching locus S?*

REMARKS.

(1) This question originated in the introduction of [**PZ05**] by Piergallini–Zuddas.

(2) Using the signature, it can be shown that a universal branching surface in S^4 is necessarily disconnected [**Vir84, IP02**].

(3) There exists an orientable ribbon surface $F \subseteq B^4$, consisting of the disjoint union of one annulus and two discs, such that every compact orientable 4-manifold M constructed by adding only 1-handles and 2-handles to B^4 admits a branched covering $M \to B^4$ with branching locus F [**PZ05**].

   QUESTION ([**PZ05**, Question 2]). *Does there exist a* connected *universal branching surface in B^4?*

(4) A link $L \subseteq S^3$ is said to be *a universal branching link* if every closed, orientable 3-manifold Y admits a branched covering $Y \to S^3$ with branching

locus L. The figure eight knot, the 9_{46} knot, the Whitehead link, the Borromean rings, and various other knots and links are known to be universal branching links [**HLM83a**, **HLM83b**, **HLM85**]. The first universal branching link was found by Thurston in unpublished work.

QUESTION ([**PZ05**, Question 4]). *Which universal links in S^3 are boundaries of universal surfaces in B^4?*

Scribed by: A. Ray

PROBLEM 4.123. *Is every closed leaf of a two dimensional co-orientable smooth taut foliation of an oriented 4-manifold smoothly genus-minimizing in its homology class?*

REMARKS.

(1) This question is due to Kronheimer [**Kro98**, Question 7.12], and is the natural generalization of the corresponding three-dimensional result of Thurston [**Thu86**]. Here we say that the foliation $\mathcal{F}$ is *taut* if there exists a 2-form ω such that:
- for every leaf L, $\omega|_L$ is an area form;
- for every $v_1, v_2 \in T_{\mathcal{F}}$ and $z \in T_M$, $d\omega(v_1, v_2, z) = 0$.

This directly generalizes the definition in dimension three (in which case the second condition simply says that ω is closed). As Kronheimer points out, the question is also interesting when one allows foliations with singularities with a suitable local model, e.g. the foliations defined by the vanishing of a holomorphic 1-form.

(2) If the foliation is calibrated to a symplectic form, then every closed leaf is a symplectic subsurface and hence genus-minimizing by the symplectic Thom conjecture [**OS00**].

(3) If "taut," is eliminated from the hypotheses, then the answer is "no." Constructions of non-genus-minimizing compact leaves of coorientable foliations of 4-manifolds were constructed by Mitsumatsu–Vogt [**MV08**] and Bowden [**Bow11**].

(4) The following specific subquestion would be an interesting first step: *Must a compact leaf of a coorientable smooth taut foliation of $S^2 \times S^2$ be a 2-sphere?*

Proposed for K3 by: P. Kronheimer
Scribed by: F. Lin

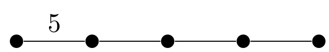
PROBLEM 4.124. *Does there exist a hyperbolic integer homology four-sphere? What about an arithmetic one? Homology four-spheres have Euler characteristic 2, so it makes sense to ask more generally if there exist any closed hyperbolic four-manifold with Euler characteristic 2.*

REMARKS.

(1) If any closed, hyperbolic manifold with Euler characteristic 2 exists, there are only finitely many. This is because the Gauss-Bonnet Theorem says that $Vol(M) = \frac{4}{3}\pi^2\chi(M)$ for a hyperbolic four-manifold M, and there are only finitely many manifolds with volume below any fixed value. Note that in general, the Euler characteristic of a closed, orientable hyperbolic four-manifold is always even, since such manifolds have zero signature (see [**LR00**]), so these would be the ones with smallest volume. The best known example seems to be the Conder-Maclachlan manifold with Euler characteristic 16 [**CM05**]. For comparison, there are one-cusped orientable hyperbolic four-manifolds with Euler characteristic one [**RT00**].

(2) It is worth pointing out that there are various constructions of aspherical 4-manifolds with Euler characteristic 2. For example, Luo constructed an aspherical rational homology 4-sphere [**Luo88**] and Tschantz constructed aspherical integer homology 4-spheres [**RT05**] (answering [**Kir97**, Problem 4.17]). The latter examples even admit metrics of non-positive curvature.

(3) If there exists an arithmetic hyperbolic integer homology sphere (or, more generally, an arithmetic, closed, hyperbolic manifold with Euler characteristic 2), then by Belolipetsky [**Bel07**, Theorem 5.5'] (see also [**Bel04**, Theorem 5.5]) it has to be an index-28800 cover of the 4-dimensional hyperbolic reflection group with the below Coxeter diagram.



Proposed for K3 by: A. Reid
Scribed by: T. Lidman

PROBLEM 4.125. *Is there a noncompact, finite volume, orientable hyperbolic four-manifold without a spin structure?*

REMARKS. All compact orientable manifolds with dimension at most three admit spin structures. Sullivan observed that every finite volume hyperbolic n-manifold has a finite cover that admits a spin structure [**Sul79b**, p.533]. Reid and Long showed in [**LR20**] that there are orientable, finite volume, non-compact hyperbolic n-manifolds with $n \geqslant 5$ that do not admit spin structures. Martelli-Riolo-Slavich showed that there are closed orientable hyperbolic four-manifolds that do not admit spin structures [**MRS20**].

Proposed for K3 by: A. Reid
Scribed by: T. Lidman

PROBLEM 4.126.

(a) If M is a closed, orientable hyperbolic 4-manifold then it always has signature 0, because its Pontryagin class vanishes [**Che55**]. *This implies that*

M bounds a compact, orientable 5-manifold. Is M always a geometric boundary?

(b) *For general n, suppose that M is a closed hyperbolic n-manifold. Is M a geometric boundary?*

Remarks.

(1) Following [**LR00**], we say that M^n is a *geometric boundary* if it is the totally geodesic boundary of a compact hyperbolic manifold W^{n+1}.
(2) Perhaps a good example to start with for (a) is the Davis manifold [**Dav85**].
(3) Every closed, orientable surface of genus at least 2 has a hyperbolic metric that is the totally geodesic boundary of a compact, orientable, hyperbolic 3-manifold by [**Fuj90**]. However, some closed, hyperbolic, orientable 3-manifolds are not the geodesic boundary of any compact, orientable, hyperbolic 4-manifold [**LR00**]. It is still open if there is a hyperbolic rational homology 3-sphere which is the totally geodesic boundary of a compact, orientable, hyperbolic 4-manifold (see Problem 3.77).
(4) We restrict to the setting of orientable 4-manifolds as a closed, nonorientable, hyperbolic 4-manifold may have odd Euler characteristic. In this case, the 4-manifold cannot bound any compact 5-manifold, without mention of geometry. Even in the setting of nonorientable 4-manifolds, it may be interesting to ask this question with the additional hypothesis that the nonorientable 4-manifold does bound some compact (nonorientable) 5-manifold.
(5) For (b), the general problem can be posed either in the orientable or nonorientable setting; one might assume that M is null-bordant to start with. In the orientable case, Long and Reid [**LR00**] observe that when $n = 4k - 1$, the η-invariant [**APS75a**] of M would have to be integral. They give examples of orientable hyperbolic 3-manifolds with non-integral η-invariant, which are therefore not geometric boundaries in the oriented category. The oriented version in higher dimensions could similarly be answered by finding hyperbolic $(4k - 1)$-manifolds with non-integral η-invariant for $k > 1$. Some constructions of higher-dimensional orientable hyperbolic manifolds that are geometric boundaries are given in [**LR01**].
(6) In the nonorientable case, J. Chen [**Che25**] claims to construct, in all dimensions $n \geqslant 4$ not of the form $n = 4k - 1$, examples of nonorientable closed hyperbolic n-manifolds that are not the boundary of any compact $(n+1)$-manifold (not assuming any geometric condition). The question of whether there are nonorientable hyperbolic manifolds that are boundaries but not geometric boundaries remains open.

Proposed for K3 by: A. Reid
Scribed by: T. Lidman

Problem 4.127. *Given an aspherical closed (or compact and bounded by flat 3-manifolds) 4-manifold M and a self-diffeomorphism f of M, find necessary and sufficient conditions on f so that the resulting 5-dimensional mapping torus M_f admits a hyperbolic structure.*

REMARKS.

