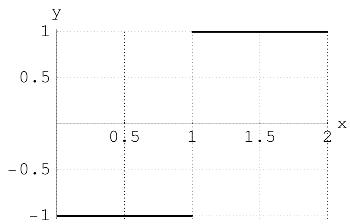
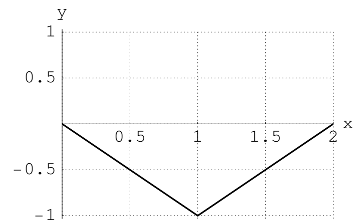
Section 6.1 – Antiderivatives Graphically and Numerically

1. For each of the following functions f, sketch two antiderivatives of f, one that satisfies $F(0) = 0$ and one that satisfies $F(0) = 1$.

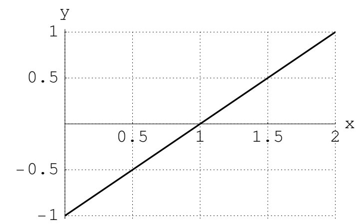
$f(x)$



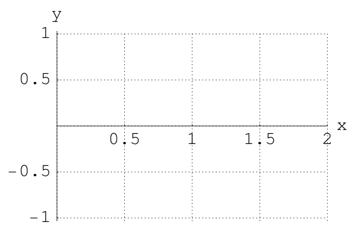
$f(x)$



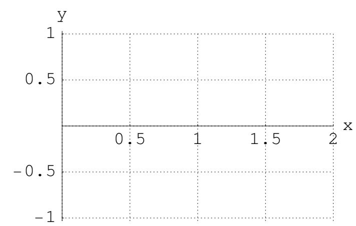
$f(x)$



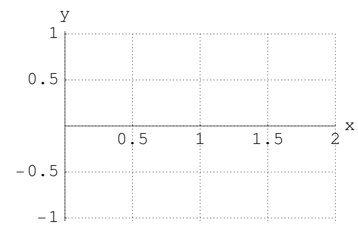
Antiderivatives:



Antiderivatives:

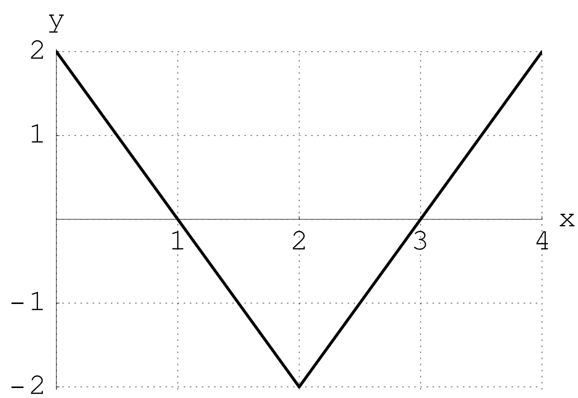


Antiderivatives:

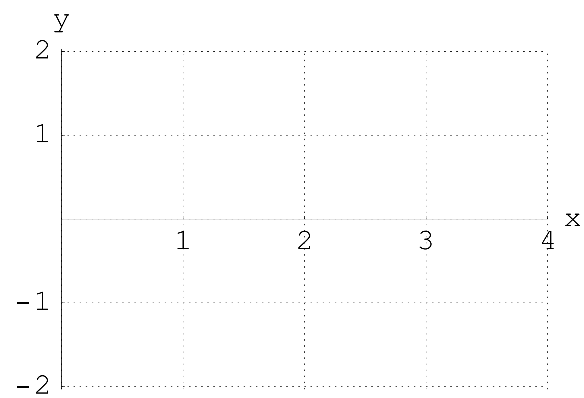


2. Given below is the graph of a function $f'(x)$. Given that $f(0) = 1$, sketch an accurate graph of $f(x)$ on the blank axes to the right. Label all critical points and inflection points of f with their x and y coordinates.

