Information Asymmetry, Market Participation, and Asset Prices

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Abstract

We derive a separation theorem: investors hold a common risk-adjusted market portfolio regardless of their information sets, and a portfolio based upon their private signals. This implies that investors have non-negligible holdings of assets they know little about, so nonparticipation remains a puzzle in a rational information setting. In contrast with the well-known prediction of a risk premium for nonparticipation, in our model risk premia satisfy the CAPM. Investors hold a fund that provides the risk-adjusted market portfolio, even if they are unaware of the fund's composition. In contrast with a literature on information risk, there is no risk premium for information asymmetry.

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1 Introduction

In his Presidential Address to the American Finance Association, Merton (1987) provided a model in which subsets of investors refrain from participating in the markets for different stocks, resulting in risk premia for stocks with more limited participation. Although nonparticipation is exogenous in the model, Merton explains this nonparticipation as being a consequence of information costs. Merton therefore offers a rational interpretation for nonparticipation, and explores its consequences.

This theory has been highly influential in guiding empirical work in financial economics and accounting. It has been appealed to as an explanation for major nonparticipation puzzles such as home bias (Foerster and Karolyi 1999), and for patterns of return predictability such as the effect of breadth of ownership (Kadlec and Mcconnell 1994; Bodnaruk and Ostberg 2009; Chen, Noronha, and Singal 2004), the effect of geographic dispersion of firm operations (García and Norli 2012), and the accrual anomaly (Lehavy and Sloan 2008).

We call the idea that an uninformed investor optimally takes a zero position when trading with informed investors the *zero holdings conjecture*. The zero holdings conjecture is quite intuitive. It seems dangerous for an investor who knows nothing about Ford Motors, for example, to take a position in Ford when in doing so he must trade with other better-informed investors.

There are at least three possible justifications for the zero holdings conjecture. First, it might actually be true that in equilibrium in frictionless markets with asymmetric information, investors optimally choose zero holdings of assets they lack information about. Second, they might optimally have very small holdings of such assets, so that when transaction costs are introduced, it becomes optimal to hold zero. Third, as suggested by Merton (1987), investors may have zero holdings of certain assets because they are *unaware* of such assets. This idea requires careful interpretation, as we discuss and model.

The first possible justification has been shown to be invalid. In settings with constant absolute risk aversion preferences, multiple risky assets, and costly information acquisition, investors in general hold nonzero quantities of all assets. In particular, in natural settings, in equilibrium investors specialize in acquiring information about only a subset of assets, and each investor tilts her portfolio toward the assets that she learns about.[1] The intuition offered for nonzero holdings of all assets is that there is a diversification benefit to holding even those assets about which the investor knows little.[2]

As these papers show, the holdings of the informed in the assets that they know about are larger than the holding of those who are uninformed about these assets, because such assets are riskier to the uninformed. This does not, however, tell us whether the holdings of the uninformed are sizable, or whether their participation generates substantial individual benefits, since being uninformed makes holding an asset riskier. So these findings do not resolve whether the second possible justification for the zero holding conjecture—that in a frictionless setting the equilibrium holdings of the uninformed are minimal and generate low individual benefits—is valid. If it is, market frictions might cause rational nonparticipation, perhaps confirming the asset pricing implications of the Merton model.

We show that it does not in general follow that uninformed investors as a group will hold positions that are close to zero. Although it would be unwise for an uninformed investor to place a bet in a stock in the hope of profit at the expense of better informed investors, if the purpose is risk sharing between different investors, not speculation, then it seems reasonable that the market would accommodate a substantial shift from the publicly known endowment position of the uninformed at low cost. So a reasonable conjecture is that, regardless of initial endowments and informational conditions, in-

[1]See Van Nieuwerburgh and Veldkamp (2009), Van Nieuwerburgh and Veldkamp (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2014) and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2015).

[2]The traditional single-asset literature on information and securities markets (e.g., Grossman and Stiglitz (1976), Kyle (1985)) suggests another reason for non-zero holdings: uninformed investors may take non-zero positions since there is a benefit to trading against price variations to absorb the trades made by noise/liquidity traders.

vestors trade toward something akin to the market portfolio (though with adjustments relating to information signal realizations, and for information-induced risk differences per the insights discussed at footnote 1).

This optimal risk-sharing argument is based on the insight that unless the informed are *very* well informed, they would still face substantial risk from holding the entire aggregate endowment of the assets they have information about. So unless the aggregate value of an asset is very small, they have a strong incentive to share its risk with uninformed investors. This is not a diversification argument; it has nothing to do with the informed investor's decision to invest in other assets (those about which the given investor lacks information). The argument is based upon an equilibrium consideration, not just portfolio optimization.

In a rational expectations setting with asymmetric information, we show that this risk-sharing intuition is correct, and therefore the weakened zero holding conjecture—even in more 'moderate' modified versions—does not follow. Instead, as one of four portfolio components, investors trade to hold what we call the *risk-adjusted market portfolio*, defined as the endowed market (i.e., the market excluding supply shock) reweighted to reflect the volatilities of the supply shocks, the average precision of investors' private information, and investors' risk aversion. This trading component is to share risk. The other three components are a contrarian position taken to accommodate shifts in market price; a non-negative position that reflects the greater safety of stocks about which they are informed; and a speculative position taken to exploit informational superiority.

In equilibrium, the position taken by an uninformed investor has just two of these components. First is a position in the risk-adjusted market portfolio, which is taken for risk sharing reasons. Second, as is standard in models of information and securities markets, is a contrarian position.[3] In addition to the two components that uninformed

[3] An uninformed trader accommodates the trades of other investors as reflected in price movements that derive in part from supply shocks. Such shocks are optimally shared among both the informed and uninformed. In trading as contrarians, the uninformed sometimes lose money to the informed, but this is offset by the risk premium benefit of accommodating the supply shock.

investors take, informed investors trade to exploit private information, and hold an additional non-negative deterministic position in assets they have information about since this knowledge makes such assets less risky.

All of this is under the assumption that investors share conventional common prior beliefs about fundamental security payoffs prior to the arrival of any private information. However, a possible justification for the zero holdings conjecture might be that this still grants the 'uninformed' some knowledge (or perceived knowledge) about assets for which they lack private signals, in the form of informative prior beliefs. We therefore examine a limiting case of the model in which prior beliefs are uninformative.

The setting with diffuse priors ought to be the best possible case for the zero holdings conjecture, but in fact highlights even more clearly that the zero holdings conjecture fails. Even when uninformed investors are extremely ignorant, the outcome is not zero holdings of stocks that they know nothing about; it is zero *deviation* of holdings from the weights in the risk-adjusted market portfolio.

In equilibrium, uninformed investors with diffuse priors refuse to accommodate noise trades, because the perceived adverse selection associated with doing so is very high. From their perspective virtually all of the variation in price comes from private information rather than supply shocks. Hence, the risk sharing benefit to trading against price variations is dominated by the expected loss to informed investors. In equilibrium uninformed investors hold the risk-adjusted market portfolio, and it is the informed investors who accommodate supply shocks.

The diffuse prior case illustrates especially clearly why it pays to hold securities one lacks information about. It is self-confirming for uninformed investors to trade to reach the risk-adjusted market portfolio and for everyone else to foresee and accommodate such trades without price pressure or cost to the uninformed.[4] All investors foresee trading to these positions for risk-sharing reasons. Incrementally, the informed trade further

[4]This is somewhat analogous to models of sunshine trading, wherein risk-sharing trades that are preannounced prior to information arrival can be made without cost since they are known to be uninformative (Admati and Pfleiderer 1991).

to absorb the supply shocks (since the informed have sufficient knowledge to absorb those shocks without bearing unduly high risk); and hold an incremental non-negative position to take advantage of the lower risk of holding assets they have information about.

We do show that the weakened zero holdings conjecture can hold in the following sense. Consider a small uninformed investor who faces a fixed transaction cost of participating in the market for one of the assets. There are parameter values such that a given transaction cost deters the investor from participating under asymmetric information, but not under symmetric information.

However, we further show that the intuition underlying the zero holdings conjecture is not robust, by performing a continuous comparative statics on the information disadvantage to an uninformed investor as captured by the fraction of informed investors. We find that there is a range of parameter values in which uninformed investors' incentives to participate increases with their information disadvantage. This shows that the idea that the information disadvantage is a deterrent to participation is a parameter-dependent possibility, not a general principle. Since greater information disadvantage to uninformed investors can encourage their participation, the pricing implications of the Merton model about the effects of information costs do not follow.

As for the frictionless rational expectations equilibrium, though investors have asymmetric information and have different asset holdings, there is full participation in all markets. So a version of security market line of the CAPM holds, where the common component of all investors' holdings, which is independent of any individual investor's information, is the pricing portfolio.

The third possible justification for the zero holdings conjecture was that 'information costs' mean a cost of becoming aware of the *existence* of a possible investment asset. There are certainly behavioral settings in which lack of awareness can cause nonparticipation, and there is evidence that awareness matters (Guiso and Jappelli 2005).

Our focus here is on a notion of unawareness that could be viewed as rational. To illustrate by example, most U.S. investors have heard of Ford and General Motors, but there are many firms that most investors do not know by name. Furthermore, investors may know almost nothing about the characteristics of such assets (their payoff distributions), nor the exact number of such assets that are available for trading. However, most U.S. investors do know that such assets exist. Many have heard of the Dow Jones Industrial Average or the S&P 500 even though they could not name the constituents of these indices nor describe the characteristics of these constituents.

We therefore interpret 'unawareness' as meaning that there are assets whose characteristics, or even names, investors know nothing about. Investors may not even know the number of such assets. We assume that investors *do* correctly understand that such assets may exist.[5] To capture the idea that investors know almost nothing about the assets that they are unaware of, we assume that investors have diffuse priors about characteristics such as an asset's mean payoff and volatility. Investors are otherwise fully rational, and have the opportunity to invest in assets that they are unaware of, in this sense, via a low-cost index mutual fund or ETF.