(1) By hyperbolization, in dimension two the sufficient and necessary condition is that f is pseudo-Anosov (see Thurston [Thu82, Thu22] and Otal [Ota01] for a complete proof). There are some analogues in strictly higher dimensions, e.g. [Far72, Theorem 6.4] gives necessary and sufficient conditions for a manifold of dimension at least six to fiber over S^1. If such a theorem worked in dimension 5, then one could potentially check such a condition against hyperbolic 5-manifolds. However, similar higher-dimensional techniques have not yet been successfully applied in the context of 5-dimensional hyperbolic manifolds.

(2) Italiano–Martelli–Migliorini recently found examples of (M, f) with M 4-dimensional that produce a hyperbolic 5-manifold M_f [IMM23].

Proposed for K3 and scribed by: B. Martelli

PROBLEM 4.128. *What is the structure of* 4*-manifolds that admit a Riemannian metric of positive scalar curvature? There are variations of this problem for different classes of manifolds.*

(a) Is every closed simply connected PSC 4*-manifold diffeomorphic to a connected sum of copies of* $\mathbb{C}\mathrm{P}^2$, $\overline{\mathbb{C}\mathrm{P}}^2$, *and* $S^2 \times S^2$*?*

(b) What is the structure of non-simply connected closed PSC 4*-manifolds?*

(c) Which 4*-manifolds with boundary have a PSC metric?*

(d) Which non-compact 4*-manifolds have a complete PSC metric with uniformly positive scalar curvature?*

REMARKS.

(1) Let us say that a smooth manifold is a *PSC manifold* if it admits a Riemannian metric of positive scalar curvature.

(2) The corresponding problem in dimension 2 is easy by the Gauss-Bonnet theorem, and is solved in dimension 3 as a consequence of Perelman's work. In particular, every orientable closed PSC 3-manifold is a connected sum of spherical space forms and copies of $S^1 \times S^2$. In dimensions at least 5 the problem is completely solved for simply connected manifolds via index theory and surgery theory, and there is a well-developed obstruction theory in the non-simply connected case; see the survey articles [Ros07b, RS01]. The classical Lichnerowicz obstruction [Lic63] states that a spin PSC $4k$-manifold has vanishing $\widehat{A}$-genus; in dimension 4 this is equivalent to having vanishing signature.

Witten [Wit94] showed that PSC 4-manifolds with $b_2^+ > 1$ have vanishing Seiberg-Witten invariants. This implies, for instance, that the existence of a PSC metric depends on the underlying smooth structure. In the non-simply connected case, there are further obstructions based on Rokhlin's theorem [RS07] and gauge theory [Lin19, Kon19, KT20, KT23].

(3) It is conjectured that the answer to (a) is positive. This seems wildly optimistic, but there are no known counterexamples. A weaker version would

allow an exotic S^4 with a PSC metric as a summand in the connected sum decomposition. The conjecture was stated as a question in [Kir97, Problem 4.143].

(4) Some standard examples of non-simply connected PSC 4-manifolds are $S^1 \times Y$ for Y a spherical space form, as well as $S^2 \times \Sigma$ and $\mathbb{RP}^2 \times \Sigma$ for any closed surface Σ. Some other constructions are described in the survey [MT21]. The decomposition theorem for PSC 4-manifolds in [BLM23] reproduces some portion of the picture in dimension 3. Problem 4.129 has a discussion of a decomposition question for non-simply connected PSC 4-manifolds, which would reduce the general problem to the classification of PSC 4-dimensional orbifolds with finite orbifold fundamental group.

(5) One standard setting for part (c) requires that the metric be a product near the boundary, in which case the boundary would have positive scalar curvature. There are obstructions to the existence of PSC metrics on bounding 4-manifolds coming from index theory [BG95] and Seiberg-Witten theory. Formulating a good conjecture here would be welcome.

Rosenberg-Weinberger [RW23] discuss a different boundary condition, and conjecture that a manifold has a PSC metric whose boundary has positive mean curvature (with respect to the outward normal) if and only if its double has a PSC metric.

(6) By [Ros07b, Theorem 0.1] every non-compact 4-manifold admits a PSC metric, typically not complete, so one needs to have additional constraints on the geometry at infinity. Using Gromov's notion of a *μ-bubble*, Chodosh-Maximo-Mukherjee [CMM24] show that there are exotic $\mathbb{R}^4$s that do not admit a complete PSC metric with uniformly positive scalar curvature. A particular case of interest, asked by A. Mukherjee, is whether the punctured $K3$ surface admits such a metric.

Scribed by: D. Ruberman

Problem 4.129. *Given a closed, 4-dimensional PSC manifold M, is there a (possibly disconnected) 4-dimensional orbifold M' with isolated singularities such that the following hold.*

(I) The orbifold M' also admits a metric of positive scalar curvature.

(II) All components of M' have finite orbifold-fundamental group.

(III) M can be obtained from M' by a series of 0 and 1 surgeries. Here we also allow 0-surgeries between two orbifold points of the same type, which amounts to a removal of two subsets of the form D^4/Γ and an addition of a copy of $S^3/\Gamma \times [0, 1]$.

Remarks.

(1) Because the orbifold M' has isolated singularities, its orbifold fundamental group is just the fundamental group of its regular part.

(2) The following converse statement is true due to the work of Gromov–Lawson [GL80]. If M' satisfies Property (I) and M can be obtained

from M' as in Property (III), then M admits a PSC metric. So if the answer to the problem is 'yes', then this would reduce the study of PSC 4-manifolds to the study of PSC 4-orbifolds with finite fundamental group. See problem 4.128 for a conjectural picture in the simply connected case.

(3) The examples of non-simply connected closed 4-manifolds admitting a PSC metric described in Question (b) of Problem 4.128 satisfy properties (I)-(III). For example, a bundle over S^1 with fiber a PSC 3-manifold can be obtained from the unreduced suspension of Y by 0-surgery at the two orbifold points. Likewise, $S^2 \times \Sigma_g$ can be obtained from a connected sum of $2g$ copies of $S^1 \times S^3$ by surgery on a circle.

(4) It seems likely that the problem can be approached using 4-dimensional Ricci flow once there is a reasonable construction of Ricci flow with surgery in this dimension. Here the 0 and 1 surgeries would correspond to geometric surgeries that excise cylindrical singularities. Other singularities, for example those modeled on non-cylindrical shrinking solitons, would contribute components to M' with finite fundamental group [**Bam21a**]. Partial progress to the problem was made via minimal surfaces in [**BLM23**], where Property (II) was proved with the weaker conclusion that every component of M' has vanishing first Betti number.

(5) Properties (I)–(III) impose nontrivial topological restrictions on M. For example, they imply that M cannot be aspherical; note, however, that non-asphericity was already shown in [**CL24**]. More generally, Properties (I)–(III) imply that any cover of M must have finite 2-dimensional Urysohn width [**Gro88**] (for the lift of one and thus any Riemannian metric on M). Here we say that a metric space (X, d) has k-dimensional Urysohn width of at most W if there is a continuous map $p : X \to A$ to a k-dimensional simplicial complex such that the diameter of any fiber $p^{-1}(a)$ is at most W. The property that any cover of a has finite 2-dimensional Urysohn width imposes a restriction on homotopy type of the manifold. See also [**CLL23, LM23a, Bol09**] for related results.

Proposed for K3 by: R. Bamler
Scribed by: D. Ruberman

Problem 4.130. *Does longitudinal knot surgery using a knot K, along a fiber in a K3 surface always yield a reducible 4–manifold? A completely decomposable 4–manifold?*

Remarks.

(1) *Longitudinal knot surgery* is a variation of Fintushel-Stern knot surgery [**FS98**] with a different gluing map. The original Fintushel-Stern version is defined using a knot K and a square-0 torus T in a 4-manifold X. Then X_K is defined as

$$(X - T \times D^2) \cup_\varphi (S^1 \times E(K)).$$

Here $E(K)$ is the exterior of the knot, and the gluing map φ sends the longitude of K to the boundary of a meridional disk of T. Under appropriate hypotheses, Fintushel and Stern show that this operation multiplies the Seiberg-Witten invariant of X by the Alexander polynomial of K. In particular, if the Seiberg-Witten invariant of X is nontrivial, the same holds for X_K.

