Section 3.1 – Powers and Polynomials

1. Find the derivative of each of the following functions. You may assume that p and q are constants.

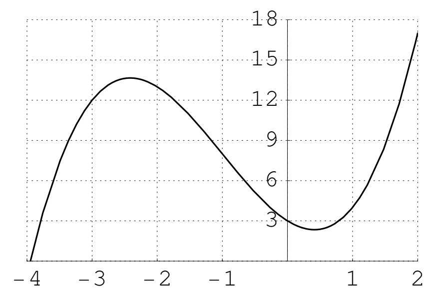
   (a) $y = 3x^2 + 2x + 1$

   (b) $y = 5x^2 + \frac{1}{x}$

   (c) $f(t) = \dfrac{t^2 + pt + q}{\sqrt{t}}$

2. Find the equation of the tangent line to the graph of the function $f(x) = 1/x$ at $x = 2$.

3. Given to the right is the graph of the function $f(x) = x^3 + 3x^2 - 3x + 3$.



   (a) Graphically estimate the value(s) of x at which f has a horizontal tangent line. Then, use derivatives to find more accurate estimates.

   (b) Find all values of x at which the tangent line to f is parallel to the line $y = 6x + 6$.

4. (Taken from *Hughes-Hallett, et. al.*) At a time t seconds after it is thrown up in the air, a tomato is at a height of $f(t) = -4.9t^2 + 25t + 3$ meters.

   (a) What is the average velocity of the tomato during the first 2 seconds? Give units.

   (b) Find the instantaneous velocity of the tomato at $t = 2$. Give units.

   (c) What is the acceleration at $t = 2$?

   (d) How high does the tomato go?

Section 3.2 – The Exponential Function

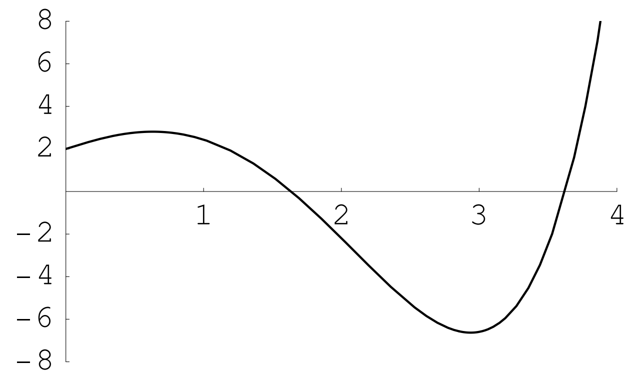
1. Find the derivative of each of the following functions. You may assume that p and q are constants.

   (a) $f(x) = \pi^x + x^\pi$

   (b) $y = qx - \dfrac{1}{\sqrt[3]{x}} + 5^x - \mathrm{e}$

   (c) $g(t) = \left(\dfrac{p}{2}\right)^t + \dfrac{p}{t^2}$

2. Let $f(x) = 2\mathrm{e}^x - 3x^2\sqrt{x}$, whose graph is given to the right. Sketch in the tangent lines to $f(x)$ at $x = 1, x = 2$, and $x = 3$, and calculate their slopes.



3. Let $f(x) = 1 + 2e^x - 3x$.

   (a) Find the equation of the tangent line to f at $x = 0$.

   (b) At what value(s) of x does f have a horizontal tangent line? Give answer(s) in exact form and as a decimal approximation.

4. The population of the world, P (in billions), is well-approximated by the function $P = 5.6(1.0117)^t$, where t represents the number of years after the beginning of 1994.

   (a) What is the population of the world at the beginning of 1999? How fast is the population of the world growing at the beginning of 1999? Include units, and make it clear which answer is which.

   (b) In what year will the population of the world be growing by 100 million people per year?

Section 3.3 – The Product and Quotient Rules

1. Find the derivative of each of the following functions. You may assume that a, b, c, and d are constants.

   (a) $f(x) = (x^2 - \sqrt{x}) \cdot 3^x$

   (b) $g(x) = \dfrac{ax + b}{cx + d}$

2. Suppose that f and h are functions and that $f(3) = 2,\ f'(3) = -2,\ h(3) = 1$, and $h'(3) = 4$.

   (a) Calculate $m'(3)$, where $m(x) = f(x)h(x)$.

