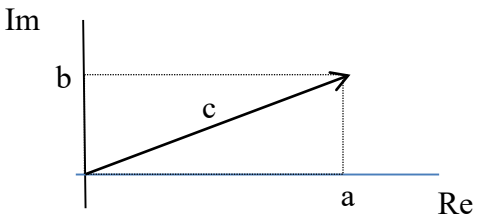
EGR 111
Complex Numbers

This lab introduces the MATLAB commands used to manipulate complex numbers.

New MATLAB commands:

```
real, imag, abs, angle, cart2pol, pol2cart
```

Complex numbers have many uses in engineering. Perhaps the most common is using complex numbers to represent vectors, such as forces and velocities. As shown below, the complex number c = a + b i can be plotted in the real (Re) and imaginary (Im) plane, where "a" is the real part, "b" is the imaginary part, and $i = \sqrt{-1}$. Like a vector, c has a magnitude and a direction. You can observe that the magnitude of c is given by $|c|=\sqrt{a^2+b^2}$, and the angle associated with c is $\angle c = \tan^{-1}(b/a)$.



While this type of calculation can be done explicitly, MATLAB has built-in support for complex numbers, which makes it relatively easy to work with them. You can see that this is the case by typing "`1i`" into the command window and hitting Enter. The lines below that start with ">>" are commands that you type into MATLAB's command window. The rest of the lines are output from MATLAB.

```
>> 1i
ans =
        0 + 1.0000i
```

In the number 0 + 1.0000i, the first part (0) is the real part and the part that is multiplied by i is the imaginary part (1.0000).

In electrical engineering, it is common to define $j = \sqrt{-1}$, and MATLAB supports this use also.

```
>> 1j
ans =
        0 + 1.0000i
```

Note: When MATLAB starts up, it predefines the value of the variables i and j to be the square root of -1, but it is good practice to use the forms 1i or 1j instead of i or j because the values of the variable i and j can be changed just like any other variable, which can cause confusion.

In order to enter a complex number that is in rectangular form, such as $z_1 = 2 + 4i$ into MATLAB, we can use the following command:

```
>> z1 = 2 + 4i
z1 =
   2.0000 + 4.0000i
```

Note that we must write "4i" and not "i4" because "i4" would be interpreted as a variable, and since there is no variable named "i4", MATLAB will give an error.

```
>> z1 = 2 + i4
Undefined function or variable 'i4'.
```

Alternatively, we could multiply the imaginary part by "1i" as follows.

```
>> z1 = 2 + 1i*4
z1 =
   2.0000 + 4.0000i
```

MATLAB stores complex numbers in rectangular form (also called Cartesian form), so a number in polar form must be converted to rectangular form to store it in a variable. For example, let's convert the number $z_3 = 4\angle\pi/4$ to rectangular form. The function `pol2cart` converts a number from polar form to rectangular form by computing the real and imaginary parts of the number. Then the complex number can be entered by multiplying the imaginary part by 1i (or 1j) and adding it to the real part as follows.

```
>> [a,b] = pol2cart(pi/4,4)
a =
    2.8284
b =
    2.8284
>> z3 = a + 1i*b
z3 =
   2.8284 + 2.8284i
```

Note that in the function `pol2cart`, the first input argument is the angle of the complex number in radians, and the second argument is the radius. The first output argument is the real part of the number, and the second output argument is the imaginary part.

Also, note that the "[a,b] =" part of the first command above tells MATLAB to put the first output in a variable named "a", and put the second output in a variable named "b".

If you don't tell MATLAB where to store the output values, then only the first output is saved in the variable named "ans" and the other output is discarded.

```
>> pol2cart(pi/4,4)      % the second output is lost!
ans =
    2.8284
```

MATLAB also has a command to convert from rectangular form to polar form. For example, to convert the number z1 = 2 - j4 to polar form, we can use the following command:

```
>> [theta,r] = cart2pol(2,-4)
theta =
   -1.1071
r =
    4.4721
```

Note that the angle is in radians for most built-in MATLAB commands.

Once the complex numbers are stored in MATLAB, we can multiply, divide, add, subtract using the usual operators (*, /, +, -). For example, to compute z1*z3, we can simply type the following command:

```
>> z = z1*z3
z =
  -5.6569 + 16.9706i
```

The real part of a complex number can be computed using the function `real` as follows.

```
>> real(z)
ans =
   -5.6569
```

Likewise, the imaginary part of a complex number can be computed using the function `imag` as follows.

```
>> imag(z)
ans =
   16.9706
```

Some of the MATLAB commands related to complex numbers are listed below where x, y, r, and theta are real numbers and z is a complex number:

real(z) - find the real part of z
imag(z) - find the imaginary part of z
abs(z) - find the magnitude or radius of z
angle(z) - find the angle (or phase) of z in radians (uses a two-argument arctan)
[theta,r] = cart2pol(a,b) – convert from rectangular (Cartesian) form to polar form
[a,b] = pol2cart(theta,r) – convert from polar form to rectangular (Cartesian) form

Exercise 1: Use MATLAB to find the value of $z = z_1 + z_2$ where $z_1 = \frac{1}{2} - \frac{1}{2}i$, and $z_2 = 0.707107\angle\pi/4$.

Exercise 2: Use MATLAB to find the value of $z = z_3 + z_4 + z_5 z_6$ where $z_3 = 1 + \frac{1}{2}i$, $z_4 = -1 + \frac{1}{2}i$, $z_5 = 2\angle - \pi/4$, and $z_6 = 0.5\angle - \pi/4$.

Checkpoint 1: Show the instructor your commands and the results for Exercises 1 and 2.