(2) In longitudinal knot surgery, the S^1 factor in $S^1 \times E(K)$ is identified with the boundary of the meridional disk of T. Denote the result by X_K^λ. Taubes shows [**Tau16**] that in contrast to the standard surgery, the Seiberg-Witten invariant of X_K^λ vanishes, even if K is nontrivial. This raises the question of whether X_K^λ splits as a connected sum, or is completely decomposable, i.e. is diffeomorphic to a connected sum $\#^m\mathbb{C}\mathrm{P}^2 \# \#^n\overline{\mathbb{C}\mathrm{P}}^2$. Taubes reports, based on communications with Akbulut, Baykur, and Fintushel, that when K is an unknot, then $(K3)_K^\lambda$ completely decomposes.

(3) One motivation comes from the search (starting with [**Poo86**], albeit with opposite orientation conventions) for 4-manifolds that admit a Riemannian metric whose self-dual Weyl curvature W_+ vanishes. Such metrics are called conformally anti-self-dual. Taubes [**Tau16**] shows that when K is hyperbolic, X is a $K3$ surface, and T is a fiber in an elliptic fibration coming from the Kummer construction, the manifold X_K^λ admits a Riemannian metric with W_+ arbitrarily small. After repeated blowing up by connected sum with $\overline{\mathbb{C}\mathrm{P}}^2$, it will have [**Tau92**] a conformally anti-self-dual metric; for 2–bridge knots, three blowups suffice. Hence it would be of interest to identify the manifold $(K3)_K^\lambda$. If K is hyperbolic and $(K3)_K^\lambda$ is completely decomposable, then $\#^3\mathbb{C}\mathrm{P}^2 \#^{19} \overline{\mathbb{C}\mathrm{P}}^2$ would admit a conformally anti-self-dual metric. Such metrics are known on $\#^3\mathbb{C}\mathrm{P}^2\#^N\overline{\mathbb{C}\mathrm{P}}^2$ when $N \geqslant 30$ by work of LeBrun [**LeB95**] and Rollin-Singer [**RS09**].

Proposed for K3 by: C. Taubes
Scribed by: D. Ruberman

PROBLEM 4.131. *Does every Lipschitz 4-manifold admit a smooth structure? Is this smooth structure unique if so?*

Some more specific, related questions are as follows.

(a) *Is there a topological, spin, closed, indefinite* 4*-manifold* X *that admits a Lipschitz structure and violates the 10/8-inequality*

$$b_2(X) \geqslant \frac{5}{4}|\sigma(X)| + 2$$

by Furuta [**Fur01**]*, or the "*$10/8 + 4$*" inequality*

$$b_2(X) \geqslant \frac{5}{4}|\sigma(X)| + 4$$

by Hopkins–Lin–Shi–Xu [**HLSX22**] *for* $X \neq S^4, S^2 \times S^2, K3$.

(b) *Let* $X \subset \mathbb{R}^4$ *be a Lipschitz embedded 4-manifold with boundary* $\partial X \simeq S^3$*. Does* X *admit a unique smooth structure?*

REMARKS.

(1) In dimension $n \neq 4$, every topological n-manifold X admits a Lipschitz structure by a theorem of Sullivan [**Sul79b**]. On the other hand, Donaldson and Sullivan [**DS89**, Theorem 2] showed that there exist closed topological 4-manifolds X that admit more than one inequivalent Lipschitz structure. Donaldson and Sullivan established that the simplest numerical invariants of smooth 4-manifolds (due to Kotschick [**Kot89**]) are quasiconformal invariants of smooth 4-manifolds.

(2) The inequalities in Problem (a) are proved for smooth 4-manifolds using the variation of Seiberg–Witten theory introduced in [**Fur01**]. Hence a Lipschitz manifold violating either of those inequalities would suggest that there is no extension of this version (or perhaps other versions) of Seiberg–Witten theory to the setting of Lipschitz 4-manifolds. One reason to suspect that there is no such extension is that the Seiberg–Witten equations on X are defined in terms of the Dirac operator associated to a Spin^c structure on X. Sullivan has conjectured [**Sul99, Sul95**] that the existence of a Dirac operator on X (as part of a full 'Dirac package') implies that X is in fact smoothable.

(3) The 4-dimensional Schoenflies conjecture (Problem 4.23) is known [**Sul79b**] to hold for Lipschitz embeddings $\phi\colon S^3 \to \mathbb{R}^4$. Thus a positive solution to Problem (b) would imply the Schoenflies conjecture.

Proposed for K3 by: J. Chaidez, D. Gabai, H. Konno
Scribed by: J. Chaidez

CHAPTER 5

Miscellany

Chapter 5 Editors: Robion Kirby and Daniel Ruberman

This short last chapter contains problems about manifolds of any dimension that interact with low-dimensional topology and geometry. We have not sought out problems on complex surfaces, although some interesting problems in this area, described in the corresponding chapter of [**Kir97**], remain unsolved.

Section 5.1 begins with two problems from the days of "analysis situs", one on the fixed point property of cellular sets and one on convergence of homeomorphisms. This latter problem is connected to the Hilbert–Smith problem and the question of whether homeomorphism groups are ANRs. We then continue in this topological vein with questions about higher dimensional pairwise connected sums, branched covers, aspherical 5-manifolds, and the classic Montgomery-Yang problem about exotic circle actions on the 5-sphere.

These are followed in Section 5.2 by three hallowed conjectures of Andrews–Curtis, Whitehead and Zeeman from [**Kir97**] (see also [**HAMS93, HHAMR20**]) relating to 2- and 3-complexes and their elementary expansions and collapses. It concludes with a problem about triangulability of aspherical 5-manifolds, and a wide-ranging problem investigating differential topology through the lens of singularity theory.

Section 5.3 contains problems of a more geometric character. This includes two questions asking about the extension to higher dimensions of known results about hyperbolic 3-manifolds: the existence of one-cusped manifolds, and whether Thurston's 'virtual' conjectures hold in higher dimensions. This section concludes with the 4 and 5-dimensional versions of the Milnor conjecture about the fundamental group of a complete Riemannian manifolds of non-negative Ricci curvature.

The final section, 5.4 has three questions related to symplectic and contact geometry: the first about the differential graded algebras that appear in contact geometry, followed by a problem about homotopies of Weinstein structures. The final problem asks about higher-dimensional analogues of lattice cohomology [**Ném05, Ném08**], currently defined for certain 3-manifolds.

5.1. Topology problems

In this section, we consider a number of problems relating to the basic topology of manifolds and their continuous automorphisms. The Hilbert–Smith Conjecture, with roots going back to Hilbert's Fifth Problem, is perhaps the most famous of these. It seeks to characterize the (locally compact) topological groups acting effectively on a topological manifold, and is related to other fundamental problems about homeomorphism groups.

Other basic problems have to do with the well-definedness of pairwise connected sum in the topological category, and the question (inspired by old work of Alexander) of whether every n-manifold is an n-fold branched cover of the n-sphere. We also discuss the Montgomery–Yang problem about circle actions on the 5-sphere, which was the inspiration for much early work on the homology cobordism classification of Brieskorn homology spheres, and address the last case for finding aspherical nontriangulable manifolds. Finally, we pose several questions about smooth maps between smooth manifolds where the maps have singularities with various restrictions on their types.

PROBLEM 5.1. *Does every cellular set in the plane have the fixed point property?*

REMARKS.

(1) A space has the fixed point property (FPP) if every self-map has a fixed point. A set in an n-manifold is cellular if it is the intersection of a countable sequence of embedded closed n-cells, each contained in the interior of the preceding one. A subset of the plane is cellular if and only if it is non-empty, compact, connected and non-separating (meaning that its complement is connected). Since a (planar) cellular set is the intersection of cells, and since the FPP holds for cells, it is natural to ask if it also has the FPP.

(2) The Mandelbrot set [**BM81**, **Man80**] in the plane is cellular, but it is unknown whether it has the FPP.

(3) The answer in dimensions$\geqslant 3$ is no; a counterexample is due to Kinoshita [**Kin53**]. See the expository article by Bing [**Bin69**].

Proposed for K3 by: R. Edwards
Scribed by: R. Kirby, D. Ruberman

PROBLEM 5.2 (Doubly-Small Morphisms of Manifolds).