It seems extremely risky for an investor to invest anything in an asset the investor is unaware of in this sense, since the investor knows nothing about the asset's factor sensitivities or volatility. Nevertheless, we show that in equilibrium, lack of information about the identity and characteristics of assets does *not* justify nonparticipation. Investors voluntarily invest in all assets, either directly, or via a fund that invests in all stocks in an uninformed way, conditioning only on equilibrium prices.

This result follows from a new portfolio separation property. Under what we call *portfolio information separation*, investors first choose to hold an *informationally passive portfolio*, and then take an addition position to exploit their own private information;

[5] More severe unawareness than this should probably not be called rational. Even an investor who does not know the name and characteristics of every asset should have some prior belief about the possible existence of such assets. This rules out extreme priors in which the investor is sure that a stock with certain characteristics does not exist, yet such a stock actually does exist. We will see that this rationality condition implies that unaware investor will take into account the possibility of diversification benefits from holding such stocks.

and finally invest the rest of their endowments in the riskfree asset. The position taken to exploit information depends on neither the prior nor on the information extracted from prices.

The uninformed investors' risky assets holdings consist of this informationally passive portfolio, which is just the risk-adjusted market portfolio combined with the position taken to trade as contrarians to market prices. It is therefore optimal for uninformed investors—even if they do not know the names, characteristics, and numbers of certain assets—to hold a fund that invests in the informationally passive portfolio. The uninformed therefore participate non-negligibly in all asset markets.

To sum up, none of the three rational justifications for the zero holdings conjecture provide clearcut support for it. So the pricing implications of Merton (1987) also do not follow as an outcome in a market with rational investors and asymmetric information—even if other market frictions are added.[6]

Empirically we do observe a remarkable lack of participation by most investors in many securities and asset classes. Our analysis therefore points to causes other than rational responses to asymmetric information. One possible explanation is other market frictions that are severe enough to block participation even when information is symmetric. Another is that investors are imperfectly rational, and in particular are fearful of stocks that they are less familiar with (see, e.g., models of ambiguity aversion, participation, and pricing (Dow and Werlang (1992), Uppal and Wang (2003), Epstein and Miao (2003)), Cao, Wang, and Zhang (2005), and Cao et al. (2011)). Alternatively, owing to narrow framing, investors may overestimate the risk of adding a security to their portfolios (Barberis, Huang, and Thaler 2006).

Explicitly modeling whether information asymmetry deters uninformed investors from trading is important for at least two reasons. First, a substantial empirical litera-

[6] A further problem with the idea that transactions cost might justify a risk premium for nonparticipation by uninformed investors is that transactions costs imply risk premia for nonparticipation even if information is symmetric (Mayshar 1979; Hirshleifer 1988). So once we appeal to fixed transaction costs, it is not obvious why the information cost part of the story is needed.

ture has taken evidence of nonparticipation or its consequences as confirming Merton's information cost conjecture. It is important to clarify that the asymmetric interpretation of this evidence lacks a clear theoretical basis.

Second, the source of nonparticipation matters for the design of policy to promote investor welfare and the efficiency of security markets. If nonparticipation results from information costs, then it can be remedied simply by providing investors with more information. For example, participation would be improved by regulation requiring additional reporting of accounting information, or more frequent disclosures.

If, however, nonparticipation derives from imperfect rationality, then policies would be needed that directly address the psychological constraints and biases of investors. In such settings, providing greater amounts of information to investors could overwhelm their limited attention, so that the intervention could make nonparticipation *more* severe.

Another important strand of research examines the relation between information asymmetry and risk premia even when all investors participate in the capital market. In an influential paper, Easley and O'Hara (2004) argue that information asymmetry creates something called *information risk*, which, in equilibrium, induces a risk premium. Specifically, suppose that in some cases the preponderance of information signals in the market for an asset are received publicly, and in other cases privately. In the model of Easley and O'Hara (2004), the asset with more private information receives higher expected returns.

Hughes, Liu, and Liu (2007) extend Easley and O'Hara (2004) to a factor setting to study the relation between information asymmetry to expected returns when there are multiple assets and investor signals. They derive conditions under which greater information asymmetry does or does not imply higher risk premia. In a setting with a single risky asset, Lambert, Leuz, and Verrecchia (2012) find that with perfect competition, the asset's expected return is determined by the average precision of investors' information conditional upon prices. They conclude that information asymmetry does not affect expected return.

In all these papers, informed investors observe the same information signals, as in Grossman and Stiglitz (1980) and Easley and O'Hara (2004). The comparative statics in these models are performed by increasing the informed investors' private signal precision while correspondingly decreasing the total precision provided by the public signals. This simultaneously increases information asymmetry and the volatility of cash flows conditional upon price, i.e., the *conventional* risk that uninformed investors face from investing in the relevant assets. So these models do not lend themselves to disentangling any possible effects of information asymmetry as contrasted with risk as conventionally defined in models with symmetric information.

This problem goes beyond the fact that existing literature offer mixed conclusions about whether we should observe a premium for information risk. The problem is a lack of a clearly defined *meaning* of such a risk premium as distinct from the premia implied by a conventional risk measures. Nevertheless, several papers argue in support of the information risk concept, and there has been extensive empirical testing of the prediction that there is a premium for information risk.[7]

Our setting permits a conceptual experiment that offers an exact meaning to the idea of a premium specifically for asymmetric information. This is a comparative statics that varies information asymmetry while holding constant both unconditional and conditional uncertainty about terminal cash flows. In this comparative statics, we increase the fractions of the informed investors who have the maximal information set (information about all assets) and the minimal information set (information about no assets) by decreasing the fraction who have intermediate amounts of information (information about a subset of assets). This unambiguously increases information asymmetry. Risk premia do not change, so this is a counterexample to the general notion that an increase in information asymmetry increases risk premia.

Intuitively, the average private information precision does not change, because of

[7] See Easley, Hvidkjaer and O'Hara (2002, 2010), Duarte and Young (2009), Aslan et al. (2011), El Ghoul et al. (2012), Hwang et al. (2013), Lai, Ng, and Zhang (2014), and Levi and Zhang (2015)).

the offsetting changes in the information possessed by different investors. Uninformed investors do not demand higher risk premium because the uncertainties they are facing (conditional on the equilibrium price) are same as before. So the risk premium estimated based on publicly available information is unchanged. This illustrates that information asymmetry per se does not induce risk premia; it is changes in conventional risk that count.

This comparative statics varies information asymmetry at the portfolio level, in the sense that we make investors more informed by giving them information about a greater number of assets, and make them less informed by giving them information about fewer assets. It is also interesting to examine what happens when the information asymmetry of an individual asset changes. This allows us to vary an information asymmetry proxy that has received greater attention in the empirical literature the probability of information-based trading (PIN). This measure of information asymmetry was introduced by Easley, Hvidkjaer, and O'Hara (2002).

In our second comparative static analysis, corresponding to PIN, we define the information asymmetry proxy in any asset's market as the measure of investors who have private information about such an asset. In varying the number of informed traders about a stock, in general we cannot avoid simultaneously varying the amount of conventional risk (conditional volatility). Given this confounding, we therefore refer to varying the information asymmetry *proxy* rather than varying information asymmetry.

We consider a shift in the composition of investors that increases the number of informed traders in one asset and decreases the mass of informed investors in another asset. We find that the risk premia are *decreasing* in the information asymmetry proxy. This is the opposite of the prediction of a positive risk premium for information risk.

Intuitively, when informed trading increases for a given asset, the information uninformed investors extract from equilibrium prices increases. So conditional on equilibrium prices, uninformed investors face less uncertainty about the asset with a greater fraction of informed traders, and hence demand a lower risk premium.[8] These conclusions may help resolve some of the differing empirical conclusions from the literature on 'information risk' (or what we would call information asymmetry). More importantly, they clarify that the concept of a risk premium for information asymmetry—as contrasted with conventional measures of consumption risk—is not helpful.

2 A Model with Asymmetric Information

There are two dates, date 0 and date 1. There is a continuum of investors with measure one, who are indexed by i and uniformly distributed over $[0,1]$. All investors trade at date 0 and consume at date 1. Any investor i invests in a riskfree asset and N risky assets. The riskfree asset pays r units, and risky asset n pays F_n units of the single consumption good. Taking the riskfree asset to be the numeraire, let P be the price vector of the risky assets and D_i be the vector of shares of the risky assets held by investor i. Let $W_i = (w_{i1}, w_{i2}, \ldots, w_{iN})'$ be the endowed shareholdings of investor i, and let $W = \int_0^1 W_i \mathrm{d}i > 0$ be the aggregate endowments of shares in the capital market. So any investor i's final wealth at Date 1 is

$$\Pi_i = r(W_i' - D_i')P + D_i'F, \tag{1}$$

where $F = (F_1, F_2, \ldots, F_N)'$. The first term in (1) is the return of investor i's investment in the riskfree asset, and the second term is the total return from her investments in risky assets. Each investor i's expected utility of consumption at date 0 is

$$\mathbb{E}_i u(\Pi_i) = \mathbb{E}_i \left[-\exp\left(-\frac{\Pi_i}{\rho} \right) \right]. \tag{2}$$

The expectation operator, $\mathbb{E}_i$, is based on investor i's information. The parameter ρ is the common risk tolerance coefficient. We verify in Appendix B that similar results hold in an extension of the model with heterogeneous risk tolerances.

[8] So this effect does not derive from information asymmetry per se, it derives from the use of a proxy for information asymmetry, PIN, which does not hold constant the total amount of uncertainty faced by uninformed investors.

We assume that F is normally distributed. Let $\bar{F}$ be the mean vector of F, and let V be the variance-covariance matrix of F. So $\bar{F}$ and V summarize the prior information of F. For now, we assume that V exists, and thus $\bar{F}$ is well-defined. Besides the prior information, any investor i's information consists of the equilibrium price vector and the realization of a private information signal S_i, which is correlated with F. In particular, $S_i = F + \epsilon_i$, where F and ϵ_i are independent; and ϵ_i and ϵ_j are also independent. Each ϵ_i is normally distributed, with mean zero and precision matrix Ω_i^{-1}. As is standard, the independence of the errors implies that in the economy as a whole signal errors will average out, so that the equilibrium pricing function does not depend on the error realizations (though it does depend on their distribution).

