University of Portland
Department of Mathematics

CALCULUS I | EXAM 1 | FALL 2017

NAME: Key

Read This First!

- Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- Calculators are allowed per our stated calculator policy, but you must show all your work in order to receive credit on the problem.

I attest that I have neither given nor received help of any kind on this exam.

SIGNATURE:________________________________

Grading - For Administrative Use Only

Question	Points	Score
1	15	
2	12	
3	6	
4	8	
5	7	
6	8	
Total:	56	

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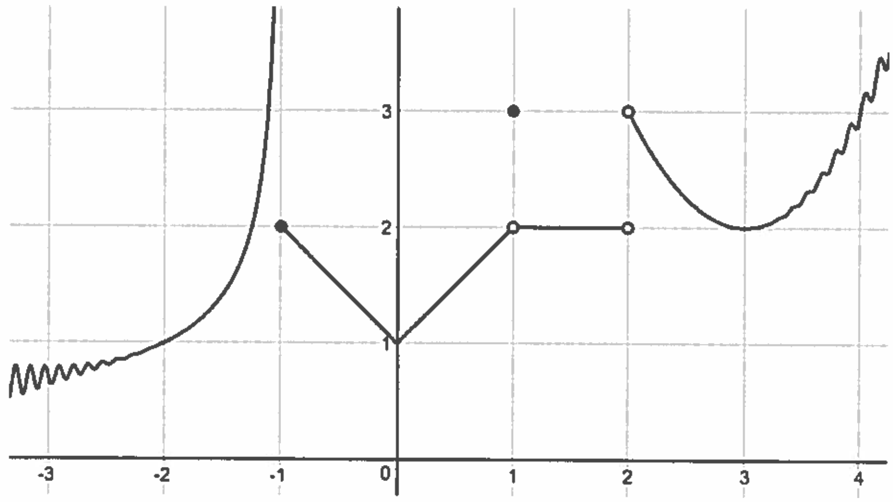
1. The graph of a function $f(x)$ is pictured below (you don't need to show work)



(a) Compute $\lim_{x\to-1^-} f(x)$, $\lim_{x\to-1^+} f(x)$, and $\lim_{x\to-1} f(x)$. Is $f(x)$ continuous at $x=-1$? [4]

∞ (DNE) 2 DNE NO

(b) Compute $\lim_{x\to0^-} f(x)$, $\lim_{x\to0^+} f(x)$, and $\lim_{x\to0} f(x)$. Is $f(x)$ continuous at $x=0$? [4]

1 1 1 yes

(c) Compute $\lim_{x\to1^-} f(x)$, $\lim_{x\to1^+} f(x)$, and $\lim_{x\to1} f(x)$. Is $f(x)$ continuous at $x=1$? [4]

2 2 2 NO

(d) For each x-value listed below, determine $f'(x)$. Write DNE if the derivative doesn't exist at that point. [3]

i. $x=-0.5$

-1

ii. $x=0$

DNE

iii. $x=3$

0

2. Evaluate the following limits. Show work and justify your answers using algebra! [12]

(a) $\lim_{x\to 2} \sqrt{x^2+x+3}$ $= \sqrt{2^2+2+3} = \boxed{3}$

its continuous!

(b) $\lim_{x\to 5} \dfrac{x^2-5x}{x^2-7x+10}$ $= \lim_{x\to 5} \dfrac{\cancel{(x-5)}x}{\cancel{(x-5)}(x-2)} = \lim_{x\to 5} \dfrac{x}{x-2} = 5/3$

(c) $\lim_{x\to 2} \dfrac{x^2-5x}{x^2-7x+10}$ $= \lim_{x\to 2} \dfrac{x}{x-2} = \infty$ DNE

(d) $\lim_{x\to\infty} \dfrac{2x^3+x-100}{-4x^3+10x^2} \dfrac{(1/x^3)}{(1/x^3)}$ $= \lim_{x\to\infty} \dfrac{2+1/x^2-100/x^3}{-4+10/x} = \dfrac{2}{-4} = -1/2$

3. For the following piecewise function $f(x)$. [6]

$$f(x) = \begin{cases} ax^2 + 1 & \text{if } x < 1 \\ \dfrac{x^2 + 7x + 1}{2x^2 + 1} & \text{if } x \geq 1 \end{cases}$$

(a) Compute $\lim\limits_{x \to 1^-} f(x)$.

$$= \lim_{x \to 1^-} ax^2 + 1 = \boxed{a+1}$$

(b) Compute $\lim\limits_{x \to 1^+} f(x)$. $= \lim\limits_{x \to 1} \dfrac{x^2+7x+1}{2x^2+1} = \dfrac{1+7+1}{2+1} = 3$

(c) Find a value for a that makes the function continuous at $x = 1$. Justify your answer using limit calculations.