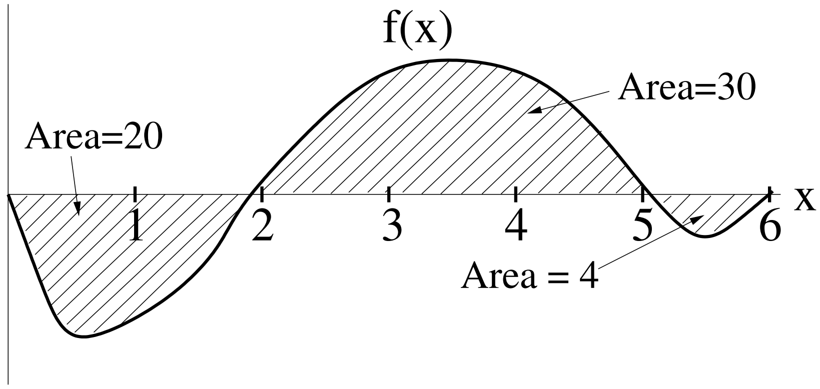
$f'(x)$

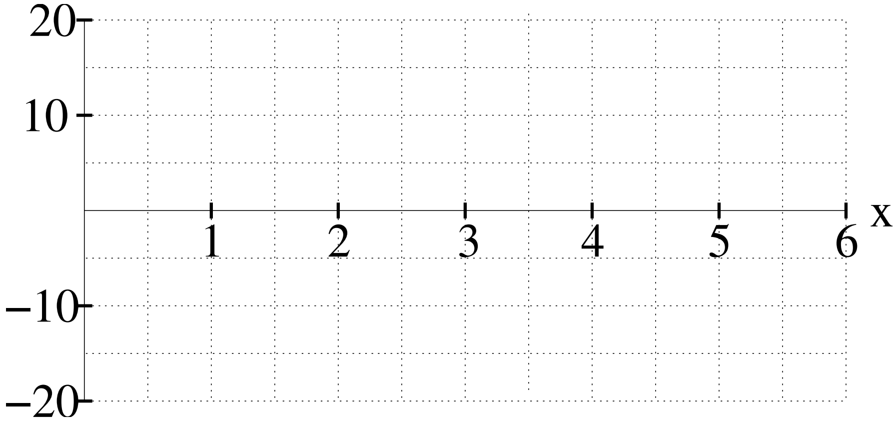


$f(x)$



3. Given below is the graph of a function $f(x)$. Sketch a graph of an antiderivative $F(x)$ of $f(x)$ that satisfies $F(0) = 5$. Label all critical points and inflection points of f with their approximate x and y coordinates.





Section 6.2 – Constructing Antiderivatives Analytically

1. Evaluate each of the of the following **exactly**.

   (a) $\displaystyle\int_1^3 \frac{10-5x}{\sqrt{x}}\,dx$

   (b) $\displaystyle\int_{-5}^0 (2e^x - 4\cos x)\,dx$

   (c) $\displaystyle\int x(x^2+1)^2\,dx$

2. Find the exact area under $y = \dfrac{1}{4x}$ between $x = 1$ and $x = 4$.

3. At Bill's Gas 'n' Snax, the most happening gas station in Dudleyville, Arizona, the demand for gasoline is changing at a rate of $r(t) = -90t - 100$ gallons per day, where $t = 0$ represents the number of days after a major jump in gas prices. Calculate $\int_1^4 r(t)\, dt$ and give a practical interpretation of this integral, including the appropriate units.

Section 6.2 – Derivatives and Antiderivatives

1. Let $f'(x) = 3 - |2x|$. Sketch the following graphs clearly and accurately and label all important points.
   (a) On one set of axes, sketch a graph of $f'(x)$ and $f''(x)$.
   (b) On another set of axes, sketch possible graphs of $f(x)$, one having $f(0) = -2$ and one having $f(0) = 0$.

2. Find derivatives and antiderivatives of the following.
   (a) e^x
   (b) $\sin x$
   (c) x^5
   (d) $5x^4$
   (e) $3e^{2x}$
   (f) $\cos 3x + \sin 4x$
   (g) $e^{-x} - \frac{x+3}{x^2} + \frac{5}{\cos^2 2x}$

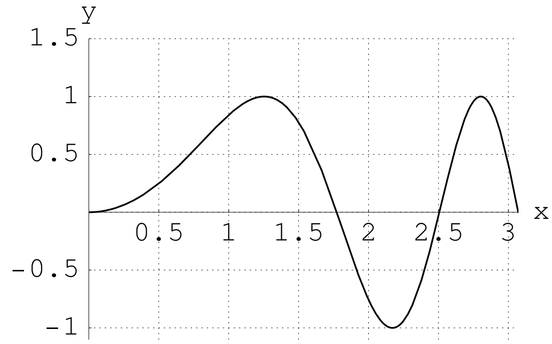
3. Explain the difference between *an* antiderivative and *the* antiderivative.