Without loss of generality, we assume that risky assets are divided into two groups, Γ_1 and Γ_2, where $\Gamma_1 = \{1, 2, \ldots, \bar{N}\}$ and $\Gamma_2 = \{\bar{N}+1, \bar{N}+2, \ldots, N\}$. The purpose of having two groups of assets is to allow for diversity of information among investors who receive information about different groups. Assets in Γ_1 could be viewed as stocks traded in the US market and assets in Γ_2 as stocks traded in a European market. For simplicity, we assume that all asset payoffs are independent. Allowing correlation of asset payoffs within each group does not change the main results. The independence assumption prevents investors who have private information about one asset group from making any inferences about assets in another asset group.

Each investor belongs to one of four groups with different types of information signals. Group $n \in \{1, 2, 12, \varnothing\}$ has $\lambda_n \in (0, 1)$ measure of investors, where $\sum_n \lambda_n = 1$. Group 1 investors possess some information about assets in Γ_1, and Group 2 investors receive private signals about assets in Γ_2; Group 12 investors have private information about all assets, while Group $\varnothing$ investors have no private information about any assets. The structure of private information in this model is summarized below.

	Group 1	Group 2	Group 12	Group $\varnothing$
Ω_i^{-1}	$\begin{bmatrix} \Sigma_1^{-1} & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & \Sigma_2^{-1} \end{bmatrix}$	$\begin{bmatrix} \Sigma_1^{-1} & 0 \\ 0 & \Sigma_2^{-1} \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Here, Σ_1^{-1} is an $\bar{N} \times \bar{N}$ matrix, and Σ_2^{-1} is an $(N - \bar{N}) \times (N - \bar{N})$ matrix. Σ_j^{-1} is the precision matrix of one investor's private information about assets in Γ_j if she is informed about assets in Γ_j. Since both Σ_1 and Σ_2 are covariance matrices, and all assets are independent, Σ_1 and Σ_2 are both diagonal and positive definite. Therefore, both Σ_1^{-1} and Σ_2^{-1} are positive definite and symmetric. For example, if the groups correspond to US versus European stock markets, Group 1 is US investors; Group 2 is European investors; Group 12 is international investors who are investing in and do private research about stocks in both markets; Group $\varnothing$ is investors who have private information about neither US stocks nor European stocks. We define the average precision matrix

$$\Sigma^{-1} = \int_0^1 \Omega_i^{-1} \mathrm{d}i = \begin{bmatrix} (\lambda_1 + \lambda_{12})\Sigma_1^{-1} & 0 \\ 0 & (\lambda_2 + \lambda_{12})\Sigma_2^{-1} \end{bmatrix}. \tag{3}$$

If we randomly draw one investor from the population, the expected precision matrix of her private information is Σ^{-1}. Since all λ's are strictly positive, Σ^{-1} is also invertible (with the inverse matrix Σ) and symmetric. We assume that Group 1 investors and Group 12 investors have the same signal precisions about assets in Γ_1, and Group 2 investors and Group 12 investors have the same precision of signals about assets in Γ_2. This assumption is purely for simplicity and does not affect results.

Finally, there are random supplies of all assets, which prevents asset prices from perfectly revealing F. Denote the random supply of the risky assets by Z. We assume that Z is independent of F and of ϵ_i (for all $i \in [0,1]$). We further assume that Z is normally distributed with mean 0 and covariance matrix U. By independence of assets, U is diagonal and positive definite.

We are interested in a linear rational expectations equilibrium defined as follows.

Definition 1 (Rational Expectations Equilibrium) *A pricing vector P^* and a profile of all investors' risky assets holdings $\{D_i^*\}_{i\in[0,1]}$ constitute a rational expectations equilibrium, if*

1. *Given P^*, $D_i^* \in \arg\max \mathbb{E}_i u(\Pi_i)$ for all $i \in [0,1]$; and*

2. *P^* clears the market, that is,*

$$\int_{i=0}^1 D_i^* \mathrm{d}i = W + Z, \quad \textit{for any realizations of } F \textit{ and } Z. \tag{4}$$

2.1 Equilibrium Characterization

As is standard in the literature of rational expectations equilibrium, we consider the linear pricing function

$$F = A + BP + CZ, \quad \text{with } C \text{ nonsingular.} \tag{5}$$

If and only if B is nonsingular, equation (5) can be rearranged to

$$P = -B^{-1}A + B^{-1}F - B^{-1}CZ, \tag{6}$$

which is the "real" pricing function. Recall that $S_i = F + \epsilon_i$, so conditional on F, P and S_i are independent. Therefore, we can make inferences one by one.

First consider investor i's belief about F conditional on P. Conditional on P, F is normally distributed with mean $A + BP$ and precision $[CUC']^{-1}$. On the other hand, conditional on S_i, investor i's belief about F is also normally distributed, with mean S_i and precision Ω_i^{-1}. Therefore, investor i's belief about F conditional on what the investor observes, P and S_i, is also normally distributed. The mean of the conditional distribution of F is the weighted average of the expectation conditional on the price P, the expectation conditional on investor i's private signal S_i, and the prior mean $\bar{F}$. Therefore, the conditional mean of F is

$$\left[(CUC')^{-1} + \Omega_i^{-1} + V^{-1}\right]^{-1} \left[(CUC')^{-1}(A + BP) + \Omega_i^{-1}S_i + V^{-1}\bar{F}\right]. \tag{7}$$

The precision of the conditional distribution of F is

$$(CUC')^{-1} + \Omega_i^{-1} + V^{-1}. \tag{8}$$

Here Ω_i^{-1} is the precision of investor i's private signal. Since some investors are uninformed, the signal noise matrix Ω_i is only well-defined for Group 12 investors.

Then, from any investor i's first order condition, investor i's demand is

$$\begin{aligned}
D_i &= \rho\left[(CUC')^{-1}+\Omega_i^{-1}+V^{-1}\right] \\
&\quad \left\{\left[(CUC')^{-1}+\Omega_i^{-1}+V^{-1}\right]^{-1}\left[(CUC')^{-1}(A+BP)+\Omega_i^{-1}S_i+V^{-1}\bar{F}\right]-rP\right\} \\
&= \rho\left\{\left[(CUC')^{-1}(A+BP)+\Omega_i^{-1}S_i+V^{-1}\bar{F}\right]-\left[(CUC')^{-1}+\Omega_i^{-1}+V^{-1}\right]rP\right\} \\
&= \rho\left\{(CUC')^{-1}(B-rI)-r\Omega_i^{-1}-rV^{-1}\right\}P \\
&\quad +\rho\Omega_i^{-1}S_i+\rho[(CUC')^{-1}A+V^{-1}\bar{F}]. \qquad (9)
\end{aligned}$$

Substituting equation (9) in to the market clearing condition (4), we can solve the pricing function, which in a rational expectations equilibrium should be same as that in equation (6), for any realizations of F and Z. Therefore, by matching all the coefficients, we can solve all coefficients and characterize the linear rational expectations equilibrium in Proposition 1 below.

Proposition 1 (Equilibrium with Supply Shocks) *In the model with supply shocks, there exists an equilibrium with pricing function*

$$P = B^{-1}\left[F-A-CZ\right], \qquad (10)$$

where

$$\begin{aligned}
A &= \left[\rho^2(\Sigma U\Sigma)^{-1}+\Sigma^{-1}\right]^{-1}\left(\frac{1}{\rho}W-V^{-1}\bar{F}\right) \qquad (11)\\
B &= rI+r[\rho^2(\Sigma U\Sigma)^{-1}+\Sigma^{-1}]^{-1}V^{-1} \qquad (12)\\
C &= \frac{1}{\rho}\Sigma=\frac{1}{\rho}\begin{bmatrix}\frac{1}{\lambda_1+\lambda_{12}}\Sigma_1 & 0\\ 0 & \frac{1}{\lambda_2+\lambda_{12}}\Sigma_2\end{bmatrix}. \qquad (13)
\end{aligned}$$

Any investor i's risky asset holding is

$$\begin{aligned}
D_i &= \left(I+\frac{1}{\rho^2}U\Sigma\right)^{-1}W+\rho\left[I+\rho^2(U\Sigma)^{-1}\right]^{-1}V^{-1}(\bar{F}-rP)+\rho\Omega_i^{-1}(S_i-rP)\\
&= \left(I+\frac{1}{\rho^2}U\Sigma\right)^{-1}W+\rho\left[I+\rho^2(U\Sigma)^{-1}\right]^{-1}V^{-1}(\bar{F}-rP)\\
&\quad +\rho\Omega_i^{-1}G+\rho\Omega_i^{-1}(S_i-rP-G), \qquad (14)
\end{aligned}$$

where

$$G = \left[I - rB^{-1} \right] \bar{F} + rB^{-1} A \tag{15}$$

is the ex-ante mean of $S_i - rP$ for any i.

3 Risky Asset Holdings and Asset Pricing Implications

3.1 Market Participation and the Portfolio Information Separation Theorem

Proposition 1 has interesting implications for investors' portfolio choices and asset pricing. We now analyze individual investors' risky asset holdings.

Owing to supply shocks, asset prices are not fully revealing, so information asymmetry persists in equilibrium and different investors have different asset holdings. Even investors from the same group have different holdings, as they receive different signals. An investor's asset holding is the sum of four components. The first term in equation (14),

$$\left(I + \frac{1}{\rho^2} U \Sigma \right)^{-1} W,$$

is the *risk-adjusted market portfolio*, which is deterministic. This portfolio is highly correlated with, but differs from, the ex-ante endowed market portfolio W, because it is also influenced by the informativeness of the equilibrium price. Investors take the informativeness of asset prices into account when trading to share risks. When the random supply shock to an asset becomes more volatile, or on average investors' private information of such an asset is less precise, the equilibrium price contains less precise information about this asset. This increases risk, which, other things equal, reduces investor holdings of this asset.[9]

The second component of any investor's risky asset holding, the second term in (14), is the contrarian position, which is taken in opposition to fluctuations in the market

[9] In this respect private information can make investing safer for uninformed investors, an effect which clashes with the idea that the presence of informed investors makes investing riskier for the uninformed.

price. Since informed investors are not perfectly informed, it is very risky for them to hold the entire supply shock. Hence, for risk sharing reasons, uninformed investors trade as contrarians to price, helping to absorb these shocks. Although sometimes they lose money to investors with superior information, they are compensated by the risk premium benefit of accommodating supply shocks. Prior information is an important determinant of this second component. In particular, how contrarian they are depends on the ex-ante mean of the assets' returns and on how risky the assets are.

