1. Evaluate the following limits [9]

(a) $\lim_{x\to 0} \frac{\sin 3x}{7x}$ $= \text{"}\frac{0}{0}\text{"}$

$= \lim_{x\to 0} \frac{3\cos 3x}{7} = 3/7$

(b) $\lim_{x\to\infty} \frac{e^{x^2}}{x^2}$ $= \text{"}\frac{\infty}{\infty}\text{"}$

$= \lim_{x\to\infty} \frac{2xe^{x^2}}{2x} = \lim_{x\to\infty} e^{x^2} = \text{"}\infty\text{"}$ DNE

(c) $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$ $= \text{"}1^\infty\text{"}$

$y = (1+x)^{1/x}$

$\ln y = \ln(1+x)^{1/x}$

$\ln y = \frac{\ln(1+x)}{x}$

$\lim_{x\to 0} \frac{\ln(1+x)}{x} = \text{"}\frac{0}{0}\text{"}$

$= \lim_{x\to 0} \frac{1/(1+x)}{1} = \frac{1/1}{1} = 1$

$\ln y = 1$

$y = e^1$

2. Consider a function $f(x)$ that has the following first and second derivatives: [12]

$$f'(x) = \frac{x^1 - 16}{x^2 + 1} \qquad f''(x) = \frac{2x(x^4 + 2x^2 + 16)}{(x^2+1)^2}$$

>0 >0

(a) Find all critical points of $f(x)$ (just the x-values)

$f'(x) = 0$ $f'(x)$ DNE

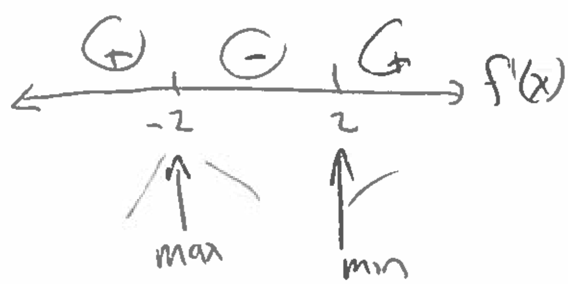
never happens

$x^4 - 16 = 0$

$x^4 = 16$

$x = \pm 2$

(b) Use the first derivative test to classify these critical points as minima, maxima, or neither (create a sign chart)

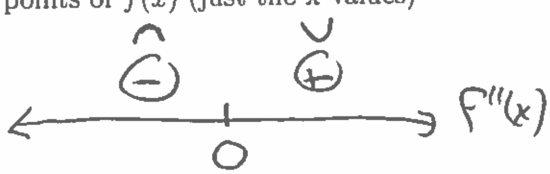


(c) Use the second derivative test to classify these critical points as minima, maxima, or neither

$f''(2) > 0$ min

$f''(-2) < 0$ max

(d) Find all inflection points of $f(x)$ (just the x-values)

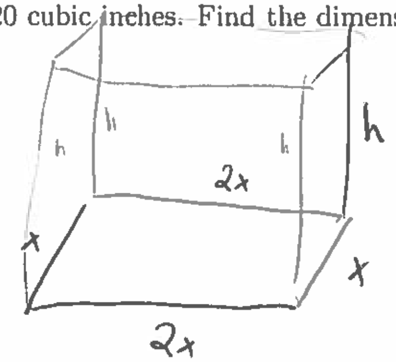


$f''(x) = 0$ $x = 0$ is an inflection pt

$2x = 0$

$x = 0$

3. You are in charge of building a very sturdy box. The base of the box is a rectangle such that the width is twice as long as the length. The box doesn't have a top. The bottom of the box costs 10 cents per square inch and the sides cost 9 cents per square inch. The volume must be 20 cubic inches. Find the dimensions that minimize the cost. What is the cost? [10]



