EGR 111
Heat Transfer

The purpose of this lab is to use MATLAB to determine the heat transfer in a 1-dimensional system.

New MATLAB commands: (none)

1. Heat Transfer 101

Heat Transfer is thermal energy in transit due to a temperature difference. One experiences heat transfer every day. Make a cup of hot cocoa or tea. Is it too hot? Just wait a bit. Thermal energy is in transit due to the difference in temperature between the lava-hot beverage and the surrounding air. In Figure 1 below, Object A has a higher temperature than Object B. If Objects A and B come into contact, heat transfers from Object A to Object B.

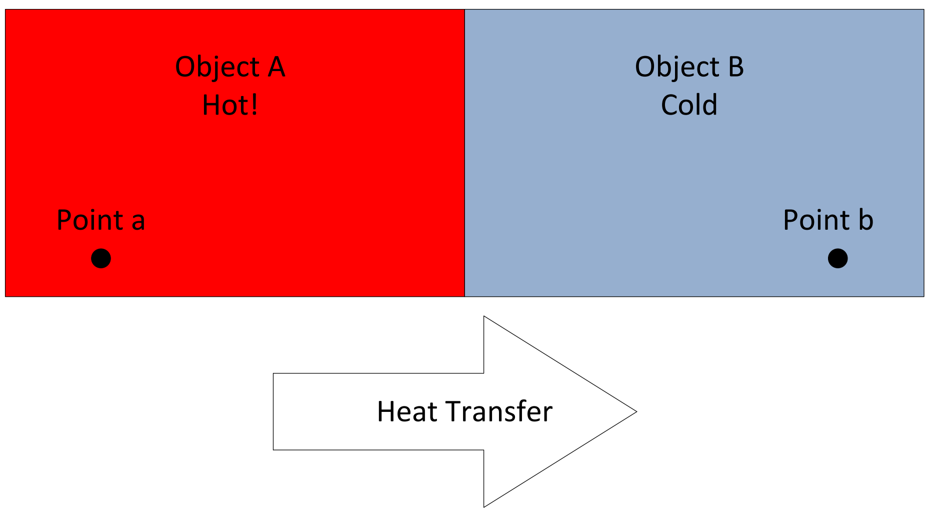

Figure 1. Heat transfer between a hot object A and a cold object B.

In the figure above, the type of heat transfer process that is taking place is called conduction. In conduction, heat transfer occurs across a stationary medium. When performing conduction analysis, engineers want to determine the temperature field in a medium. That is, they want to know the temperature distribution which represents how temperature varies with position in the medium. For example, in the figure above, in the initial moment that Object A comes into contact with Object B, the temperature at Point a is much greater than the temperature at Point b. However, if left in contact for a few minutes or a few hours, the temperatures at Points a and b would likely be equal.

In an engineering context, conduction analysis is important for many reasons. From a structural perspective, a temperature distribution can say something about the thermal stresses in a material. For example, ovens should not be built from materials susceptible to thermal stresses. Moreover, a temperature distribution can offer insight into the thickness of an insulating material that is required in a winter jacket.

To help with conduction analysis, engineers use Fourier's Law which states that heat transfer through a material is proportional to the negative gradient in the temperature and to the area through which the heat is flowing. This means that for a three dimensional object, one must consider the temperature gradient in the x, y, and z axes as well as the volume of the object. This analysis can get tricky, and so here we will only consider a 1-dimensional object.

2. Analyzing an Aluminum Bar

Analyzing a 1-dimensional object gives some insight into heat transfer as well as how to use MATLAB to perform a conduction analysis. Consider Figure 2 below which shows a 0.2 m long, pure aluminum bar, perfectly insulated at the top and bottom, so that heat transfer may occur only in the horizontal direction. Also observe that it has five nodes, T_1 through T_5 which are reference points for the analysis.

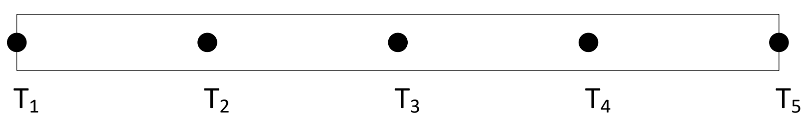

Figure 2. A 0.2 m long, pure aluminum bar, perfectly insulated at the top and bottom.