The third component of any investor's risky asset holding, the third term in (14), is what we call the knowledge safety position (Van Nieuwerburgh and Veldkamp 2009; Van Nieuwerburgh and Veldkamp 2010; Kacperczyk, Van Nieuwerburgh, and Veldkamp 2014; Kacperczyk, Van Nieuwerburgh, and Veldkamp 2015). This position, $\rho\Omega_i^{-1}G$, consists of extra holdings in the securities about which the investor has information, because possessing an information signal about an asset reduces its conditional volatility (independent of the signal realization). This portfolio component is deterministic and is the same for all investors who are in the same group.

The fourth component of an investor's risky asset holding, the fourth term in (14), is the speculative position, which is taken to exploit superior information. This term, $\rho\Omega_i^{-1}(S_i - rP - G)$, has mean zero, since this information could be either favorable or unfavorable. Different investors, even if they are in the same group, hold different speculative portfolios, because they receive heterogeneous private signals.

A critical feature of the first two components of investors' risky asset holdings is that they are both independent of any investor's private information. Any investor includes these two components as part of the investor's asset holding, regardless of whether the investor is a member of Group 12—investors who have the most superior private information, or Group $\varnothing$—investors who have no private information. As a result, information asymmetry does not cause investors with informational disadvantage to have zero holdings.

Corollary 1 (Market Participation) *In the model with random supply shocks and heterogeneous risky assets endowments, if the ex-ante endowments of risky assets are positive, all investors include a common portfolio in their risky asset holdings, which is on average strictly positive. Therefore, even investors who are at an informational disadvantage participate in the market.*

We call the common portfolio, which is the sum of the risk-adjusted market portfolio and the contrarian trading portfolio, the *informationally passive portfolio*, because it can be formed by an investor based only on equilibrium prices without any private information. Since Group $\varnothing$ investors do not have any private information (their $\Omega^{-1} = 0$) and can get information through the equilibrium prices only, their risky asset holdings are exactly the informationally passive portfolio.

Since this portfolio includes the risk-adjusted market portfolio, which has strictly positive holdings of all assets and is a risk-modified version of the market, there is no presumption that asset holdings should be close to zero even in assets the investor has no information about, and even as an approximation. We analyze the effect of transaction costs explicitly in Section 3.4. Of course, in realization investors will occasionally hold low or negative holdings of assets. But this does not justify the zero holdings conjecture, which is about investors holding none of certain assets on a continuing basis (which in our static model, would mean on average) owing to lack of information.

All other investors possess some private information. The third term and the fourth term in equation (14), the *knowledge safety* and *speculative* components, sum to a portfolio $\rho\Omega_i^{-1}(S_i - rP)$. To form such a portfolio, investor i neither extracts information from the price nor uses prior beliefs. The investor only uses private information, so we call it investor i's *information-based portfolio*. Obviously, if the k^{th} diagonal element of Ω_i^{-1} is 0, the k^{th} element of the vector $\rho\Omega_i^{-1}(S_i - rP)$ is also 0, implying that if investor i does not have private information about asset k, then i does not trade asset k beyond holding the informationally passive portfolio. This argument is summarized in Corollary 2 below.

Corollary 2 (Information-Based Trading) *Investors trade assets beyond the informationally passive portfolio for further reducing risks and for speculation if and only if they have private information about those assets.*

Merton (1987) assumes that if investors do not have information about some assets, they will have zero holdings of those assets, because they are afraid of losing money to investors who have superior information. However, as shown in Corollary 1 and Corollary 2, the zero holdings conjecture does not hold in an explicit model of information and securities trading, nor in general will it hold even as an approximation. Uninformed investors trade to share risks, even though they understand that they are at a position of information disadvantage. Informational disadvantage does deter investors from including assets in the information-based portfolio. So the effect of informational disadvantage in a set of assets is zero deviation from the informationally passive portfolio rather than zero holdings of these assets.

We next derive a new portfolio separation theorem under asymmetric information. This will turn out to be important for understanding investors' behavior when they are unaware of certain assets, as analyzed in Subsection 3.2.

Proposition 2 (Portfolio Information Separation Theorem) *When all assets' characteristics are common knowledge, equilibrium asset portfolios have three components: an informationally passive portfolio based only upon equilibrium prices; an information-based portfolio based upon private information and equilibrium prices; and the riskfree asset.*

The informationally passive portfolio is just the portfolio consisting of the risk-adjusted market portfolio and the contrarian portfolio as characterized in Proposition 1. The information-based portfolio combines the speculative portfolio and the knowledge safety portfolio as characterized in Proposition 1.

Uninformative Priors about Payoffs

The analysis above was under the assumption that investors share a common prior belief that is well-defined. An informative prior implicitly grants the investor some information about the payoffs of all assets. Hence, a possible defense for the zero holdings conjecture might be that investors participate because even the uninformed do not see themselves as completely ignorant—they possess knowledge (or at least, opinions) in the form of informative prior beliefs. This raises the question of whether the conclusion of Corollary 1 would hold with uninformative priors.

To address this issue, we next examine uninformative priors. In other words, we consider a limiting case of the model in which investors have uniform diffuse beliefs about all risky assets, i.e., $V^{-1} = 0$ and thus $\bar{F}$ is not well-defined. So there is no information embedded within investors' priors about risky assets' returns. Because all steps to solve the model are continuous in V^{-1}, Corollary 3 below shows that the zero holdings conjecture does not follow in this setting.

Corollary 3 (Holdings under Uniform Diffuse Prior) *In the limiting case in which investors hold uniform diffuse prior beliefs, investors' portfolios contain three components:*

1. *The risk-adjusted market portfolio, which is held for risk sharing;*
2. *The knowledge safety portfolio component; and*
3. *The speculative component.*

The last two components place nonzero weight on an asset if and only if the investor has private information about the asset.

In contrast to the case with conventional priors, with a uniform diffuse prior belief, investors do not trade as contrarians to the market price; the second term in equation (14) vanishes. This is because the adverse selection problem is extremely severe for an uninformed investor with a diffuse prior. From equation (10), the prior variance of the equilibrium price is extremely high, because the variance of risky assets' returns are extremely high. Consequently, given that the variance of random supply shocks is finite and well-defined, an increase in the price is inferred to be the result almost entirely of higher realized assets' payoffs. Hence, for uninformed investors, the expected return from trading as a contrarian is zero, and taking such a position is risky. Therefore, uninformed investors do not trade as contrarians to price.

3.2 Investor Unawareness

We now analyze the third possible justification for the zero holding conjecture of Merton (1987): that investors are not even aware of certain assets by name or by their specific characteristics. For example, if an investor has never heard of FLIR Systems (an S&P 500 firm), does not know the market of its stock or its price, it seems natural for the investor not to participate in this market. A possible counterargument that we explore is that in equilibrium even investors who are 'unaware' in this sense may invest in an informationally passive low-cost mutual fund or ETF, in order to acquire a share of the risk even of securities they know nothing about. In this account, they are in equilibrium offered an adequate price by better-informed investors to bear this risk.

A possible objection to this argument is that in general, even if an investor understands that assets exist that are unknown to the investor, and that a fund can be operated cheaply, different investors might need different funds to implement their optimal portfolios. The portfolio constructed by the fund may not meet all investors' needs, because investors are aware of different sets of assets and have heterogeneous information about the assets they are aware of. Moreover, without knowledge of the portfolio the fund provides, investors cannot assess the expected return and the risk of buying the fund, so holding the fund could be extremely risky. Consequently, according to this argument, investors may optimally choose not to buy the fund and thus not participate in the markets of stocks of which they are unaware. Such 'unawareness' may also affect how an investor uses private information to trade the assets about which the investor does have private information.[10]

[10] In a standard rational expectations equilibrium, investors make use of all assets' characteristics and prices to extract information about any particular asset's payoff from its equilibrium price.

Nevertheless, we can tractably analyze the effect of investors' unawareness of a subset of assets in the sense considered here—that investors lack knowledge about the specific names and characteristics of certain assets, and the number of such assets, but know about the existence of such assets.

Formally, we first assume $\lambda_{12} = 0$, so all investors are unaware of some risky assets. We also assume that investors hold diffuse prior beliefs about assets' payoffs, so with full awareness, the common component of investors' asset holdings is the risk-adjusted market portfolio only.

Consider for example an arbitrary investor i in Group 1. Investor i is aware of the set of assets Γ_1: she knows that the number of assets is $\bar{N}$, that the vector of total endowments is W_1, that the precision of her private information is Σ_1^{-1}, that the average precision of investors' private information is $\lambda_1 \Sigma_1^{-1}$, and that the random supplies have the variance U_1. Investor i in Group 1 can also observe the prices of all assets in Γ_1.

However, investor i in Group 1 knows that there exists a set of assets, namely Γ_2, which she is unaware of. Investor i believes that the number of assets in Γ_2 is $1, 2, 3, \ldots$ with equal probability. For each asset $j \in \Gamma_2$, investor i holds uniform diffuse prior beliefs (with the support $\mathbb{R}^{++}$ for each) about its total endowment, the average precision of investors' private information about it, and the variance of its random supply shock. Investor i cannot observe asset j's price either.

Investors in Group 2 are aware of assets in Γ_2 but unaware of assets in Γ_1, and investors in Group $\varnothing$ are unaware of any asset in $\Gamma_1 \cup \Gamma_2$, as defined similarly. We further assume that all investors are aware of the riskfree asset, and that there is a fund that invests in the risky assets. Investors know of the existence and name of the fund and are aware of its price, but are unaware of (have diffuse priors about) its return characteristics. So an investor who is unaware of some assets may potentially have very poor information about the distribution of returns on this fund.