Section 6.4 – The Second Fundamental Theorem of Calculus

Example. Given to the right is a graph of the function $y = \sin(x^2)$. Define a new function

$$F(x) = \int_0^x \sin(t^2)\,dt.$$

$y = \sin(x^2)$



1. Use your calculator to help you fill in the following table entries.

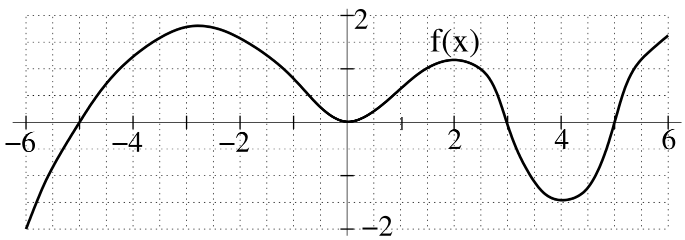
x	0	0.5	1	1.5	2	2.5	3
$F(x)$							

2. On the same set of axes as the above graph, sketch the graph of $F(x)$. Do you notice a relationship between f and F? How are these two functions related?

Exercises.

1. For each of the following, find $F'(x)$.
   (a) $F(x) = \int_3^x (2t+1)\,dt$
   (b) $F(x) = \int_1^{\ln x} \frac{1}{1+t^4}\,dt$
   (c) $F(x) = x\,\mathrm{Si}(x)$, where $\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt$

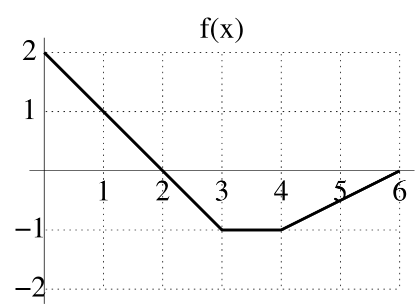
2. Let $F(x) = \int_0^x f(t)\,dt$, where $f(x)$ is the function whose graph is given below.



   (a) What are the critical points of $F(x)$?
   (b) Where is $F(x)$ increasing? decreasing?
   (c) Locate all places where $F(x)$ has a local maximum or a local minimum, and make it clear which are which.
   (d) Where is $F(x)$ concave up? concave down?
   (e) Where does F attain its maximum value? Use a method of your choice to estimate this maximum value.

3. Let $F(x) = \int_2^x \frac{1}{\ln t}\,dt$ for $x \geq 2$.
   (a) Find $F'(x)$.
   (b) Is F increasing or decreasing? What can you say about the concavity of F?

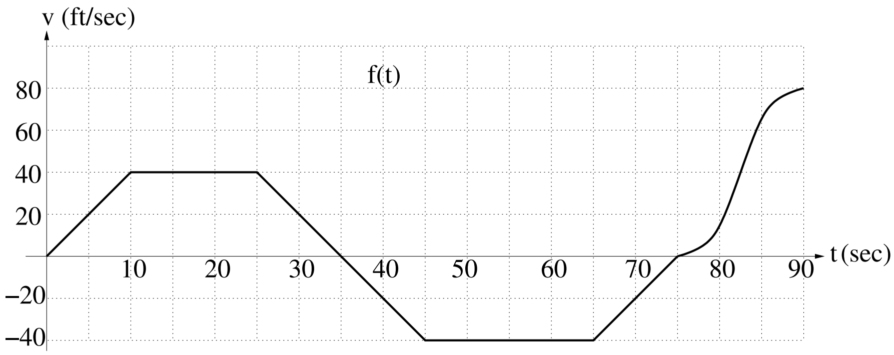
4. Let $f(x)$ be the function whose graph is given to the right. Define
$$F(x) = \int_0^x f(t)\,dt.$$
Fill in the entries in the table that follow:



x	1	2	3	4	5
$F(x)$					
$F'(x)$					
$F''(x)$					

The Calculus Workout Sheet

1. A motorcyclist traveling along a straight east/west road begins travel at her home at $t = 0$. Below you are given a graph of her velocity, v, as a function of time, where positive velocities indicate easterly travel, and negative velocities indicate westerly travel.