(a) Suppose that $h\colon \mathbb{R}^n \to \mathbb{R}^n$ is a homeomorphism (or diffeomorphism) which satisfies two smallness hypotheses:

(i) every orbit of h (of any $x \in \mathbb{R}^n$, under all powers of h) is uniformly bounded in diameter (by 1 say), and

(ii) some subsequence of powers of h converges to $\mathrm{Id}_{\mathbb{R}^n}$ in (say) the compact-open topology.

Then must h be the identity map?

(b) A special case of this question is: let h be a homeomorphism of B^n that is the identity on ∂B^n. If there is a subsequence of powers of h which converge to the identity, must h itself be the identity?

REMARKS.

(1) The case $n = 1$ is trivial, $n = 2$ seems likely to be true, as a consequence of results of Brouwer and Cartwright-Littlewood (nicely and succinctly reproved in [**Bro84**] and [**Bro77**]), and $n \geqslant 3$ is open.

(2) The question is meant to be local in nature, i.e. the question can be adapted to any open subset of $\mathbb{R}^n$. It also applies to any manifold, where in Condition (i) one would assume that every orbit of h has diameter less than some ϵ in the compact-open topology.
(3) The answer is 'yes' if h is periodic. This is Newman's Theorem, with an excellent exposition in [**Dre69**].
(4) For h a diffeomorphism, and using C^∞ convergence, the answer seems likely to be yes.
(5) Since a homeomorphism $h\colon \mathbb{R}^n \to \mathbb{R}^n$ generates a homomorphism $\phi\colon \mathbb{Z} \to \mathrm{Homeo}(\mathbb{R}^n)$ (and vice-versa) by $m \mapsto h^m$, the Question can be rephrased in terms of such a ϕ. Condition (i) becomes: Assume that image(ϕ) lies in a suitably small neighborhood of $\mathrm{Id}_{\mathbb{R}^n}$, and Condition (ii) becomes: Assume that ϕ accumulates at $\mathrm{Id}_{\mathbb{R}^n}$. And the Question becomes: Must ϕ be the trivial homomorphism?
(6) The question is 'stronger' than the Hilbert–Smith Conjecture, discussed in Problem 5.3 below. That is, an affirmative answer to it would imply the Hilbert–Smith Conjecture. The Hilbert–Smith Conjecture (in its Question form) is equivalent to the Question above if in addition one assumes that the closure of the union of the powers of h in $\mathrm{Homeo}(\mathbb{R}^n)$ is compact.

Proposed for K3 by: R. Edwards
Scribed by: R. Edwards, R. Kirby, D. Ruberman

PROBLEM 5.3 (Hilbert–Smith Conjecture).

(a) The Hilbert–Smith Conjecture [**Smi41**] *asserts that a locally compact subgroup of the homeomorphism group of a connected manifold must be a Lie group.*
(b) CONJECTURE: *The free-action subset of any Cantor group action on an ENR is a homology-Z-subset (of the ENR).*

REMARKS.

(1) We first define the terms in Conjecture (b).
  (i) A *Cantor group* is a topological group G whose underlying space is a Cantor set (space). That is, G is a profinite group that is non-finite and 2^{nd} countable (hence metrizable). A universal example of such a group is the direct product of all (of the countably many) finite groups (or, if you wish, the finite symmetry groups). Other important examples of Cantor groups are the p-adic integers.
  (ii) An *ENR* is a Euclidean Neighborhood Retract, that is, a subset of some $\mathbb{R}^n$ that has a neighborhood that retracts onto it. Such subsets are characterized as stably having manifold mapping cylinder neighborhoods (like tubular neighborhoods for manifolds, or regular neighborhoods for polyhedra).
  (iii) A subset A of X is a *homology-Z-set* (in X) if for any $a \in A$ and any open neighborhood U of a in X, the relative homology $H_*(U, U - A)$ is 0. If X is a manifold, the only such A are subsets of ∂X.
(2) Pardon [**Par13a**] proved the Hilbert–Smith Conjecture for dimension 3. The conjecture has been reduced to whether the locally compact subgroup

in question can be the p-adic integers. See Pardon's papers [Par13a, Par19] for more discussion and additional references.

(3) Conjecture (b) is known as the *Free-Set (is a) Z-Set Conjecture* (FSZSC) and is due to R. Edwards. It is a natural and stronger version of the Hilbert–Smith Conjecture. As a special case, the FSZSC asserts that a Cantor group cannot act freely on an ENR. Like the HSC, the FSZSC has been reduced to the case of proving it for the p-adic integers.

(4) The Hilbert–Smith Conjecture is closely related to the preceding problem 5.2, as discussed above.

Proposed for K3 by: R. Edwards
Scribed by: R. Edwards, R. Kirby, D. Ruberman

PROBLEM 5.4. *Is the homeomorphism group of a manifold an absolute neighborhood retract (ANR)?*

REMARKS.

(1) This is [Kir97, Problem 5.27] and is also listed in [Wes90]. In dimension 2 it was shown to hold by Mason [Mas71] and Luke-Mason [LM72]; it is unknown for manifolds of dimension greater than 2.

(2) It is not easy to distinguish between ANRs and more general locally contractible spaces, so it is worth recalling that the homeomorphism group of a manifold is locally contractible [EK71, Čer69].

Proposed for K3 by: R. Edwards
Scribed by: R. Kirby, D. Ruberman

PROBLEM 5.5. *Is the connected sum of (locally flat) pairs*

$$(M_1^{n+k}, N_1^n)\#(M_2^{n+k}, N_2^n)$$

well-defined in the topological category?

REMARKS. When $k = 1$, this is easily true from Brown's paper showing that locally flat codimension one submanifolds are flat [Bro62]. According to [Liv24], it is well-defined in any dimension when $k = 2$. This uses the uniqueness of normal bundles in codimension 2, which fails in higher codimensions; see [Liv24, Appendix C]. However this question appears to be open for codimension > 2 and $n + k > 4$. It would follow from a pairwise version of the 'torus trick' [KS77] if a pairwise version of Wall's non-simply connected surgery [Wal99] were known and in print.

Proposed for K3 and scribed by: C. Livingston

PROBLEM 5.6.

(a) *Does every closed, PL, orientable n-manifold admit an n-fold branched covering map over S^n?*

Assuming that the answer to this problem is "yes", the following follow-up questions would be natural to ask.

(b) *Does every n-manifold admit an n-fold branched covering over S^n with branch locus a codimension-2 embedded submanifold?*
(c) *If the answer to (b) is "yes", then one could naturally ask if the cover can additionally be taken to be* simple*, meaning the covering monodromy sends every meridian of the embedded submanifold to a transposition. For $n = 2, 3, 4$, the above papers show the answer is, "yes".*
(d) *If the answer to (b) is "yes", then in a separate direction one could ask when the branch locus can be taken to be orientable.*

Remarks.

(1) A classical theorem of Alexander [**Ale20**] says that every n-manifold admits a branched covering map over S^n with no restriction on degree of the covering. An easier version of this problem would be to show that for every n, there exists some natural number m_n such that every n-manifold admits an m_n-fold branched covering over S^n. Certainly m_n cannot be smaller than n: for example, because the n-torus T^n has reduced cohomology ring of length n, any branched covering $f : T^n \to S^n$ has degree at least n [**BE78**].
(2) It is well-known that every orientable surface is a 2-fold branched cover over S^2. Hilden [**Hil74**], Hirsch [**Hir74**], and Montesinos [**Mon74**] showed that every 3-manifold is a 3-fold branched cover over S^3. Piergallini [**Pie95**] showed that every PL 4-manifold is a 4-fold branched cover over S^4. The answer to this question is unknown in all higher dimensions.
(3) Note by Berstein–Edmonds [**BE78**] the answer to Question (b) is "no" if we additionally require that this submanifold be locally flat. For $n = 2, 3$ the answer is known to be "yes," but for $n = 4$, the best known result to date, by Piergallini in [**Pie95**], produces branched loci that are immersed surfaces. Iori–Piergallini [**IP02**] later showed that every 4-manifold admits a 5-fold simple branched covering over S^4 with branch locus an embedded surface. Whether every 4-manifold admits a 4-fold cover over S^4 with embedded branch locus remains open.
(4) The answer to Question (d) is generally negative – for example, when $n = 4k$ the existence of such a covering implies the n-manifold has signature zero [**Vir84**]. For this question, it may be simpler to restrict to the case that the n-manifold is null-cobordant.