The fund management observes the characteristics and prices of all assets and therefore is able to offer the risk-adjusted market portfolio to investors as specified in the setting with full awareness (in which asset characteristics are common knowledge). It is common knowledge that this is the portfolio offered by the fund.

We propose an equilibrium in which all investors, regardless of their information sets, buy the fund, and in which informed investors combine this with additional information-based portfolio. We now show that in this setting, investors hold all assets, either directly or via the fund.

Proposition 3 (Participation under Unawareness) *When investors have diffuse priors about the characteristics of different subsets of assets (including the variances of random supply shocks and prices), and there is a low-cost mutual fund or ETF that provides the informationally passive portfolio, there is an equilibrium in which asset prices and investors' risky assets holdings are identical to those in the model with full awareness.*

From the information separation theorem, the portfolios described in Proposition 3 are implementable. Specifically, if a fund company wants to provide investors with the informationally passive portfolio, it does not need to know the private information of any investor. For investors, buying the fund's shares is the same as holding the informationally passive portfolio, the first component described by the information separation theorem. Therefore, intuitively, all investors are satisfied to buy the fund's shares, even if they do not know how many assets there are and do not know by name and characteristic exactly what assets the fund provides. So all investors participate in the markets of all risky assets, in contrast with the unawareness interpretation of the zero holdings conjecture of Merton (1987).

In addition, investors do not even need to know the number of assets traded in the market when forming their information-based trading portfolios. Consider Group 1 investors as an example. For any given $N \geq \bar{N}$, except the $\bar{N} \times \bar{N}$ block Σ_1^{-1}, all other blocks in the $N \times N$ matrix Ω_1^{-1} are 0. So lack of knowledge about the number N does not affect investors' information-based trading.

Despite the potentially extremely high risk to an investor of investing in an asset the investor is unaware of, and of investing in the fund, Proposition 3 shows that investors, directly or indirectly, hold portfolios that are identical to what they hold in the setting with full awareness; and market prices are identical in the two settings.

To see this, conjecture a strategy profile in which all investors buy the fund, and then, if they possess private information, hold their information-based portfolio and the riskfree asset according to the information portfolio separation theorem. We further conjecture that market prices are identical to those in the setting with full awareness. To verify that this strategy profile and pricing function is an equilibrium, we analyze any investor i's optimal portfolio choice, given that all other investors behave as described in the strategy profile.

Investors can reason based on all possible worlds that they might face. Let's again take an investor i from Group 1 as an example, and the arguments for Group 2 investors and Group $\varnothing$ investors are similar. Investor i is aware of assets in Γ_1 and knows that there exists a set of assets Γ_2. Investor i can form a possible world by first constructing a possible set of assets Γ_2: the number of assets ($\tilde{N}$), an $\tilde{N}$ dimension vector of the total endowments that is strictly positive (W_2), the average precision of investors' private information ($\lambda_2 \Sigma_2^{-1}$), and the variance matrix of random supplies (U_2). She then completes a hypothetical world by combining the characteristics about assets in Γ_1 that she knows and the characteristics about assets in Γ_2 that she hypothesizes.

Investor i knows that in the hypothesized world, the fund provides the risk-adjusted market portfolio, and all investors buy the fund and hold their information-based portfolio and the riskfree asset according to the portfolio information separation in the quantities specified in Proposition 1. Then, aggregating all other investors' demands, investor i can derive the pricing function from the market clearing condition. Such a pricing function must be same as the equilibrium pricing function with full awareness in investor i's hypothetical world, because the market clearing conditions are same. This implies that investor i extracts information from any realized price vector in exactly the same way that i extracts information in a setting with full awareness, and i's optimal risky asset holding in the hypothetical world is exactly the one characterized in equation (14). Then, the information portfolio separation theorem implies that investor i will also buy the fund.

This argument holds for any possible world. So if all other investors buy the fund and hold their information-based portfolio and the riskfree asset according to the information portfolio separation theorem, then it is optimal for any individual investor to do so. Therefore, the strategy profile we propose is an equilibrium.

3.3 CAPM Pricing with Supply Shocks

Returning to the basic setting with full awareness, with random supply shocks, asset prices in equilibrium are not perfectly revealing. Consequently, in equilibrium there are information asymmetries among investors, resulting in investors having different risky asset holdings. So our setting is very different from the CAPM setting, which assumes identical beliefs and has the implication that all investors hold the same portfolio. Since holding the market is equivalent to the CAPM pricing relation, it seems intuitive that in our setting the CAPM pricing relationship would fail. Formally, the CAPM Security Market Line relation can be represented by

$$R - r\mathbb{1} = \alpha + \beta \left(R_M - r \right), \tag{16}$$

where R is the $n \times 1$ vector of assets' gross rates of returns, R_M is the market portfolio's gross rate of return, and α is the $n \times 1$ vector of extra expected returns in deviation from the CAPM. Because of information asymmetry and investors' heterogeneous asset holdings in the equilibrium, it is natural to conjecture that α in equation (16) is not equal to zero.

Nevertheless, in the case of diffuse uniform prior beliefs, even with information asymmetry, α in equation (16) is equal to zero for an appropriately defined market portfolio, so that a version of CAPM pricing holds. This shows that the pricing predictions of the Merton model do not apply to a setting in which information asymmetry is explicitly modeled, which should not be surprising since there is full participation in our model.

From Corollary 3, we know that all investors in this case hold the risk-adjusted market portfolio as a common component of their holdings. Therefore, it is natural to consider the risk-adjusted market portfolio, M, as a candidate for CAPM pricing, defined as

$$M = \left(I + \frac{1}{\rho^2} U\Sigma\right)^{-1} W. \tag{17}$$

From equation (10), the equilibrium pricing function is

$$P = \frac{1}{r}\left[F - A - \frac{1}{\rho}\Sigma Z\right], \tag{18}$$

where $A = [\rho^2(\Sigma U \Sigma)^{-1} + \Sigma^{-1}]^{-1}\frac{1}{\rho}W$.

Given any realized equilibrium price P, the volatility of asset payoffs derives from the random supply shock only. Let $\text{diag}(P)$ be an $N \times N$ diagonal matrix, whose off-diagonal elements are all zero and whose k^{th} diagonal element is just the k^{th} element of the vector P. Generically, as no asset has a zero price, $\text{diag}(P)$ is invertible. Then, by the definition of $\text{diag}(P)$,

$$\text{diag}(P)^{-1}P = \mathbb{1}. \tag{19}$$

From the equilibrium pricing (equation (18)), we have

$$\text{diag}(P)^{-1}\mathbb{E}(F) - r\mathbb{1} = \text{diag}(P)^{-1}A. \tag{20}$$

The LHS of equation (20) is just the vector of the risky assets' equilibrium risk premia.

Given a realized equilibrium price, the risk-adjusted market portfolio M has value $P'M$. Then the vector of the weights of risky assets in the risk-adjusted market portfolio is

$$\omega = \frac{1}{P'M}\text{diag}(P)M.$$

Hence, conditional on the price P, the difference between the risk-adjusted market portfolio's expected rate of return and the riskfree asset's rate of return is

$$\begin{aligned}
\mathbb{E}(R_M) - r &= \omega' \mathrm{diag}(P)^{-1}\mathbb{E}(F) - r \\
&= \frac{1}{P'M} M' \mathrm{diag}(P) \mathrm{diag}(P)^{-1}(A + rP) - r \\
&= \frac{1}{P'M} M' A, \qquad (21)
\end{aligned}$$

where the expectations are all conditional on the equilibrium price.

The variance of the risk-adjusted market portfolio is

$$\mathbb{V}(R_M) = \mathbb{E}\left[\left(\omega' \mathrm{diag}(P)^{-1} CZ\right)\left(\omega' \mathrm{diag}(P)^{-1} CZ\right)'\right] = \left(\frac{1}{P'M}\right)^2 M'CUCM, \qquad (22)$$

and the covariance between all risky assets and the risk-adjusted market portfolio is

$$Cov(R, R_M) = \frac{1}{P'M} \mathrm{diag}(P)^{-1} CUCM. \qquad (23)$$

Let α be the CAPM alpha. From equations (20)-(23), and since $M = \rho(CUC)^{-1}A$, we have the following proposition.

Proposition 4 (No Extra Risk Premia with Supply Shocks) *In the model with random supply shocks and a uniform diffuse prior belief, assets do not have extra risk premia beyond those predicted by the CAPM where the relevant market portfolio for pricing is the risk-adjusted market portfolio. So even with information asymmetry, in equilibrium, the α with respect to the risk-adjusted market portfolio is zero.*

This result may seem surprising, since investors have heterogeneous asset holdings, and since the portfolios held by informed investors are not mean-variance efficient with respect to the public information set. Nevertheless, $\alpha = 0$ independent of the information asymmetry. In equilibrium, there are no extra risk premia incremental to those predicted by the CAPM using the risk-adjusted market.

Saying that the CAPM pricing relation holds is equivalent to asserting that the risk-adjusted market portfolio is mean-variance efficient conditional on the assets' prices only. This efficiency can be seen from Group $\varnothing$ investors' utility maximization problem.

Group $\varnothing$ investors balance the expected returns and the risks of their holdings, and their information consists of the equilibrium price only. In equilibrium, Group $\varnothing$ investors all hold the risk-adjusted market portfolio, implying that the risk-adjusted market portfolio is mean-variance efficient conditional on the equilibrium price only.

Privately informed investors also hold the risk-adjusted market portfolio as a component of their portfolios; this is the piece that does not depend upon their private signals (except to the extent that their signals are incorporated into the publicly observable market price). In addition they have other asset holdings taking advantage of the greater safety of assets they have more information about, and for speculative reasons based upon their private information. The risk-adjusted market portfolio is not mean-variance efficient with respect to their private information sets, but it is efficient with respect to the information set that contains only publicly available information.

Proposition 4 contrasts with the pricing implications of Merton (1987). In Merton (1987), owing to information costs, investors are assumed not to trade assets they don't have private information. Then, since some investors do not trade some assets, these assets may have extra risk premia incremental to those predicted by the CAPM. In our model, in equilibrium, there are still information asymmetries among investors, resulting in different information-based trades. However, all investors hold the risk-adjusted market portfolio for risk sharing purposes, which is mean-variance efficient conditional on equilibrium asset prices only. Hence, assets do not have extra risk premia beyond those predicted by the CAPM.