   (b) Calculate $p'(3)$, where $p(x) = \dfrac{f(x)}{x^2 h(x)}$.

Sections 3.4 and 3.5 – The Chain Rule and Trigonometric Functions

The Chain Rule (Version 1). If y is a function of u, and u is a function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

The Chain Rule (Version 2). Let $f(x)$ and $g(x)$ be differentiable functions. Then

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x).$$

Notes:

Derivatives of Trigonometric Functions

$\frac{d}{dx} \sin x =$	$\frac{d}{dx} \cos x =$
$\frac{d}{dx} \tan x =$	$\frac{d}{dx} \cot x =$
$\frac{d}{dx} \sec x =$	$\frac{d}{dx} \csc x =$

1. Find the derivative of each of the following functions.

(a) $f(x) = \sqrt{\dfrac{x^2+9}{x+3}}$

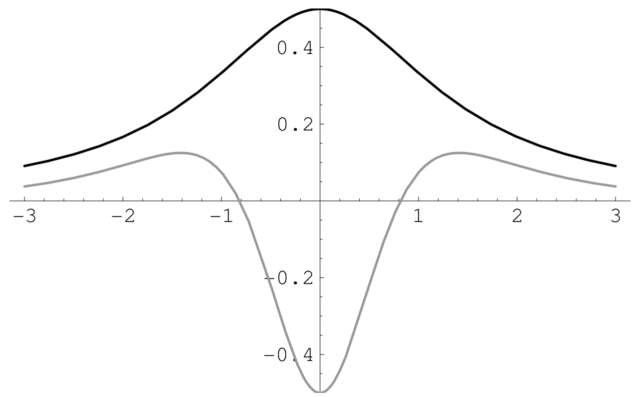
(b) $y = x \cdot 2^{-x^2}$

(c) $f(t) = \sin(t^2)$

(d) $g(t) = \tan^2 t$

(e) $f(x) = \sqrt{\cos(\sin^2 x)}$

2. Given to the right is the graph of the function $f(x) = (x^2+2)^{-1}$ (dark graph) and its second derivative $f''(x)$ (lighter graph).



   (a) Find the equation of the tangent line to f at $x = 2$.

   (b) Find a formula for $f''(x)$.

   (c) Graphically estimate the interval on which f is concave down. Then, use your formula for the second derivative to find the exact interval on which f is concave down.

3. The population of the world, P (in billions) is well-modeled by the equation $P = 6e^{0.013t}$, where t is the number of years after the beginning of 1999. First, estimate the population of the world in 2009. Then, predict the rate at which the world's population was growing in 2009. Include units with your answers.

4. Let f be a differentiable function, and let $g(x) = (f(\sqrt{x}))^3$.

   (a) Calculate $g'(x)$.

   (b) Use the values in the table below to calculate $g'(4)$.

x	$f(x)$	$f'(x)$
2	1	-2
4	-3	4

Section 3.4 – The Chain Rule

Part I - Find the indicated derivatives.

1. Find $\dfrac{d^2y}{dx^2}$ where $y = 2x - \dfrac{1}{\sqrt[3]{x}} + 3^x - e$.
2. Find $f'(3)$ where $f(z) = \dfrac{z^2+1}{\sqrt{z}}$.
3. Find $h'(y)$ where $h(y) = (3y^2 + 7y)(2(1.3)^y + 5)$
4. Find $h'(2)$ where $h(x) = 2f(x) \cdot g(x)$ and $f(2) = 7$, $f'(2) = -2$, $g(2) = -1$, $g'(2) = 3$.

