

*University of Portland*  
*Department of Mathematics*

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CALCULUS I

EXAM 1

FALL 2017

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NAME: \_\_\_\_\_

**Read This First!**

- Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- Calculators are allowed per our stated calculator policy, but you must show all your work in order to receive credit on the problem.

I attest that I have neither given nor received help of any kind on this exam.

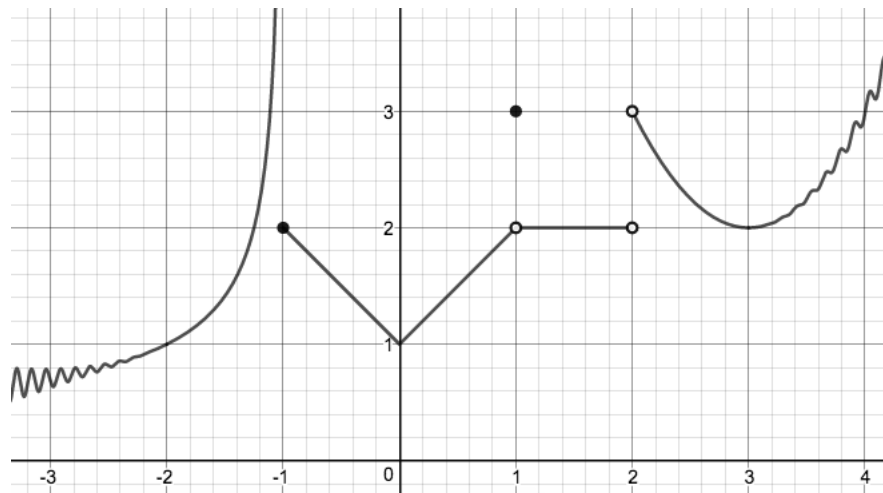
SIGNATURE: \_\_\_\_\_

**Grading - For Administrative Use Only**

Question	Points	Score
1	15	
2	12	
3	6	
4	8	
5	7	
6	8	
Total:	56	

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1. The graph of a function  $f(x)$  is pictured below (you don't need to show work)



- (a) Compute  $\lim_{x \rightarrow -1^-} f(x)$ ,  $\lim_{x \rightarrow -1^+} f(x)$ , and  $\lim_{x \rightarrow -1} f(x)$ . Is  $f(x)$  continuous at  $x = -1$ ? [4]
- (b) Compute  $\lim_{x \rightarrow 0^-} f(x)$ ,  $\lim_{x \rightarrow 0^+} f(x)$ , and  $\lim_{x \rightarrow 0} f(x)$ . Is  $f(x)$  continuous at  $x = 0$ ? [4]
- (c) Compute  $\lim_{x \rightarrow 1^-} f(x)$ ,  $\lim_{x \rightarrow 1^+} f(x)$ , and  $\lim_{x \rightarrow 1} f(x)$ . Is  $f(x)$  continuous at  $x = 1$ ? [4]
- (d) For each  $x$ -value listed below, determine  $f'(x)$ . Write DNE if the derivative doesn't exist at that point. [3]
- i.  $x = -0.5$
  - ii.  $x = 0$
  - iii.  $x = 3$

2. Evaluate the following limits. Show work and justify your answers using algebra!

[12]

(a)  $\lim_{x \rightarrow 2} \sqrt{x^2 + x + 3}$

(b)  $\lim_{x \rightarrow 5} \frac{x^2 - 5x}{x^2 - 7x + 10}$

(c)  $\lim_{x \rightarrow 2} \frac{x^2 - 5x}{x^2 - 7x + 10}$

(d)  $\lim_{x \rightarrow \infty} \frac{2x^3 + x - 100}{-4x^3 + 10x^2}$

3. For the following piecewise function  $f(x)$ .

[6]

$$f(x) = \begin{cases} ax^2 + 1 & \text{if } x < 1 \\ \frac{x^2 + 7x + 1}{2x^2 + 1} & \text{if } x \geq 1 \end{cases}$$

(a) Compute  $\lim_{x \rightarrow 1^-} f(x)$ .