Need $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = f(1)$

$$a+1 = 3 = 3$$

$$a+1 = 3$$

$$\boxed{a=2}$$

4. Consider the function $f(x) = \dfrac{1}{3x-2}$

(a) Find the domain of $f(x)$ [1]

$3x-2 \neq 0$

$\boxed{x \neq 2/3}$

(b) Use the limit definition of the derivative to compute $f'(x)$ [6]

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{(3x-2)}{(3x-2)}\frac{1}{3x+3h-2} - \frac{1}{3x-2}\frac{(3x+3h-2)}{(3x+3h-2)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(3x-2)-(3x+3h-2)}{(3x+3h-2)(3x-2)}}{h} = \lim_{h \to 0} \frac{\frac{\cancel{3x} - \cancel{2} - \cancel{3x} - 3\cancel{h} + \cancel{2}}{(3x+3h-2)(3x-2)}\left(\frac{1}{h}\right)}{\cancel{h}\left(\frac{1}{h}\right)}$$

$$= \lim_{h \to 0} \frac{-3}{(3x+3h-2)(3x-2)} = \boxed{\frac{-3}{(3x-2)^2}}$$

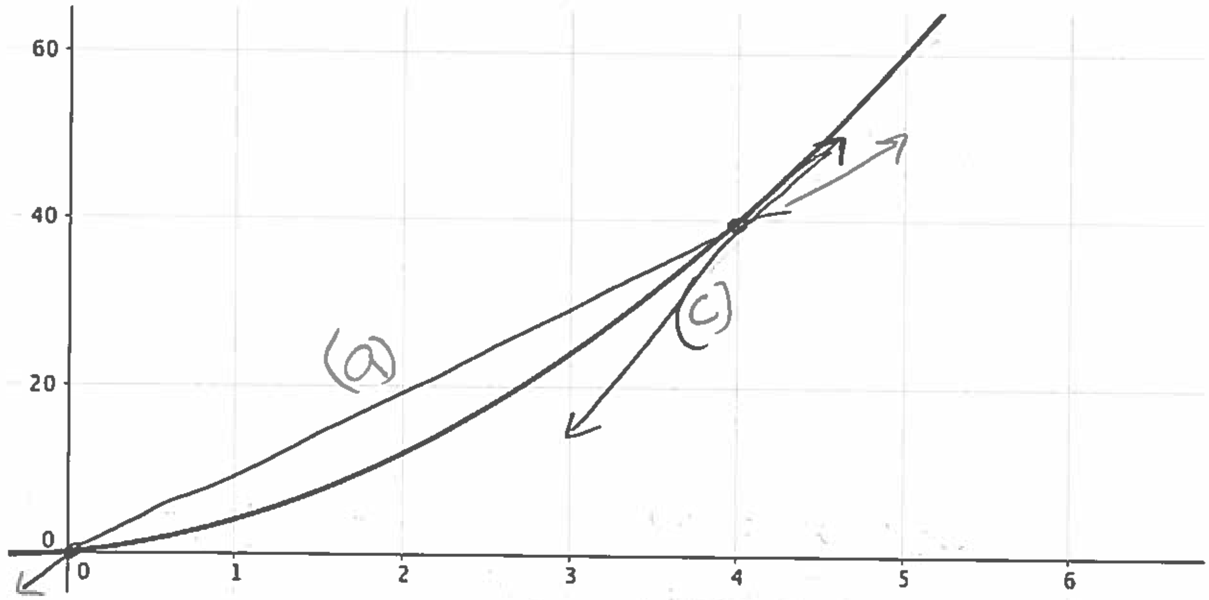
(c) Is $f(x)$ differentiable for all values of x? Explain why or why not in one sentence. [1]

NO! $x \neq 2/3$

$f'(x)$ is undefined there

above limit DNE if $x = 2/3$

5. The height (in feet above the ground) of a rocket t seconds after launch is given by the function $s(t) = 2t^2 + 2t$, whose graph is given below.



(a) On the graph, draw the secant line between $(0, s(0))$ and $(4, s(4))$. Compute the slope of this line. [2]

$$\frac{s(4)-s(0)}{4-0} = \frac{(2\cdot 4^2+2\cdot 4)-(2\cdot 0^2+2\cdot 0)}{4} = \frac{40}{4} = \boxed{10}$$

(b) Explain how your answer in (a) relates to the velocity of the rocket. [1]

Its the average velocity between 0 seconds & 4 seconds

(c) On the graph, sketch the tangent line at the point $(4, s(4))$. [2]

(above)

(d) Explain how the slope of the line you sketched in part (c) relates to the velocity of the rocket. [2]

Its the insteneous velocity at $t=4$ seconds

6. Let I be the number of professors at the University of Portland who currently have the flu. Let $P = f(I)$ be the percentage of U of P students that are infected by a professor. [8]

(a) Interpret with units the statement $5 = f(20)$

When 20 professors have the flu, they will infect 5% of UP students

(b) Interpret with units the statement $f^{-1}(35) = 100$

If 35% of the students are infected by profs, that means 100 professors have the flu.

(c) Interpret with units the statement $(f^{-1})'(35) = 6$

* Every percentage increase of students infected by profs means 6 more profs have the flu. (when 35% are infected)

* The rate of change of professors infected with the flu when 35% of the students are infected is 6.

* A small change in percentage of students infected (when 35% are infected) causes an increase of six times as many professors (as that % change)

(d) Use the above information to approximate $f^{-1}(37)$

$$\underbrace{f^{-1}(37)}_{} = \underbrace{f^{-1}(35)}_{y_1} + \underbrace{(37-35)}_{x-x_1} \cdot \underbrace{(f^{-1})'(35)}_{m}$$

$$= 100 + 2 \cdot 6 = \boxed{112}$$