Part II - Given the table below, find the indicated derivatives at $x = 1$ and $x = -2$ where possible.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	3	−2	−1
−2	−2	−5	1	7

1. $\dfrac{d}{dx}\left[(f(x))^2 - 3g(x^2)\right]$ at $x = 1$
2. $\dfrac{d}{dx}\left[f(x) \cdot g(x)\right]$ at $x = 1$
3. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]$ at $x = -2$
4. $\dfrac{d}{dx}\left[f(g(x))\right]$ at $x = 1$
5. $\dfrac{d}{dx}\left[g(f(x))\right]$ at $x = -2$
6. $\dfrac{d}{dx}\left[g(g(x))\right]$ at $x = -2$
7. $\dfrac{d}{dx}\left[f(g(4-6x))\right]$ at $x = 1$
8. $\dfrac{d}{dx}\left[(g(x))^2\right]$ at $x = 1$

Section 3.4 – Differentiation Practice

USE THE VALUES IN THE FOLLOWING TABLE TO ANSWER THE QUESTIONS BELOW.

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$	$f''(x)$
0	0	1	2	-1	4	-5	0
1	3	2	1	3	-2	-4	-4
2	1	0	3	-2	3	2	1
3	2	3	0	4	2	-3	2

1. Determine if $y = f(x)g(x)$ has a horizontal tangent at $x = 1$.

2. Determine if $y = h(g(x))$ is increasing or decreasing at $x = 3$.

3. Find the equation of the tangent line to $y = f(g(x))$ at $x = 2$.

4. Find $u'(1)$ if $u(x) = \sqrt{h(x) + 3}$

5. Determine if $y(x) = (f(x))^2$ is concave up or down at $x = 1$.

6. Find the slope of $y = \dfrac{g(x)}{x^3}$ at $x = 2$.

7. Find $\frac{dy}{dx}$ for $y = f(g(3))$.

8. Find $u'(4)$ if $u(x) = h(\sqrt{x})$.

9. Find the slope of the tangent line to $y = e^{g(x)}$ at $x = 0$.

Section 3.6 – The Chain Rule and Inverse Functions

1. Calculate $\frac{d}{dx}\left(\arcsin(e^{x^2})\right)$.

2. Calculate $\frac{d}{dx}\left(x \arctan(x^3)\right)$.

3. Find all points on the curve $f(x) = \ln(x^2 - 4x + 5)$ where the tangent line is horizontal.

4. Air is being blown into a spherical balloon at a constant rate of 30 cm^3 per second. Find the rate at which the radius of the balloon is increasing when the radius is 1 cm and the rate at which the radius is increasing when the radius is 2 cm.

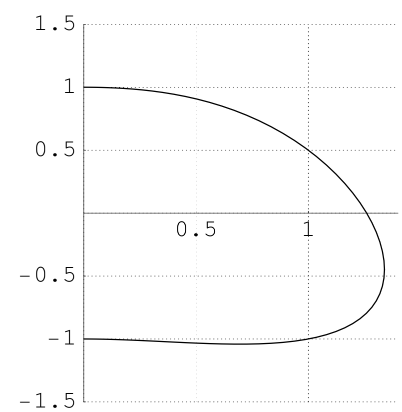
Section 3.7 – Implicit Functions

1. Find y' if $4 \cos x \cos y = 3y$.

2. Find y' if $e^{xy} + y^2 = 2x$.

3. Find the equation of the tangent line to the curve $x^3 + x^2y + 2y^2 = 2$ at the point $(1,\ 0.5)$. Then, sketch this line on the diagram to the right.

$x^3 + x^2y + 2y^2 = 2$



Section 3.9 – Linear Approximation and the Derivative

1. Show that $\dfrac{1}{\sqrt{x+1}} \approx 1 - \dfrac{x}{2}$ near $x = 0$.

2. (a) Show that $1 + kx$ is the local linearization of $(1+x)^k$ at $x = 0$.

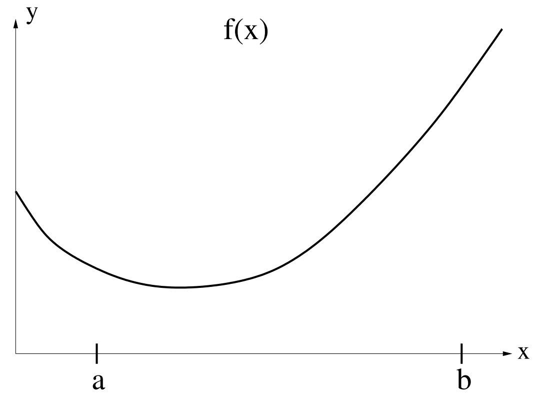
   (b) Someone claims that the square root of 1.1 is about 1.05. Without using a calculator, do you think this estimate is about right? **Hint:** Use the linearization you calculated in part (a).