Fourier's law and partial differential equations yields the following equation which governs how heat transfer takes place over time in a 1-dimensional object.

$$T_i^n = T_i^{n-1} + \frac{\alpha \Delta t}{(\Delta x)^2}\left[T_{i+1}^{n-1} - 2T_i^{n-1} + T_{i-1}^{n-1}\right]$$

The temperature T at node i at time step n depends on the temperature from the previous time step n-1 at node i, the node to the right (node i+1), the node to the left (node i-1), the thermal diffusivity of the material α, the change in time between time steps Δt, and the distance between nodes Δx. The thermal diffusivity of aluminum is $\alpha = 97.1 \times 10^{-6}$ m^2/s.

Let's use a 2-dimensional matrix called T (Temperature Matrix) to store the temperature data. Each row of the matrix T will store the temperatures for each time step n, and each column will store the temperatures for each of the five nodes. Thus T_i^n, which is the temperature for a given time step n and node i, will be stored in the matrix at T(n,i), which is the n^{th} row and i^{th} column.

The number of rows in T must be the same as the number of time steps, and the number of columns in T must be equal to the number of nodes.

Suppose that we want to analyze the bar using 5 nodes from t = 0 to t = 300 seconds, with time slices $\Delta t = 0.1$ seconds. Open a new script file and initialize the following values:

```
clear

% Define thermal diffusivity for aluminum.
alpha = 97.1e-6;

% Define the total time, in seconds.
totalTime = 300;

% Define delta t, in seconds.
dt = 0.1;

% Define number of nodes in the bar
nodes = 5;

% Define the length of the bar, in meters
len = 0.2;

% Calculate the distance between nodes, delta x
dx = len / (nodes - 1);

% Define the number of time steps
% The plus 1 is necessary because the time starts at t = 0 sec
timeSteps = totalTime/dt + 1;

% Define the Temperature Matrix, T
T = zeros(timeSteps, nodes);
```

Suppose that the initial temperature of the bar is 20°C, but starting at time t = 0 seconds, the ends of the bar are heated to 200°C and kept at a constant 200°C for the duration of the experiment. Initialize T to reflect the initial temperature conditions.

```
% Initialize the temperature matrix
% The first row is the initial 20 degrees C.
% The first and last columns are 200 degrees C.
T(1, :) = 20;       % initial temperature
T(:, 1) = 200;      % constant temperature on left side
T(:, nodes) = 200;    % constant temperature on right side
```

The first few rows of the initialized T matrix are shown below.

```
T =
 T1   T2    T3    T4    T5
200  20    20    20    200  <- temperatures at t = 0.0 sec
200   0     0     0    200  <- temperatures at t = 0.1 sec
200   0     0     0    200  <- temperatures at t = 0.2 sec
200   0     0     0    200  <- temperatures at t = 0.3 sec
200   0     0     0    200  <- temperatures at t = 0.4 sec
200   0     0     0    200  <- temperatures at t = 0.5 sec
200   0     0     0    200  <- temperatures at t = 0.6 sec
200   0     0     0    200  <- temperatures at t = 0.7 sec
200   0     0     0    200  <- temperatures at t = 0.8 sec
```

Let's start by calculating the temperature of the bar at time step n = 2 (t=0.1 sec) for node 2. This value will be stored in the second row and second column of the T matrix, which is T(2,2).

```
T(2,2) = T(1,2) + alpha*dt/dx^2*(T(1,3)-2*T(1,2)+T(1,1));
ans =
   20.6991
```

We would calculate the temperature at time step n = 2 (t=0.1 sec) for node 3 as follows:

```
T(2,3) = T(1,3) + alpha*dt/dx^2*(T(1,4)-2*T(1,3)+T(1,2));
ans =
20.0000
```

It would be very tedious to compute all of the required values in this way, so we need to set up a nested loop to compute the temperature values for each node in each time step.

Note that since the temperature at a given time step depends on the temperatures at the previous time step, **the nested loop needs to compute the values for all of the nodes in each time step before going on to the next time step**.

Exercise 1: Write a program that uses nested `for` loops to compute the temperature for an aluminum bar whose initial temperature is 20°C at time t = 0 seconds. At time t = 0, both ends of the bar are instantly brought up to a temperature of 200°C and held constant for the rest of the time. The goal is to determine the temperature at the center of the bar at t = 300 seconds using $\Delta t = 0.1$ seconds and 5 nodes along the length of the bar. Use the above program to help you formulate your solution. Check to make sure that your program gives the correct values for T(2,2) and T(2,3) as given above. Then plot the temperature of the center node (node 3) as a function of time (in seconds). Are the initial and final values of the plot what you expect?

Checkpoint1: Show the instructor your script file and plot from Exercise 1.