Scribed by: M. Miller

Problem 5.7 (Montgomery–Yang problem). *Does there exist a pseudo-free, smooth, S^1 action on S^5 with more than three multiple orbits?*

Remarks.

(1) This is Problem 4.123 in [Kir97]; see the surveys by Kollár [Kol08] and Şavk [Şav24]. Recall that an S^1 action is pseudo-free if there are no fixed points and the orbits of finite isotropy are isolated.
(2) There is a good understanding of the situation for other odd dimensional spheres: S^1 actions on S^3 are linear by Seifert [Sei33], whereas every homotopy $2k-1$ sphere (for $k \geqslant 4$) admits pseudo-free S^1 actions with arbitrarily many multiple orbits by Montgomery-Yang [MY72] (for $k=4$) and Petrie [Pet75] (for $k>4$).
(3) The answer is positive if a Seifert fibered homology 3–sphere Σ with more than 3 multiple fibers bounds an acyclic 4-manifold W, where the induced homomorphism by the inclusion of boundary is surjective on the π_1. Then $\Sigma \times D^2 \cup W \times S^1 = S^5$ and S^1 acts diagonally on $\Sigma \times B^2$ and trivially on W. See also Problem 4.58.

Proposed for K3 by: R. Fintushel, R. Stern
Scribed by: I. Baykur

Problem 5.8. *Is there a closed aspherical* 5*-manifold that is not triangulable?*

Remarks.

(1) In this context, 'triangulable' means homeomorphic to a simplicial complex. Davis and Januszkiewicz [DJ91] constructed aspherical non-triangulable 4-manifolds, by applying the process of 'hyperbolization' to Freedman's E_8 manifold. Subsequent to Manolescu's (negative) solution to the triangulation conjecture [Man16b] in dimensions $\geqslant 5$, Davis-Fowler-Lafont [DFL14] used a hyperbolization procedure to construct aspherical non-triangulable n-manifolds for all $n \geqslant 6$. They explain why their procedure breaks down in dimension 5, and explicitly asked [DFL14, §3] about the 5-dimensional case.
(2) It follows from [Sie70] (using the double suspension theorem [Edw06, Can79]; compare [GS80, Mat78]) that any orientable 5-manifold is triangulable. Hence any aspherical non-triangulable 5-manifold would have a triangulable cover. This suggests the following.

Question. *Are there examples of non-triangulable aspherical 4-manifolds in dimension 4 that are virtually triangulable, i.e., that admit a finite cover that can be triangulated?*

The examples of [DJ91, DFL14] have residually finite fundamental groups, so they would be a good place to start.

Scribed by: D. Ruberman

Problem 5.9. *Let M_1 and M_2 be smooth manifolds of dimension n. Suppose M_1 admits an $\mathcal{S}$-map into $\mathbb{R}^p$. If M_2 is homeomorphic to M_1, then does M_2 admit an $\mathcal{S}$-map as well?*

REMARKS.

(1) For this problem, we consider C^∞ maps $f\colon M \to \mathbb{R}^p$ of smooth n–dimensional manifolds into $\mathbb{R}^p$ with $n \geqslant p \geqslant 1$. Let $\mathcal{S}$ be a set of (equivalence classes of) singularities of smooth map germs $(\mathbb{R}^n, 0) \to (\mathbb{R}^p, 0)$. A smooth map $f\colon M \to \mathbb{R}^p$ is called an *$\mathcal{S}$-map* if all its singularities are equivalent to a singularity belonging to $\mathcal{S}$.

(2) If M_2 is diffeomorphic to M_1, then by composing a diffeomorphism $M_2 \to M_1$ and an $\mathcal{S}$-map $M_1 \to \mathbb{R}^p$, we get an $\mathcal{S}$-map on M_2. Therefore, if the answer to the above question is negative, then M_2 is not diffeomorphic to M_1; in other words, (M_1, M_2) is an exotic pair of manifolds.

In this sense, for $n \leqslant 3$, the answer is always affirmative.

(3) Let $\mathcal{S}$ be the singleton consisting of the definite fold singularity represented by the map germ

$$(x_1, x_2, \ldots, x_{p-1}, x_p^2 + x_{p+1}^2 + \cdots + x_n^2).$$

In this case, an $\mathcal{S}$-map is called a *special generic map*. Then there are many examples of manifold pairs (M_1, M_2) for which the answer is negative as follows.

(i) For $n \geqslant 7$ and $p = n-1, n-2$ and $n-3$, the pair of the standard n–sphere and an exotic n–sphere is such an example [**Sae93b**].

(ii) For $n = 4$ and $p = 1, 2$ and 3, the pair of the standard $\mathbb{R}^4$ and an exotic $\mathbb{R}^4$ is such an example, provided that we consider *proper* special generic maps [**Sae10**].

(iii) For $n = 4$ and $p = 3$, there are quite a few such examples (M_1, M_2), where M_1 is a connected sum of $S^2 \times S^2$ and $\mathbb{C}\mathrm{P}^2 \sharp \overline{\mathbb{C}\mathrm{P}}^2$ [**Sae93a, SS99**].

(4) According to the solution to the Poincaré Conjecture in high dimensions due to Smale [**Sma61**], for special generic functions with $p = 1$, the answer is affirmative for $n \geqslant 5$. In other words, only by the existence of a special generic function with $n \geqslant 5$ and $p = 1$ one cannot detect exotic differentiable structures. Note also that for $n = 4$ and $p = 1$, the problem for special generic functions is equivalent to the 4-dimensional smooth Poincaré Conjecture.

On the other hand, for $p = 1$, let $\mathcal{S}$ be the set of non-degenerate critical points of indices in the set $\{0, 2, 3, \ldots, n-2, n\}$. Then, for $n \geqslant 5$, by Smale's result [**Sma62a**], the answer to the problem is always affirmative, whereas for $n = 4$, the problem seems to be still open.

(5) Let us consider C^∞ stable maps of closed 4–manifolds into $\mathbb{R}^4$. They have, in general, fold, cusp, swallowtail, butterfly and umbilic singularities. It is known that when the 4–manifold is oriented, each umbilic singularity can be given a sign $+1$ or -1 and the total number of umbilic points counted with signs is equal to 3 times the signature. Furthermore, if the signature vanishes, then all the umbilic singularities can be eliminated by homotopy [**And82, Sti95**]. Hence, for $\mathcal{S}$ consisting of fold, cusp, swallowtail and butterfly singularities, the answer to the above question is affirmative in this case.

(6) Let F be a set of (equivalence classes) of singular fibers of C^∞ maps in the following sense [**Sae04**]. Let $f_i : M_i \to N_i$ be smooth maps, $i = 0, 1$. For

$y_i \in N_i$, we say that the fibers over y_0 and y_1 are equivalent if for some open neighborhoods U_i of y_i there exist diffeomorphisms $\tilde{\varphi}\colon f_0^{-1}(U_0) \to f_1^{-1}(U_1)$ and $\varphi\colon U_0 \to U_1$ with $\varphi(y_0) = y_1$, which make the following diagram commutative:

$$\begin{array}{ccc} (f_0^{-1}(U_0), f_0^{-1}(y_0)) & \xrightarrow{\tilde{\varphi}} & (f_1^{-1}(U_1), f_1^{-1}(y_1)) \\ {\scriptstyle f_0}\downarrow & & \downarrow{\scriptstyle f_1} \\ (U_0, y_0) & \xrightarrow[\varphi]{} & (U_1, y_1) \end{array}$$

When the fibers over y_0 and y_1 are equivalent, we also say that for $i = 0, 1$, the map germs $f_i : \big(M_i, f_i^{-1}(y_i)\big) \to (N_i, y_i)$ are right-left equivalent. A smooth map $f : M \to \mathbb{R}^p$ is called an F-map if all its singular fibers are equivalent to a singular fiber belonging to F. Then, we can ask the same question as Problem 5.9 for F-maps.

(7) It is known that for maps of smooth closed oriented 4–manifolds into $\mathbb{R}^3$, each singular fiber of type III^8 can be given a sign $+1$ or -1, and for a certain class of generic maps (so-called C^∞ stable maps), the number of III^8–fibers counted with signs coincides with the signature of the source 4–manifold [**SY06**]. Therefore, it is a natural question if singular fibers of type III^8 with opposite signs can be eliminated by homotopy. So far, we do not know if there exists a smooth closed oriented 4–manifold M with zero signature such that an arbitrary C^∞ stable map $M \to \mathbb{R}^3$ necessarily has a singular fiber of type III^8. Though for connected sums of copies of $S^2 \times S^2$ and $\mathbb{C}\mathrm{P}^2 \sharp \overline{\mathbb{C}\mathrm{P}}^2$, the answer is known to be affirmative.