Section 3.10 – Theorems About Differentiable Functions

Mean Value Theorem. If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c, with $a < c < b$, such that

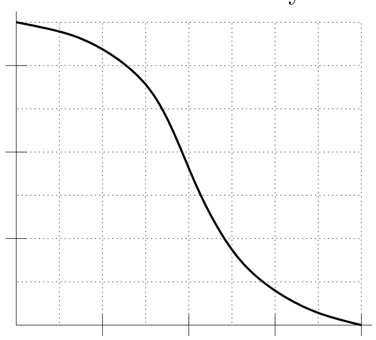
$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

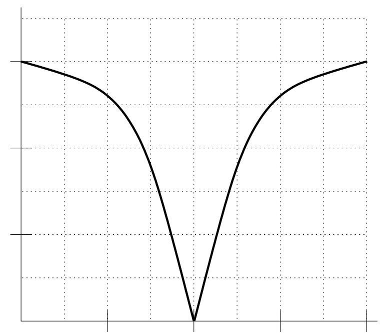
In other words, $f(b) - f(a) = f'(c)(b - a)$.

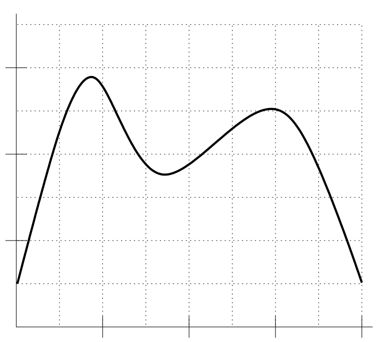


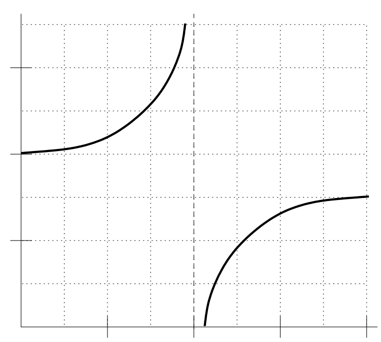
Exercises

1. Which of the following appear to satisfy the hypotheses of the Mean Value Theorem? Which appear to satisfy the conclusion of the Mean Value Theorem? For those that satisfy the conclusion, graphically estimate all values of c that satisfy the conclusion. Use $a = 0$ and $b = 4$ for each function.









2. Consider the function $f(x) = x^3 - 4x$ on the interval $[1, 2]$. Does there exist a value of c that satisfies the conclusion of the Mean Value Theorem on this interval? If so, find such a value. If not, explain why no such value exists.

3. Consider the function $f(x) = \dfrac{x^2 - 4}{x^2}$ on the interval $[-1, 1]$. Does there exist a value of c that satisfies the conclusion of the Mean Value Theorem on this interval? If so, find such a value. If not, explain why no such value exists.

Chapter 3 – Derivative Practice and Summary

1. (a) Find the equation of the tangent line to $f(x) = x + \frac{4}{x}$ at the point $(1, 5)$.

   (b) Use your calculator or computer to graph $f(x)$ and the tangent line you found to check your work.

   (c) Do you expect the tangent line approximation to $f(x)$ at $x = 1$ to be an over- or under-estimate? Why?

   (d) Use the tangent line to estimate the value of the function at $x = 1.1$.

   (e) Compare the actual value of $f(1.1)$ to your estimate in part (d). Does your result confirm your prediction in part (c)?

2. Find the first derivative of the following functions.

   (a) $y = \dfrac{\tan x}{x}$

   (b) $y = \sin(e^x)$

   (c) $y = e^{\sqrt{x}}$

   (d) $y = \ln(\cos(\theta^2))$

3. Find $\frac{dy}{dx}$ by implicit differentiation: $x^4 + y^4 = 16$.

4. Find an equation of the tangent line to the curve $y^2 = x^3(2 - x)$ at the point $(1, 1)$.