Our focus on uniform diffuse priors provides a very simple and clear contrast in implications between the Merton model pricing implications and a setting that explicitly models information asymmetry and minimizes the knowledge of the uninformed. This provides an especially clearcut counterexample to the idea that there is a risk-premium for information costs, because even investors who are maximally informationally handicapped (by their uninformative priors) still hold the risk-adjusted market portfolio, and there is no extra risk premium associated with information costs.

More generally, with informative priors as well, the CAPM security market line still holds with respect to the informationally passive portfolio instead of the risk-adjusted market portfolio. The intuition is exactly same as in the case with uniform diffuse priors. Since Group $\varnothing$ investors, whose information set consists solely of equilibrium prices, hold the informationally passive portfolio, the informationally passive portfolio is mean-variance efficient conditional on the assets' prices only.[11]

The online appendix of Van Nieuwerburgh and Veldkamp (2010) provides a similar model setup and shows that a different version of the CAPM holds.[12] The result they derive uses as the market portfolio for CAPM pricing the ex-post total supply of the risky assets, the sum of the endowed risky assets and the random supply of risky assets ($W + Z$ in our model). Hence, they derive that the market portfolio is mean-variance efficient conditional on the average investor's information set. In our model, the informationally passive portfolio is a natural candidate for the market portfolio for CAPM pricing, because it is the common component in all investors' risky asset holdings. And we show that the informationally passive portfolio is mean-variance efficient unconditional on any investor's private information. Therefore, using the informationally passive portfolio for CAPM pricing, we show that the CAPM security market line relation holds unconditional on any investor's private information. One of the further contributions here is to establish that increasing information asymmetry, and its effect on investor participation, does not clearly predict whether there will be an increase versus decrease in risk premium.

[11] A possible objection to this argument is that Merton's model does not have a distinction between the risk-adjusted market portfolio and the market portfolio inclusive of supply shocks. So even though a version of the CAPM holds in our setting, there could be deviations from the CAPM pricing with respect to this inclusive market portfolio, and perhaps these deviations are what the Merton model is describing as premia for nonparticipation. However, such a claim is not tenable, for the simple reason that in our model there is full participation. The main insight here is robust with respect to the definition of market portfolio: rational uninformed investors fully participate in the stock market, so there is no risk premium for information-asymmetry-driven nonparticipation.

[12] Biais, Bossaerts, and Spatt (2010) derive a similar version of the CAPM in a dynamic model in which the market portfolio used for CAPM pricing is also the ex-post total supply of the risk, and the security market line holds *conditional* on the average investor's information set.

3.4 Transaction Costs, Information Asymmetry, and Participation

In the model of Section 2, the separation theorem of Proposition 2 indicates that in a frictionless setting with information asymmetry, rational investors participate in all asset markets. In this section, we employ a simple version of the model to analyze in a stylized way how participation varies with asymmetric information and other transaction costs. In sharp contrast with the theory that asymmetric information causes nonparticipation, we will see that greater information asymmetry sometimes causes *greater* participation.

Consider an individual investor i in Group $\varnothing$ who has no private information about asset payoffs. There are N assets (which can also be viewed as funds). For simplicity, we assume that investor i can costlessly trade all assets except asset 1. Investor i needs to pay a fixed cost $C > 0$ to take a positive position in asset 1, and must decide whether to incur this cost before observing prices. This can be viewed as a time cost of setting up a brokerage account or placing an order, and the commission for placing an order for a stock or ETF. Also for simplicity, we assume that investor i has zero endowment of any of the risky asset.

As before, the equilibrium pricing function is given by equation (10). If investor i pays the fixed cost, i participates in all asset markets (including asset 1). Exactly as in the frictionless model, i holds the informationally passive market portfolio (by the separation theorem), which places positive weights on all assets. Alternatively, i can avoid the fixed cost, and abstain from buying asset 1, and investing optimally in other assets taking this into account. The critical transaction cost, denoted by C^* makes the investor indifferent ex ante between participating and not participating. So i will participate for any cost $C \leq C^*$.

We now assume that there are $1,000$ independent risky assets. As before, the assets are divided into two groups, but for this calibration the specific grouping does not matter. For specificity, group 1 including assets 1 through 400 and group 2 including asset 601 through $1,000$. Fraction $\lambda_1 > 0$ of investors receive private signals about group 1 assets only, and fraction $\lambda_2 > 0$ receive private signals about group 2 assets only, where $\lambda_1 + \lambda_2 = 1$. Investors without any private information have zero measure, but we can still discuss the participation decision of a single uninformed investor (who is very small relative to the capital market) who needs to pay cost C to participate in the market for asset 1. Table 1 shows the parameter values we use in the numerical calibration, which come from Leland (1992) and Easley, O'Hara, and Yang (2013).

Each asset's ex-ante mean payoff is normalized to 1, and the variance is 0.04, so that the annual volatility of the asset's payoff is 20%. If an investor receives a signal about one asset, the signal's precision is $(\sigma^2)^{-1} = 5$, making the ratio of the private signal precision to an asset's ex-ante precision 0.2. We normalize the per capita total endowment of each asset to 1, so $W = (1, 1, \ldots, 1)'$. Since the annual volatility of liquidity trading is equal to 50% of the total supply, the precision of the liquidity trading in any particular asset is set to $(u^2)^{-1} = 4$. Table 2 describes the calibration results. It shows that when we vary λ_1 from 0 to 0.8, the critical fixed transaction cost (C^*) lies between 0.126 and 0.031.[13]

Table 2 shows that when there is zero information asymmetry ($\lambda_1 = 0$), the critical transaction cost is highest. This shows, consistent with our weakened version of the zero holdings conjecture, that there exists a set of parameters such that participation is lower under information asymmetry than under symmetric information. However, notably, the critical transaction cost is a non-monotonic function of λ_1. This implies that there is a range of parameters in which greater information asymmetry *encourages* market participation. This demonstrates that the basic intuition underlying the zero holdings hypothesis, that information asymmetry is a deterrent to participation, is not robust.

Specifically, Table 2 shows that the critical transaction cost first decreases when λ_1 increases from 0 to 0.1, then increases when λ_1 increases from 0.1 to 0.4, and then decreases as λ_1 increases further. Intuitively, an increase in λ_1, the informational disadvantage of an uninformed investor, has two opposing effects. First, when λ_1 increases, there are

[13] When we fix λ_2 and vary λ_1 with $\lambda_1 + \lambda_2 < 1$, we find the same pattern, and the critical transaction cost is even larger.

more investors who are informed about asset 1, so prices more fully reflect information. This reduces the conditional variance (risk) of the uninformed, *encouraging* participation.

Second, the fraction of price volatility that comes from liquidity trading decreases, which reduces the potential profits from trading as a contrarian to price. An uninformed investor who does so tends to make less money off of the noise traders and loses more money to the informed. This tends to reduce the expected profits of an investor who participates. The net effect of an increase in λ_1 on the critical transaction cost thus depends on which effect dominates.[14]

As a practical matter, non-informational fixed transactions costs for many assets are extremely small. The current brokerage commission for buying a stock or ETF at some online brokerages is is $1.50, and the Robin Hood brokerage charges $0 per trade.[15] The time cost to a rational investor of setting up an online brokerage is only a few minutes (the Robin Hood brokerage states on its web site, "Signing up takes less than 4 minutes"). The time cost of transferring funds into the online brokerage is also small, as is the cost of placing an order. Of course, the cost of acquiring information about a stock can be non-negligible, but our focus is on the decision of an investor who chooses whether to trade without acquiring the private signal possessed by informed investors.

The very low level of non-informational fixed transactions costs raises doubts about the idea that such costs are the key catalyst which, when combined with asymmetric information, cause both zero holdings by large sets of investors for many assets, and

[14]A subtlety about our comparative statics on information asymmetry is that it is impure in the sense that it varies conventional risk at the same time. When we increase the fraction of the informed, price becomes more informative, so return volatility conditional upon price decreases. However, a comparative statics on the information asymmetry for a single asset inherently must change either this conditional volatility, or else prior volatility (if prior volatility is varied together with λ_1 so as to hold conditional volatility constant). Since there is no way to increase the information asymmetry for a single asset without changing either the exogenous prior risk or the endogenous conditional risk, the notion that a 'pure' variation in information asymmetry affects participation is not a well-defined concept. In Section 4, we perform a pure variation in information asymmetry without varying conventional risk, which becomes feasible when the focus is on information asymmetry among multiple assets.

[15]Other transactions costs are also fairly small. Hasbrouck (2009), Figure 3, estimates NYSE, AMEX, and NASDAQ effective transaction costs as of approximately 2006-8 as under 1%, and well under 0.5% for all but the smallest size quintile on each exchange. He also observes that "After the Depression, however, average effective costs don't rise above 1% for the three highest capitalization quartiles."

substantial risk premia for nonparticipation. It seems more plausible that the effective barriers to trading are psychic costs, such as a disutility of thinking about investing, or an irrational fear of certain assets.

4 Risk Premia and Information Asymmetry Proxies

Our finding that equilibrium prices satisfy a version of the CAPM seems to contrast with the conclusion of an influential paper by Easley and O'Hara (2004) that there is a risk premium for information asymmetry. So it is argued that different securities have different *information risk*, and that in equilibrium investors earn a premium for bearing information risk. This idea has been tested in an extensive empirical literature.

In the model of Easley and O'Hara (2004), there is a fixed number of signals about a given risky asset's payoff. Investors are either informed or uninformed about the asset. The comparative statics consists of an increase in the fraction of signals that are received only by the informed. This is viewed as an increase in information asymmetry.

For example, in one market there might be 8 distinct public signals received by all, and 2 private signals that are both observed by every informed investors; and in another market only 2 public signals, and 8 private signals, all of which are observed by every informed investor. With identical signal precisions and conditionally independent signals, as the publicly available information decreases, the informed investors' information set remains constant. In contrast, the uninformed investors' information becomes less precise owing to the reduction in public information. Uninformed investors demand higher risk premia as compensation for information risk.