   (a) What are the location, velocity, and acceleration of the rider after 20 seconds?

   (b) Estimate the time(s) when her speed is the greatest, the time(s) when her acceleration is the greatest, and the time(s) when her distance from home is the greatest.

   (c) At what time does she pass her house going west?

2. Between noon and 6 p.m., the electrical power in use by a family, in kilowatts, is given by the function $P = 0.6t^2 - 0.1t^3 + 0.2$, where t is the number of hours after 12:00 noon.

   (a) How fast is the family's power usage increasing, in kilowatts per hour, at 2:30 p.m.?

   (b) What is the family's power usage, in kilowatts, at 2:30 p.m.?

   (c) How much total energy is consumed by the family, in kilowatt hours, between noon and 6 p.m.?

3. The volume of water remaining in a leaking tank is given by the function $V = 50e^{-0.1t}$, where V is measured in gallons and t in minutes.

   (a) How fast is the tank leaking after 10 minutes? Include appropriate units.

   (b) How much water remains in the tank after 10 minutes? Include appropriate units.

4. The rate at which water is leaking from an initially full 100-gallon tank is given by the function $r = -7e^{-0.1t}$, where r is measured in gallons per minute and t is measured in minutes. (The negative sign indicates a decrease as opposed to an increase in volume.)

   (a) How fast is the tank leaking after 10 minutes? Include appropriate units.

   (b) How much water remains in the tank after 10 minutes? Include appropriate units.

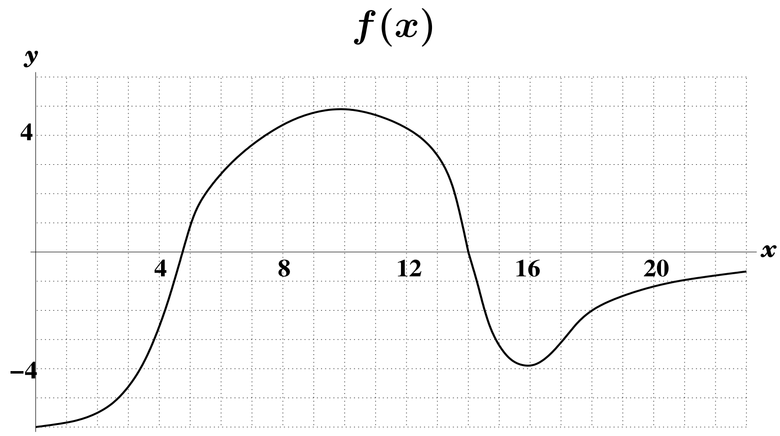
5. Let f be a differentiable function on the interval $0 \leq x \leq 4$, and suppose that the derivative of f is also continuous. Suppose that the table below is the only information about f that we have.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
$f(x)$	1.1	2.1	2.9	3.6	4.2	4.6	4.9	5.1	5.2

Estimate each of the following as accurately as you can based on the data given.

(a) $f'(1)$ (b) $f'(3)$ (c) $\int_0^4 f(x)\,dx$ (d) $\int_1^3 f'(x)\,dx$

6. Let f be the function whose graph is given below, and define a new function F by the equation $F(x) = \int_0^x f(t)\,dt$.



Given below are several lists of numbers. Rank each list in order from smallest to largest.

(a) $0,\ f'(3),\ f'(4),\ f'(9),\ f'(14),\ f'(15)$

(b) $0,\ F(3),\ F(4),\ F(6),\ F(13),\ F(14)$

(c) $0,\ F'(0),\ F'(4),\ F'(7),\ F'(10),\ F'(15),\ F'(18),\ F'(23)$

7. Find the slope of the tangent line to each of the following functions at $x = 2$. Give the exact slope if possible; otherwise, round to the nearest 0.01.

(a) $f(x) = \sin(\cos(x))$

(b) $F(x) = \int_0^x e^{-t^2}\,dt$

(c) $f(x) = x^x$