$C = 10 \cdot 2x \cdot x + 9(2xh) + 9(2 \cdot 2xh)$

$C = 20x^2 + 18xh + 36xh$

$C = 20x^2 + 54xh$

$V = 20$

$2x \cdot x \cdot h = 20$

$x^2 h = 10$

$h = \frac{10}{x^2}$

$C = 20x^2 + 54 \cdot x \cdot \frac{10}{x^2}$

$C = 20x^2 + \frac{540}{x}$

$C' = 40x - \frac{540}{x^2} = 0$

$40x = \frac{540}{x^2}$

$x^3 = 54/4 = 27/2$

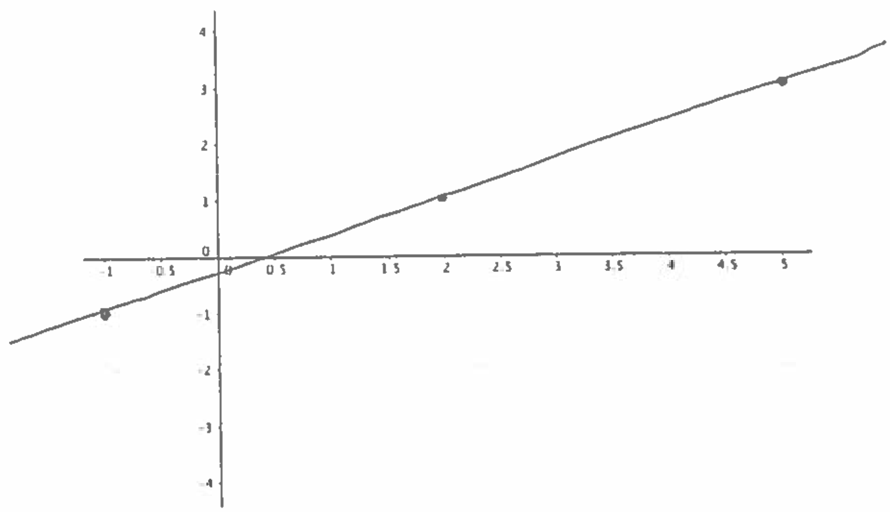
$x = \sqrt[3]{27/2} =$

$h =$

$C =$

4. Consider the parametric equation: [5]

$$x(t) = 5 - 3t$$
$$y(t) = 3 - 2t$$
$$-\infty < t < \infty$$



(a) Sketch the parametric equation on the graph above

(b) Write a different parametric equation that sketches the same line

$x(t) = 2 - 3t$
$y(t) = 1 - 2t$ } Many other options!

5. Consider the parametric equation describing the motion of a particle: [7]

$$x(t) = t^3 - 3t$$
$$y(t) = t^2 - 2t$$
$$-\infty < t < \infty$$

(a) Find the speed of the particle at time $t = 2$

$V_x = 3t^2 - 3$ $V_x(2) = 9$ $S = \sqrt{8^2 + 2^2} = \sqrt{68}$

$V_y = 2t - 2$ $V_y(2) = 2$

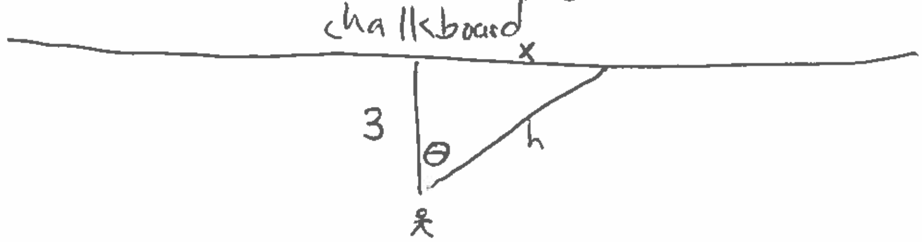
(b) Is the particle ever stopped? When? Where?

V_x { $3t^2 - 3 = 0$, $t^2 = 1$, $t = \pm 1$

V_y { $2t - 2 = 0$, $t = 1$

stopped at $t = 1$
$x(1) = -2$
$y(1) = -1$

6. Recall the in-class example we did where I shined a laser pointer on the chalkboard while rotating in my chair. I am sitting 3 meters from the wall and rotating at 4 radians per minute. How fast is the red dot moving when the angle between the the laser and the wall is $\pi/4$? [10]



$\frac{d\theta}{dt} = 4$

$\tan\theta = \frac{x}{3}$

$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}(x/3)$

$(\sec\theta)^2 \frac{d\theta}{dt} = \frac{1}{3}\frac{dx}{dt}$

$\frac{3}{(\cos\theta)^2}\frac{d\theta}{dt} = \frac{dx}{dt}$

$\frac{3}{(\sqrt{2}/2)^2} \cdot 4 = \frac{dx}{dt}$

$\frac{dx}{dt} = 24$ meters/min