(b) Compute  $\lim_{x \rightarrow 1^+} f(x)$ .

(c) Find a value for  $a$  that makes the function continuous at  $x = 1$ . Justify your answer using limit calculations.

4. Consider the function  $f(x) = \frac{1}{3x-2}$

(a) Find the domain of  $f(x)$

[1]

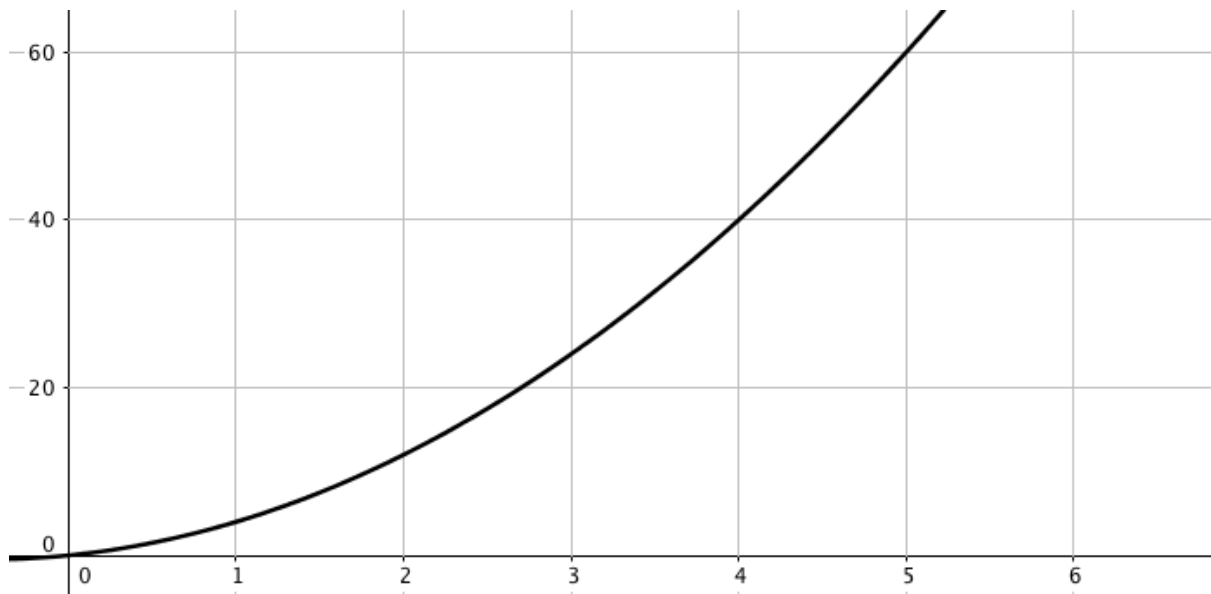
(b) Use the limit definition of the derivative to compute  $f'(x)$

[6]

(c) Is  $f(x)$  differentiable for all values of  $x$ ? Explain why or why not in one sentence.

[1]

5. The height (in feet above the ground) of a rocket  $t$  seconds after launch is given by the function  $s(t) = 2t^2 + 2t$ , whose graph is given below.



- (a) On the graph, draw the secant line between  $(0, s(0))$  and  $(4, s(4))$ . Compute the slope of this line. [2]
- (b) Explain how your answer in (a) relates to the velocity of the rocket. [1]
- (c) On the graph, sketch the tangent line at the point  $(4, s(4))$ . [2]
- (d) Explain how the slope of the line you sketched in part (c) relates to the velocity of the rocket. [2]

6. Let  $I$  be the number of professors at the University of Portland who currently have the flu. Let  $P = f(I)$  be the percentage of U of P students that are infected by a professor. [8]

(a) Interpret with units the statement  $5 = f(20)$

(b) Interpret with units the statement  $f^{-1}(35) = 100$

(c) Interpret with units the statement  $(f^{-1})'(35) = 6$

(d) Use the above information to approximate  $f^{-1}(37)$