(8) In the problem, one can also consider the problem by replacing "homeomorphic to" by "homotopy equivalent to". For example, for $n \geqslant p \geqslant 1$, let $\mathcal{S}$ be the set of fold singularities of all (absolute) indices. Then, the answer to the homotopy version of the problem is affirmative for closed orientable 4–manifolds and $1 \leqslant p \leqslant 4$ [**Sae03**, **Sad04**].

Proposed for K3 and scribed by: O. Saeki

5.2. Andrews–Curtis, Whitehead, and Zeeman Conjectures

The conjectures and questions discussed in this section touch on the interaction between group theory and the topology of low-dimensional cell complexes. The Andrews–Curtis Conjecture posits that a certain collection of algebraic operations suffices for passing between finite presentations of the trivial group; it has intriguing relations to the 4-dimensional Poincaré Conjecture. Whitehead's Asphericity Question is disarmingly simple, and asks if subcomplexes of an aspherical 2-complex are always aspherical. A positive answer would imply, among other things, that ribbon disk complements are aspherical. Zeeman's Conjecture posits that a (finite) contractible 2-complex '3-collapses', i.e., collapses to a point after taking the product with the interval. It is closely connected to the Andrews–Curtis conjecture, and to older approaches to the Poincaré Conjecture.

PROBLEM 5.10. *The Andrews–Curtis Conjecture* [AC65] *for the trivial group: a presentation of the trivial group can be changed to the trivial presentation by Andrews-Curtis moves.*

REMARKS.

(1) This problem appears in Problem 5.2 in [Kir97].
(2) Let $\mathcal{P}$ be a finite presentation of a given group π. The stabilized - Andrews–Curtis moves (abbreviated AC moves) change the presentation $\mathcal{P} = \{x_1, \dots, x_n : R_1, \dots, R_m\}$ as follows:
   (i) $R_i \to R_i^{-1}$,
   (ii) $R_i \to R_i R_j, i \neq j$
   (iii) $R_i \to w R_i w^{-1}, w$ any word,
   (iv) add generator x_{n+1} and relation $w x_{n+1}$.
   The AC Conjecture for the trivial group states that the presentation $\mathcal{P}$ can be changed to the trivial presentation $(x_1, \dots, x_n : x_1, \dots, x_n)$ by AC moves.

   Note that redundant relations cannot be added, so that $m - n$ is unchanged. Also note that the broader conjecture that any two presentations of an arbitrary finitely presented group are equivalent by AC moves is false for some nontrivial groups, e.g., the trefoil group [HAMS93]. (See also [AC66].)
(3) Given a presentation $\mathcal{P}$ of the trivial group, we can construct a 5-dimensional handlebody Y from a 0-handle, 1-handles for each generator, and 2-handles for each relation; Y is unique because the attaching maps are isotopic because they are homotopic.

   Then if the AC Conjecture is true, Y is diffeomorphic to the 5-ball, because the AC moves correspond to handle moves (in particular the move $R_i \to R_i R_j, i \neq j$ corresponds to sliding the i^{th} handle over the j^{th} handle).

Scribed by: R. Kirby

PROBLEM 5.11 (Whitehead's Asphericity Question). *Is every subcomplex of an aspherical 2-complex aspherical?*

REMARKS.

(1) This was problem 5.4 in [Kir97]; a thorough discussion of older progress on this problem, also known as the Whitehead Conjecture, may be found in [HAMS93, Chapter X]. It was stated as a question on page 428 in [Whi41], with the implicit assumption that the complex is finite, but the question still makes sense without that assumption.
(2) The fundamental group of a finite-dimensional aspherical complex is torsion free, so an a priori easier question is whether the fundamental group of a subcomplex of an aspherical two-complex is necessarily torsion-free. For partial results relating to fundamental group structure, see [Coc54, Ada55, How79].

(3) Bestvina and Brady [BB97] showed that the Whitehead conjecture and the Eilenberg–Ganea conjecture cannot both be true. The Eilenberg–Ganea conjecture [EG57] is that a group with cohomological dimension 2 has a 2-dimensional Eilenberg-Mac Lane space.

More concretely, Bestvina and Brady showed that the (Bestvina–Brady) group H_L associated with a flag triangulation L of a spine of the Poincaré homology sphere is either a counterexample to the Eilenberg–Ganea conjecture, or there exists a contractible 2-complex Y that contains a non-aspherical subcomplex [BB97, Theorem 8.7]. Note that the potential counterexample Y to the Whitehead conjecture that they construct is infinite (since the group H_L acts freely and cellularly on it).

(4) In [How79], J. Howie reduced the problem to finding counterexamples $K \subset L$ of two types: (i) L is finite and contractible and $K = L - e$ for some 2-cell e; or (ii) L is the union of an infinite chain of non-aspherical subcomplexes $K = K_0 \subset K_1 \subset \dots$ such that each inclusion is null-homotopic.

According to Howie's results in [How79] and [How83], if the Andrews–Curtis conjecture (Problem 5.10) holds, then the standard 2-complexes associated with LOT presentations account for all test cases of type (i). Recall that a version of the Andrews–Curtis conjecture asserts that every finite contractible 2-complex can be 3-deformed to a point. A LOT presentation is a group presentation described by a (finite) labeled oriented tree, and the associated 2-complex has the homotopy type of a ribbon disc complement [How83].

It remains unknown whether all LOT complexes are aspherical, though significant progress has been made in the last decades, establishing the asphericity of various subfamilies. For instance, Harlander and Rosebrock [HR17] showed that alternating ribbon disk-complements, the ones that can be encoded by injective labeled oriented trees, are aspherical.

(5) Costa and Farber [CF17] give a model for random simplicial complexes in which aspherical 2-complexes satisfy the Whitehead conjecture with probability 1.

Scribed by: R. Kirby

Problem 5.12 (Zeeman Conjecture). *If K is a finite contractible 2-complex, then $K \times I$ collapses to a point* [Zee64, Conjecture (1)].

Remarks. This problem appears as part of Problem 5.2 in [Kir97].

The Zeeman Conjecture for a special polyhedron that is the spine of a compact 3-manifold is equivalent [GR83] to the Poincaré Conjecture, and thus true by Perelman. The Zeeman conjecture for special polyhedra that do not embed in compact 3-manifolds is equivalent to the Andrews–Curtis Conjecture [Mat87].- A nice discussion of this general area may be found in Chapters I, XI and XII of [HAMS93] and the exposition in [Kup21].

Scribed by: R. Kirby

5.3. Geometry

The first two problems in this section are concerned with hyperbolic manifolds of dimension greater than 3. Thurston's 'virtual' conjectures (see [**Ago08**] and Problem 3.29) asking about properties of finite covers of hyperbolic 3-manifolds, have been highly influential in the study of 3-manifolds in recent years, and it is natural to ask the corresponding questions for hyperbolic manifolds of any dimensions. Similarly, there are many explicit examples of one-cusped hyperbolic 3-manifolds, and we ask for such examples in higher dimensions.

Milnor's conjecture, discussed in the next problem, proposed that the fundamental group of a manifold admitting a complete Riemannian metric of nonnegative Ricci curvature is finitely generated. It holds for 2- and 3-manifolds, and there are counterexamples in every dimension starting with 6. So we ask about the status of this conjecture in the remaining dimensions 4 and 5.

PROBLEM 5.13. *Let M be a finite-volume hyperbolic n-manifold.*

(a) Does it always have a finite cover with $b_1 > 0$?
(b) Does it always have a finite cover with fundamental group that surjects onto a non-abelian free group?
(c) Does it have a finite cover with cubulated fundamental group?
(d) When n is odd, does it always have a finite cover that fibers over the circle?

REMARKS.