However, an increase in the fraction of private signals in the model of Easley and O'Hara (2004) does not just cause an increase in information asymmetry. It also increases the volatility of the asset payoff for uninformed investors conditional upon what they observe. As is standard in models without information asymmetry, we expect greater uncertainty about payoff outcomes to be associated with greater risk premia. This insight does not require a new form of risk (information risk). This raises the question of whether there is any clearcut and distinctive sense in which information asymmetry induces rational risk premia.

To probe this issue more deeply, we perform comparative statics on the model to vary information asymmetry. We first consider a portfolio concept of a shift in information asymmetry, wherein some informed investors receive signals about a greater number of stocks, and others about a smaller number of stocks. Specifically, we consider a shift in the composition of the investors from $\lambda = (\lambda_1, \lambda_2, \lambda_{12}, \lambda_\varnothing)$ to $\lambda' = (\lambda_1 - \epsilon, \lambda_2 - \epsilon, \lambda_{12} + \epsilon, \lambda_\varnothing + \epsilon)$, where $\epsilon > 0$ is arbitrarily small. This increases the fraction of investors who have the maximal information set and the fraction who have the minimal information set, and decreases the fraction who have intermediate amounts of information. Hence, the information asymmetry in the economy unambiguously increases. It also holds constant the fraction of investors who are informed about any given security.

From equation (20), ex ante risk premia are

$$\mathbb{E}\left[\mathrm{diag}(P)^{-1}\mathbb{E}(F) - r\mathbb{1}\right] = \left[\mathrm{diag}\left(B^{-1}(\bar{F} - A)\right)\right]^{-1} A, \tag{24}$$

where A and B are defined in equation (11) and equation (12). Since $\lambda_1 + \lambda_{12}$ and $\lambda_2 + \lambda_{12}$ do not change when λ changes to λ', the average private information precision Σ^{-1} does not change. It immediately follows that risk premia do not change.

This is a counterexample to the general notion that an increase in information asymmetry increases risk premia. The key intuition is that although the ex ante information asymmetry increases, the average private information precision does not change, because of the offsetting between the investors with more information and the investors with less information. As a result, the precision of information that uninformed investors (Group $\varnothing$ investors) extract from equilibrium prices is the same as before. Hence, uninformed investors do not demand higher risk premia, because the uncertainties they are facing (conditional on the equilibrium price) are the same as before. This risk premium demanded by uninformed investors is the usual risk premium studied by econometricians—the premium that conditions only on publicly available information.

This comparative statics is in a sense a purer means of evaluating whether there is a premium for information asymmetry than that of Easley and O'Hara (2004), because their comparative statics varies both information asymmetry and the total conventional uncertainty (fundamental volatility conditional upon price) faced by uninformed investors. Since it is well known from models with symmetric information that risk premia increase with conventional risk, this does not isolate the effect of information asymmetry. Instead, by fixing total uncertainty (both unconditionally and conditional on price), our comparative statics focuses specifically on the effect of varying information asymmetry. We therefore conclude that there is no general principle that there is a positive risk premium associated with information asymmetry per se. In brief, it is best not to think of information asymmetry as a type of risk.

Proposition 5 *In a comparative statics in which information asymmetry is increased by increasing the sets of securities about which signals are received by the best informed investors and reducing the sets of securities about which signals are received by less well-informed investors, assets' risk premia remain unchanged.*

In the first comparative statics, information asymmetry was defined at the portfolio level. Investor i is more informed than investor j, if and only if the set of assets about which investor i receives private signals contains the set of asset about which investor j receives private signals. But for any particular asset, the measure of informed investors does not change, so the information asymmetry at the individual asset level does not change.

We next examine the effects of varying information asymmetry at the individual asset level to see whether securities with higher measured information asymmetry earn higher risk premia. The proxy for information asymmetry that we vary is much like the probability of information-based trading (PIN), developed by Easley, Hvidkjaer, and O'Hara (2002), which has been applied extensively in empirical studies.[16]

We define the information asymmetry proxy in any risky asset's market as the measure of investors who have private information about such an asset. So the proxy for information asymmetry of assets in Γ_1 is $\lambda_1 + \lambda_{12}$, while that of assets in Γ_2 is $\lambda_2 + \lambda_{12}$. This corresponds to PIN, defined by Easley, Hvidkjaer, and O'Hara (2002) as the ratio of the arrival rate for information-based orders to the arrival rate for all orders. In our model, all investors—informed and uninformed—participate in all assets' markets (Corollary 1), so the arrival rate for all orders is 1. For Γ_1 assets, the arrival rate for information-based orders is $\lambda_1 + \lambda_{12}$, since Group 1 investors and Group 12 investors possess private information about these assets. Similarly, for Γ_2 assets, the arrival rate for information-based orders is $\lambda_2 + \lambda_{12}$. Therefore, PIN of any asset in Γ_1 is $\lambda_1 + \lambda_{12}$, while PIN of any asset in Γ_2 is $\lambda_2 + \lambda_{12}$.

Suppose now that the composition of investors shifts from λ to $\lambda' = (\lambda_1 - \epsilon, \lambda_2 + \epsilon, \lambda_{12}, \lambda_\varnothing)$. By the definition of the information asymmetry proxy, in Γ_1 assets' markets it decreases, and in Γ_2 assets' markets it increases.

From equation (24), we can calculate the risk premium of asset i in Γ_1 as

$$\frac{W_i \left(1 + \frac{v_i^2}{\rho^2(\lambda_1+\lambda_{12})^2\sigma_i^4 u_i^2 + (\lambda_1+\lambda_{12})\sigma_i^2}\right)}{r\left[\rho \bar{F}_i \left(\rho^2(\lambda_1+\lambda_{12})^2\sigma_i^4 u_i^2 + (\lambda_1+\lambda_{12})\sigma_i^2 + v_i^2\right) + W_i\right]}, \tag{25}$$

where σ_i^2, u_i^2, and v_i^2 are the i^{th} diagonal elements of Ω_{12}^{-1}, U^{-1}, and V^{-1}, respectively.

It is then obvious that when the composition of investors shifts from λ to λ', the expression (25) increases, because λ_1 decreases to $\lambda_1 - \epsilon$. So the risk premium of asset i in Γ_1 increases. Similarly, when the composition of investors shifts from λ to λ', the risk premium of asset i in Γ_2 decreases. It follows that an asset's risk premium is *decreasing* in its information asymmetry proxy. This is the opposite of the prediction of the model

[16]PIN in our model does not, however, hold constant the uncertainty faced by uninformed investors about a security conditional on price. In that sense it is not a pure measure of information asymmetry, so we refer to it as the information asymmetry *proxy*.

of Easley and O'Hara (2004).

Proposition 6 *In a comparative statics where the fraction of informed traders in one security increases and the fraction of informed traders in another security decreases, the risk premium of the asset with an increased number of informed traders decreases and the risk premium of the other asset increases.*

It is not hard to reconcile the differing predictions. In the comparative statics of Easley and O'Hara (2004), an increase in the fraction of signals about an asset that is known only to informed investors is accompanied by greater uncertainty faced by uninformed investors about that asset based on what they observe. This is because, by assumption, when there are more private signals, there are fewer public signals.

In contrast, the key intuition in Proposition 6 is that when there are fewer informed investors about the assets in Γ_1 and more for Γ_2, the information investors extract from the equilibrium prices about assets in Γ_1 becomes less precise, and the information investors extract from the equilibrium prices about assets in Γ_2 becomes more precise. So conditional on equilibrium prices, uninformed investors face more uncertainty about assets in Γ_1 and less uncertainty about assets in Γ_2. In consequence, they demand higher risk premia of assets in Γ_1 and less risk premia of assets in Γ_2.

These results may help explain the mixed evidence in the empirical literature about whether information risk (or what we would call information asymmetry) is priced. As Proposition 5 showed, a pure variation of information asymmetry that does not shift total risk implies no shift in risk premium. In Proposition 6, when PIN is increased in a way that decreases the volatility of the asset payoff conditional on price (by increasing the amount of information incorporated into price), the risk to the uninformed is reduced, implying lower risk premia. Finally, if PIN is increased in a way that increases the volatility of the asset payoff conditional upon price (by reducing the number of public signals, as in the model of Easley and O'Hara (2004)), the risk premium increases. We conclude that it is best not to think of asymmetric information as a kind of risk.

Some empirical tests use proxies for information risk that confound variations in information asymmetry with variations in total amount of *ex ante* uncertainty (not conditioning upon price). This is an additional source of possible ambiguity in what outcome we would expect to observe. For example, several empirical papers use as proxies for information uncertainty (lack of) analyst following, total or residual volatility, or measures of (low) quality of a firm's financial reporting and disclosure environment. It is indeed plausible that each of these proxies is correlated across firms with information asymmetry. However, it is equally plausible that each of these is correlated with total ex ante uncertainty about the firm's future cash flows. Securities with high uncertainty will tend to have high factor loadings and/or idiosyncratic risk. So from the viewpoint of traditional asset pricing theory, higher risk premia would be expected for such securities even if there were no information asymmetry in any asset market.

5 Concluding Remarks

A leading theory of capital market trading and pricing contends that investors refrain from participating in the market for stocks for which it is too costly to acquire information, resulting in a risk premium for stocks in which participation is low. We show that in rational settings with asymmetric information, the zero holdings conjecture does not follow, even in weakened or approximate form.

In natural rational settings with asymmetric information, investors take nonzero positions in all assets (e.g., Van Nieuwerburgh and Veldkamp (2009)), the intuition offered being that there is a diversification benefit to holding many assets. However, since assets that an investor knows little about are especially risky to such an investor, it is less clear whether the positions held by the uninformed are small. If so, the zero holdings conjecture might hold as an approximation.

We find that in equilibrium, for risk-sharing reasons, investors trade toward the market portfolio, with some adjustment to this position to accommodate noise trades or the information trades of other investors. The portfolio prediction of the zero holdings conjecture do not follow even as an approximation; uninformed investors can have large holdings of assets they know nothing about. The asset pricing implication that there is a risk premium in compensation for the nonparticipation of uninformed investors also does not follow.

