Derivative and Integral Reference Guide

Differentiation Rules

Linearity

$$\frac{d}{dx}\big[u+v\big] = u' + v'$$

$$\frac{d}{dx}\big[cu\big] = cu'$$

Product & Quotient Rules

$$\frac{d}{dx}\big[uv\big] = u'v + v'u$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - v'u}{v^2}$$

Chain Rule

$$\frac{d}{dx}\big[f(u)\big] = f'(u)\cdot u'$$

Derivative Identities

$$\frac{d}{dx}\big[c\big] = 0 \qquad \frac{d}{dx}\big[x\big] = 1 \qquad \frac{d}{dx}\big[u^n\big] = nu^{n-1}u'$$

$$\frac{d}{dx}\big[e^u\big] = u'e^u \qquad \frac{d}{dx}\big[b^u\big] = \ln(b)b^u u'$$

$$\frac{d}{dx}\big[\ln u\big] = \frac{u'}{u} \qquad \frac{d}{dx}\big[\log_b u\big] = \frac{1}{\ln b}\cdot\frac{u'}{u}$$

$$\frac{d}{dx}\big[\sin u\big] = u'\cos u \qquad \frac{d}{dx}\big[\cos u\big] = -u'\sin u \qquad \frac{d}{dx}\big[\tan u\big] = u'\sec^2 u$$

$$\frac{d}{dx}\big[\csc u\big] = -u'\csc u\cot u \qquad \frac{d}{dx}\big[\sec u\big] = u'\sec u\tan u \qquad \frac{d}{dx}\big[\cot u\big] = -\csc^2 u$$

$$\frac{d}{dx}\big[\arcsin u\big] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}\big[\arccos u\big] = -\frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}\big[\arctan u\big] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx}\big[\operatorname{arccsc} u\big] = -\frac{u'}{u\sqrt{u^2-1}} \qquad \frac{d}{dx}\big[\operatorname{arcsec} u\big] = \frac{u'}{u\sqrt{u^2-1}} \qquad \frac{d}{dx}\big[\operatorname{arccot} u\big] = -\frac{u'}{1+u^2}$$

Fundamental Theorems of Calculus

$$F'(x) = f(x) \quad\Longrightarrow\quad \int_a^b f(x)\,dx = F(b) - F(a)$$

$$\frac{d}{dx}\left[\int_{a(x)}^{b(x)} f(t)\,dt\right] = f\big(b(x)\big)\cdot b'(x) - f\big(a(x)\big)\cdot a'(x)$$

Integration Rules

<u>Linearity</u>

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int af(x)\,dx = a\int f(x)\,dx$$

<u>Integration by Parts</u>

$$\int u\,dv = uv - \int v\,du$$

Integral Identities

$$\int 0\,dx = C$$

$$\int dx = x + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \ln x\,dx = x\ln(x) - x + C$$

$$\int \log_b x\,dx = \frac{1}{\ln(b)}[x\ln(x) - x] + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

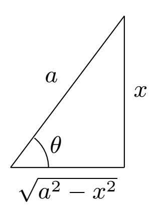
$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C$$

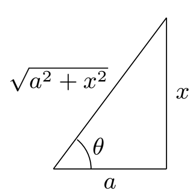
$$\int \frac{1}{x\sqrt{x^2 - 1}}\,dx = \operatorname{arcsec} x + C$$

Trig Sub

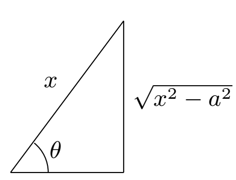
$$x = a\sin\theta$$
$$dx = a\cos\theta\,d\theta$$
$$\sqrt{a^2 - x^2} = a\cos\theta$$



$$x = a\tan\theta$$
$$dx = a\sec^2\theta\,d\theta$$
$$\sqrt{a^2 + x^2} = a\sec\theta$$



$$x = a\sec\theta$$
$$dx = a\sec\theta\tan\theta\,d\theta$$
$$\sqrt{x^2 - a^2} = a\tan\theta$$



Source: Stewart, J. (2020). *Calculus, 9e.* Cengage Learning.