(1) When $n = 3$, these questions all have positive answers, by work of Agol [**Ago13**] and Wise [**Wis21**]. However, their methods break down when $n > 3$. The crucial steps in their argument are as follows. By work of Kahn-Markovic [**KM12**], $\pi_1(M^3)$ contains lots of surface subgroups. These are 'codimension 1' subgroups and hence $\pi_1(M^3)$ has a finite index subgroup that is the fundamental group of a compact non-positively curved cube complex, i.e., it is *cubulated*. Agol showed that such cubulated groups are virtually special, and Wise showed that virtually special groups have many excellent virtual properties. In particular, they have a finite index subgroup with $b_1 > 0$ and indeed a finite index subgroup that surjects onto a non-abelian free group. Using a 3-dimensional argument involving sutured manifolds, Agol [**Ago08**] was able to show that hyperbolic 3-manifolds virtually fiber. An alternative and more general argument using group rings was given by Kielak [**Kie20**].

(2) This argument fails at the first step when $n > 3$. The methods of Kahn-Markovic have been extended to all odd dimensions by Hamenstädt [**Ham15**], and hence $\pi_1(M)$ is known to contain many surface subgroups. However, these groups are not codimension one, and therefore do not establish that the group is cubulated. However, it is known that some hyperbolic n-manifolds are cubulated when $n > 3$. Indeed, any arithmetic hyperbolic n-manifold containing a totally geodesic $(n-1)$-dimensional (possibly immersed) submanifold is cubulated. When n is even, this includes all arithmetic hyperbolic n-manifolds.

(3) There are further results for arithmetic hyperbolic n-manifolds. There are 3 types of arithmetic lattices in $\mathrm{SO}(n,1)$: those arising from quadratic forms (type I), those arising from quaternion algebras (type II), and those arising from octonion algebras, and type III - trialitairian lattices. The first type appears in all dimensions and are known to be cubulated after work of Bergeron–Wise [**BW12**]. The second type are not known to be cubulated, but only appear in even dimensions. The third type only occurs in dimension 7; for these it is known [**BC17a**] that the first Betti number of every congruence subgroup equals 0. Moreover, it is still open if the lattices have the congruence subgroup property.

(4) If a manifold fibers over the circles, its Euler characteristic is zero. In even dimensions, the volume of a hyperbolic manifold is proportional to its Euler characteristic, and hence its Euler characteristic is necessarily non-zero. This explains the restriction to n odd in the final question above. The first examples of hyperbolic 5-manifolds that fiber over the circle were given by Italiano-Martelli-Migliorini [**IMM23**].

(5) When a hyperbolic 3-manifold fibers over the circle, the universal cover $\widetilde{F}$ of the fiber has a circle at infinity, and the inclusion $\widetilde{F} \to \mathbb{H}^3$ is known to extend continuously to a map from the circle at infinity of $\widetilde{F}$ to the sphere at infinity of $\mathbb{H}^3$. This forms a space-filling curve. This was established by Cannon and Thurston [**CT07**]. It would be interesting to know whether there is any analogue of this in higher dimensions. The fiber will not in general have word hyperbolic fundamental group, and so part of the challenge would be to define its space at infinity appropriately.

(6) By work of Delzant and Gromov [**DG05**], complex hyperbolic manifolds do *not* have cubulated fundamental group.

Proposed for K3 and scribed by: M. Lackenby

Problem 5.14. *Does there exist a 1-cusped finite-volume hyperbolic n-manifold for any $n \geqslant 5$?*

Remarks.

(1) Hyperbolic n-manifolds with one cusp are abundant when $n = 2$ or 3, and Kolpakov–Martelli [**KM13a**] constructed 1-cusped hyperbolic 4-manifolds. However, there are far fewer constructions of hyperbolic n-manifolds when $n \geqslant 4$ and they tend to produce manifolds with a large number of cusps. There are no known examples in dimensions $\geqslant 5$ with a single cusp; the question of their existence was raised in [**LR02**].

(2) Stover [**Sto13**] proved that there are no 1-cusped arithmetic hyperbolic n-dimensional orbifolds when $n > 30$. In fact, he showed that for each $m \geqslant 1$, there is a $c_m \geqslant 1$ such that there are no m-cusped arithmetic hyperbolic n-dimensional orbifolds when $n > c_m$.

Proposed for K3 by: A. Reid
Scribed by: M. Lackenby

PROBLEM 5.15. *Suppose M is a manifold with a complete Riemannian metric with nonnegative Ricci curvature. Is the fundamental group of M finitely generated?*

REMARKS.

(1) This is a conjecture of Milnor, following on his paper [Mil68a]. The conjecture was known to hold in dimension 2 by work of Cohn-Vossen [CV35] and in dimension 3 by work of Liu [Liu13]. Bruè-Naber-Semola construct counterexamples in dimensions at least 7 [BNS25], and 6 [BNS23]. The issue is therefore to determine the status of the conjecture in dimensions 4 and 5. This question is explicitly posed in [BNS23, Question 1.1], along with some other interesting open problems in the area.

    Previous work of Wilking [Wil00] shows that the fundamental group of any counterexample could be assumed to be abelian; the fundamental group of the example from [BNS25] in dimension 7 is $\mathbb{Q}/\mathbb{Z}$.

(2) A crucial point in the Bruè-Naber-Semola paper [BNS25, Lemma 9.1] is that the orbit of the mapping class group of $S^3 \times S^3$ acting on the standard product metric lies in a single path component of the space of positive Ricci curvature metrics on $S^3 \times S^3$. This raises several questions.

    (i) What can be said about the action of the mapping class group of $S^2 \times S^2$ on the path components of the space of positive Ricci curvature metrics on $S^2 \times S^2$?
    (ii) Is the space of positive Ricci curvature metrics on $S^2 \times S^2$ path connected?
    (iii) One could ask the same questions for $S^2 \times S^3$; see [BNS23, §6] for a particular diffeomorphism of $S^2 \times S^3$ such that the pullback of the standard metric is connected to the standard metric through Ricci curvature metrics.

Proposed for K3 and scribed by: D. Ruberman

5.4. Symplectic and contact topology

These last three problems deal with aspects of symplectic and contact topology in dimensions greater than 4 (respectively, 3).

PROBLEM 5.16. *Let R be $\mathbb{Z}$ or a field. Let A and B be differential graded algebras so that either:*

- *As a graded algebra, A (respectively B) is isomorphic to the free, non-commutative R-algebra on finitely many homogeneous generators x_i, i.e., to the tensor algebra on the free R-module generated by $x_1, \ldots, x_n$.*
- *As a graded algebra, A (respectively B) is isomorphic to a free graded-commutative algebra on finitely many homogeneous generators x_i. (So, if the x_i have even gradings, A is a polynomial algebra.)*

(*Different generators can have different gradings, A and B may have different numbers of generators, and gradings of generators may be negative.*)

Are the following questions decidable?

(a) *Is A stable tame isomorphic to B?*
(b) *Is A quasi-isomorphic to B?*
(c) *Is A derived Morita equivalent to B?*

Remarks.

(1) This question is inspired by contact topology, where many invariants take the form of a finitely-generated differential graded algebras up to a notion of equivalence.
(2) The kinds of dgas described in the problem are often called *semifree* in the literature. The information in a semifree dga A consists of the grading of the generators $x_1, \ldots, x_n$ and the elements $d(x_i)$, which are either polynomials in $x_1, \ldots, x_n$ (in the commutative case) or linear combinations of words in $x_1, \ldots, x_n$ (in the non-commutative case).
(3) A dga homomorphism is a *quasi-isomorphism* if it induces an isomorphism on homology; two dgas are *quasi-isomorphic* if there is a dga C and quasi-isomorphism $C \to A$ and $C \to B$. *Stable tame isomorphism* of semifree dgas was introduced in [**Che02**] (see also [**ENS02**, Section 3.3] and [**EN22**, Section 3.2]); it allows introducing a pair of canceling generators (stabilizing) and isomorphisms sending some x_i to x_i plus a word in the other variables. Two dgas are *derived Morita equivalent* if the derived categories of differential modules over them are equivalent (as triangulated categories). Stable tame isomorphism implies quasi-isomorphism implies derived Morita equivalence.
(4) This question arises from contact homology and related invariants. For example, to a Legendrian knot Λ in $\mathbb{R}^3$, one can associate a dga $\mathcal{A}_\Lambda$, the *Legendrian contact dga*, whose stable tame isomorphism class is an isotopy invariant of Λ (see the citations above). The Legendrian contact dga can also be defined for knots in other manifolds and higher-dimensional Legendrian knots. Contact homology can also be defined for (certain) closed contact manifolds; in this case, one is forced to work with a commutative dga. In practice, it seems to be hard to tell whether two dgas are stable tame isomorphic or quasi-isomorphic.
(5) One strategy that has been developed to distinguish dgas is to study the set of *augmentations* of A, i.e., dga maps $A \to R$ (where R lies in grading 0 and has trivial differential); this set is called the *augmentation variety* of A. Given an augmentation of A, one can form the linearized homology with respect to that augmentation, which is a finitely generated R-module [**Che02**].
(6) For Legendrian knots in $\mathbb{R}^3$, the set of augmentations form the objects of a category, the *augmentation category*, which in the case of Legendrian contact homology is equivalent to a certain category of sheaves [**NRS+20**]. The set of augmentations and the linearized contact homology depend only on the abelianization of the dga, but the augmentation category needs the non-commutative version.

(7) Contact homology can also be applied to study smooth objects in low-dimensional topology, for instance by considering the unit cotangent bundle or unit conormal bundle. An instance is Ng's knot contact homology [**Ng05**] (a variant of which is a complete knot invariant [**ENS18**]); another is given in Problem 4.106. It is possible that a result along these lines could also be applied to decision problems in symplectic or smooth topology.

(8) In some of the applications, one actually works over $\mathbb{Z}\{t, t^{-1}, x_1, \ldots, x_n\}$, say, but this seems unlikely to affect the question.

(9) In the special case that A and B are commutative with all of their generators in positive gradings, Sullivan's theory of minimal models gives an algorithm for answering the question; in particular, that case is well studied in rational homotopy theory. Note also that there is a unique augmentation in this case; in particular, the dgas arising from contact topology rarely have this property.

(10) Given that the question has some similarity to Hilbert's tenth problem, the answer might be different for R a finite field from $R = \mathbb{Q}$ or $\mathbb{Z}$, say.

Proposed for K3 and scribed by: R. Lipshitz

PROBLEM 5.17. *Let (W, ω, V_i, ϕ_i) be two Weinstein structures on a fixed symplectic manifold (W, ω) (or equivalently consider two Weinstein handle decompositions). Is there a Weinstein homotopy from (W, ω, V_1, ϕ_1) to (W, ω, V_2, ϕ_2)?*

REMARKS.

(1) The convex combination $V_t = (1-t)V_2 + tV_2$ is a Liouville vector field positively transverse to the boundary, but it may fail to be gradient-like for some t. The question is whether we can find a family of convex Liouville vector fields V_t that are all gradient-like.

(2) There are various additional hypotheses one can add which will ensure a Weinstein homotopy exists. For example, by [**CE12**, Proposition 11.22], if we have a fixed complex Stein structure J on (W, ω), any two J-convex (plurisubharmonic) functions ϕ_1 and ϕ_2 will yield Weinstein homotopic Weinstein structures. There are numerous other specific Weinstein homotopies constructed in [**CE12**, Chapter 12], that assume some specific properties or estimates about the Liouville form and/or gradient-like function.

For Weinstein manifolds of dimension strictly greater than 4, there is a notion of "flexible Weinstein manifolds." In this case any two flexible Weinstein structures are Weinstein homotopic [**CE12**, Theorem 14.5].

(3) A special case, which may be easier, is the following. Suppose (W, ω) admits a symplectomorphism that restricts to a contactomorphism on the boundary, but which is not isotopic to the identity in the class of symplectomorphisms that are contact on the boundary. Let λ be one Liouville form on (W, ω), corresponding to a Liouville vector field V_1 which is gradient-like for some Morse function ϕ_1. Consider the Liouville form $f^*\lambda$ and denote its corresponding Liouville vector field by V_2. Then V_2

is gradient-like for $\phi_2 = f^*\phi_1$. We can ask if (W, ω, V_1, ϕ_1) Weinstein homotopic to (W, ω, V_2, ϕ_2) in this case.

(4) It may be helpful to think about the Liouville form λ_i rather than the Liouville vector field V_i (which are related by $\iota_{V_i}\omega = \lambda_i$). While λ_i is not closed (since $d\lambda_i = \omega$ by the Liouville condition), $\lambda_1 - \lambda_2$ is closed and thus represents an element of $H^1(W)$. It may be easier to construct a counterexample for manifolds where $H^1(W)$ is nontrivial, in which case, one should refine the question to consider manifolds where $H^1(W) = 0$, or further that W is simply connected.

(5) This question is open in arbitrary dimensions > 2, but dimension 4 could be a good place to start.

Proposed for K3 by: Y. Eliashberg
Scribed by: L. Starkston

Problem 5.18.

(a) In higher dimensions, find the 'non-analytic' cohomology module $\mathbb{H}^*_?$ *analogous to the analytic lattice cohomology* $\mathbb{H}^*_{an}$*, and a natural functor* $\mathbb{H}^*_{an} \to \mathbb{H}^*_?$ *connecting them.*

(b) Define a version ECH of Embedded Contact Homology for isolated complex singularity links (in any dimension) associated with the canonical contact structure of the link, together with a natural graded* $\mathbb{Z}[U]$*–module morphism* $\mathbb{H}^*_{an} \to \mathrm{ECH}^*$*. More precisely,*

(i) Show that this morphism is injective.

(ii) Fix the diffeomorphism type of a link, and also a contact structure on it that can be realized as the canonical contact structure associated with some singularity analytic structure. Then characterize the family $\{\mathbb{H}^*_{an}\}$*, indexed by all the possible analytic germs inducing this contact link, via those graded sub–*$\mathbb{Z}[U]$*–modules of* ECH^* *which satisfy certain specific properties.*

Remarks.

(1) As background, consider a complex normal surface singularity with a rational homology sphere link. The (topological) lattice cohomology $\mathbb{H}^*_{top}$, associated with the link (which is a plumbed 3–manifold associated with a connected negative definite graph), was introduced by Némethi in [**Ném05, Ném08**]. It has an analytic analogue $\mathbb{H}^*_{an}$ constructed recently in [**ÁN21a, ÁN21b**]. Both theories are multigraded, $\mathbb{H}^*_{top} = \oplus_{q \geqslant 0}\mathbb{H}^q_{top}$, and $\mathbb{H}^q_{top}$ is a $2\mathbb{Z}$–graded $\mathbb{Z}[U]$–module, with an additional grading indexed by the spin^c structures of the link. The same is valid for $\mathbb{H}^*_{an}$ as well. Even more, the existence of a graded $\mathbb{Z}[U]$–module morphism $\mathbb{H}^*_{an} \to \mathbb{H}^*_{top}$ was also established.

Following on calculations in [**OS03b**], it was conjectured in [**Ném08**] that the topological version can be identified with Heegaard Floer homology. Further results from [**Ném08, OSS14b**] support the conjecture, whose full proof was announced in [**Zem25**].

In particular, it is also isomorphic with any other (co)homology theory of 3-manifolds that agree with Heegaard Floer homology, e.g. with

Embedded Contact Homology (introduced in [**Hut10, Hut14**], for its equivalence with the Heegaard Floer homology see [**CGH20**]).

One of the spin^c structures (determined from the analytic structure) is distinguished; it is called the 'canonical spin^c structure'. The analytic lattice cohomology associated with this canonical structure has an extension to any higher dimension, for complex isolated singularities (again, with certain restrictions).

(2) Part (b) addresses the important issue that in higher dimension the link of the isolated singularity contains essentially less information than might be needed to construct such an invariant (e.g. the link can be even the usual sphere). In particular, in the above context, the 'non-analytic' version shouldn't be 'topological' or 'smooth'. However, the smooth structure enhanced with its canonical contact structure (induced by the analytic structure of the germ) might produce the desired cohomology.

A good candidate for this is some version of Embedded Contact Homology associated with the canonical contact structure of the link. For 3-dimensional singularity links the canonical contact structure can be uniquely determined from the link itself [**CNPP06**]. However, in higher dimensions, it is an essential enhancement of the smooth structure. For results regarding ECH in higher dimensions see e.g. [**CHT24**].

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Problems carried over from K2

As mentioned in the introduction, some problems from **K2** reappear in this volume, about 50 in all. Some are essentially unchanged from the original, but in many cases there has been some progress in the intervening years and we cite only part of the original problem. Here is a list of those problems, with the **K2** problem listed first, followed by the corresponding problem number (in parentheses) in this volume.

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