Furthermore, when there is a transaction cost to investing in an asset, we find that there is a range of parameters in which greater information asymmetry *increases* participation. This shows that the intuition underlying the zero holdings conjecture is not robust. So the conclusion that there is a risk premium for the fact that some investors choose not to incur costs of acquiring information about a security does not follow.

A different possible justification for the zero holdings conjecture is that investors are literally *unaware* of certain securities. We make the idea of unawareness more specific by assuming that investors do not know the names, payoff characteristics, and prices of some assets, but do know that such assets may exist. In particular, investors have diffuse priors over the characteristics of stocks that they are unaware of. Few investors could name all the stocks in the S&P500, yet many investors are aware of the fact that the stocks composing S&P500 exist, and that they can invest in those stocks by buying an index fund. We also allow for the possibility that investors do not know how many assets they are unaware of.

We consider a setting in which there is a mutual fund whose manager is aware of all assets and which offers an informationally passive portfolio, and in which investors know of this fund and its price. We show that investors will delegate their investment to the fund, even if they have diffuse priors over its characteristics. In particular, there is an equilibrium under unawareness that is essentially identical to the equilibrium under full awareness. Investor hold the same overall portfolios, which are attained by investing part or all of their wealths in informationally passive funds. Market prices are also identical.

To sum up, these findings suggest that nonparticipation puzzles are not resolved by costs of acquiring information. Instead, their resolution requires an appeal either to substantial market frictions (other than information costs), or to imperfect rationality of market participants. Similarly, non-informational frictions or imperfect rationality underlie any pricing implications of nonparticipation, such as those derived in the Merton (1987) model.

Understanding the source of nonparticipation is important, since a policy remedy that could effectively address information costs can be ineffective or even counterproductive in addressing investor behavioral biases. For example, a natural solution to the problem of information costs about an asset is to provide investors with more information about it. But if the source of nonparticipation is psychological bias, this solution could make the problem worse. More information does *not* always debias decision makers, since extraneous information can be distracting or overwhelming.

For example, providing extensive information about numerous assets could make investors feel less competent about evaluating their investments. This could exacerbate ambiguity aversion, which is a plausible source of nonparticipation. Similarly, such information might push investors toward the use simple decision heuristics such as narrow framing, which is another leading possible explanation for nonparticipation. Such a policy intervention, by intensifying behavioral biases and reducing participation, can also make securities less efficient, with greater deviations of prices from the CAPM of the sort modeled by Merton (1987).

Behavioral approaches suggest that providing information *per se* is often not helpful. What is most likely to help is forms of presentation that make the benefits to a diversified portfolio more salient and easier to grasp. For example, a graph showing the historical mean and variance of a portfolio containing only well-known stocks, in comparison with the mean and variance of a wider portfolio that includes many less familiar stocks, could help drive home the advantages of diversifying beyond more familiar assets. This would help address the feeling that naive investors may have that assets they are not familiar with are dangerous. Providing detailed information about many specific assets might distract from this key point, weakening this message.

In this example, the calculations would be based on information that is already publicly available, so this is not a matter of improving investors' information set. Rather, it is a matter of focusing investor attention on certain subsets of available information, processed and framed in a meaningful way, that provides cues for good decisions. This difference highlights the importance of understanding the actual sources of nonparticipation—non-informational market frictions or behavioral bias, rather than information costs per se—in order to determine how to address it.

Finally, our model provides insight about another important strand of research that examines the relation between information asymmetry and risk premia even with full investor participation in the capital market. In an influential paper, Easley and O'Hara (2004) provide a model in which assets with high information asymmetry earn high risk premia. It is therefore argued that there is a premium for information risk. Many empirical papers have performed tests of this hypothesis.

However, in the model of Easley and O'Hara (2004), the comparative statics shift that increases information asymmetry also increases total unconditional uncertainty about cash flows that uninformed investors face. So the apparent premium for information asymmetry could instead be a premium for bearing risk as conventionally defined in models without information asymmetry.

To evaluate this, we perform a comparative statics to vary information asymmetry without varying total uncertainty. To do so, we increase the fraction of the population of informed investors who have the maximal information set (gain signals about the largest number of securities) and those with the minimal information set (gain signals about the smallest number of securities), while reducing the fraction who have intermediate amounts of information. This unambiguously increases information asymmetry, while holding constant the fraction of investors who are informed about any given security. We find that the risk premia do not change. So it is not in general the case that an increase in information asymmetry increases risk premia.

Intuitively, the average private information precision does not change, because of the offsetting changes in the information possessed by different investors. Uninformed investors do not demand higher risk premia because the uncertainties they are facing (conditional on the equilibrium price) are same as before. So the risk premium estimated based on publicly available information is unchanged. This illustrates that information asymmetry per se does not induce risk premia; what matters is conventional consumption risk.

Instead of defining information asymmetry at the portfolio level, we can vary information asymmetry of individual assets, to examine the effect of varying an information asymmetry proxy that has been applied extensively in the empirical literature. So in our second comparative static analysis, we define an information asymmetry proxy in any asset's market as the measure of investors who have private information about such an asset. This corresponds to the probability of information-based trading (PIN), the measure of information asymmetry used by Easley, Hvidkjaer, and O'Hara (2002).

We consider a shift in the composition of investors that increases informed trading in one asset and decreases informed trading in the other asset. We find that an asset's risk premium is *decreasing* in its information asymmetry proxy. This is the opposite of the prediction of a positive risk premium for information risk.

Intuitively, when informed trading increases for a given asset, the information uninformed investors extract from equilibrium prices increases. So conditional on equilibrium prices, uninformed investors face less uncertainty about the asset with a greater fraction of informed traders, and hence demand a lower risk premium. This effect derives from the use of a proxy for information asymmetry, PIN, which does not hold constant the total amount of uncertainty faced by uninformed traders. These conclusions may help resolve some of the differing empirical conclusions from the literature on information risk (or what we would call information asymmetry). More importantly, they suggest that it is best not to think of information asymmetry as a type of risk that is distinct from conventional measures of consumption risk.

Appendices

A Proofs

Proof of Proposition 1:

Integrating across all investors' demands gives the aggregated demand as

$$\int_0^1 D_i \mathrm{d}i = \rho \left\{ (CUC')^{-1}(B - rI) - r \left(\int_0^1 \Omega_i^{-1} \mathrm{d}i \right) - rV^{-1} \right\} P + \rho \left(\int_0^1 \Omega_i^{-1} S_i \mathrm{d}i \right) + \rho[(CUC')^{-1}A + V^{-1}\bar{F}]. \tag{26}$$

By equation (3), we have $\int_0^1 \Omega_i^{-1} \mathrm{d}i = \Sigma^{-1}$. Also, note that

$$\begin{aligned} & \int_0^1 \Omega_i^{-1} S_i \mathrm{d}i \\ = & \int_{\text{Group 1}} \Omega_i^{-1} S_i \mathrm{d}i + \int_{\text{Group 2}} \Omega_i^{-1} S_i \mathrm{d}i + \int_{\text{Group 12}} \Omega_i^{-1} S_i \mathrm{d}i \\ = & (\lambda_1 + \lambda_{12}) \Omega_1^{-1} F + (\lambda_2 + \lambda_{12}) \Omega_2^{-1} F \\ = & \Sigma^{-1} F. \end{aligned} \tag{27}$$

Therefore, from the market clearing condition, we have

$$\int_0^1 D_i \mathrm{d}i = Z + W. \tag{28}$$

In an equilibrium, both equation (5) and equation (28) hold simultaneously for any realized F and Z, therefore, by matching coefficients in these two equations, we have

$$\rho \left[(CUC')^{-1}A + V^{-1}\bar{F} \right] - W = -C^{-1}A \tag{29}$$

$$\rho \left[(CUC')^{-1}(B - rI) - r\Sigma^{-1} - rV^{-1} \right] = -C^{-1}B \tag{30}$$

$$\rho\Sigma^{-1} = C^{-1} \tag{31}$$

Therefore, from equation (31), we have

$$C = \frac{1}{\rho}\Sigma = \frac{1}{\rho} \begin{bmatrix} \frac{1}{\lambda_1 + \lambda_{12}} \Sigma_1 & 0 \\ 0 & \frac{1}{\lambda_2 + \lambda_{12}} \Sigma_2 \end{bmatrix}$$

Obviously, C is positive definite and symmetric. Then from equation (29), we have

$$[\rho^2(\Sigma U \Sigma)^{-1} + \Sigma^{-1}]A = \frac{1}{\rho}W - V^{-1}\bar{F}.$$

Because both $(\Sigma U \Sigma)^{-1}$ and Σ^{-1} are both positive definite, we have

$$A = [\rho^2(\Sigma U \Sigma)^{-1} + \Sigma^{-1}]^{-1}\left(\frac{1}{\rho}W - V^{-1}\bar{F}\right).$$

From equation (30), we have

$$[\rho^2(\Sigma U \Sigma)^{-1} + \Sigma^{-1}](B - rI) = rV^{-1}.$$

Again, because $[\rho^2(\Sigma U \Sigma)^{-1} + \Sigma^{-1}]$ is positive definite, we have

$$B = rI + r[\rho^2(\Sigma U \Sigma)^{-1} + \Sigma^{-1}]^{-1}V^{-1}.$$

Obviously, B is invertible. By substituting A, B, and C into equation (6), we solve the equilibrium pricing function.

Now, let's look at any investor i's holding. Substituting the coefficients into investor i's holding function (9), we have

$$D_i = \left(I + \frac{1}{\rho^2}U\Sigma\right)^{-1}W + \rho\left[I + \rho^2(U\Sigma)^{-1}\right]^{-1}V^{-1}(\bar{F} - rP) + \rho\Omega_i^{-1}(S_i - rP).$$

Take expectation about $S_i - rP$, we have

$$\begin{aligned}
& \mathbb{E}(S_i - rP) \\
= \; & \mathbb{E}\left[S_i - rB^{-1}(F - A - CZ)\right] \\
= \; & \mathbb{E}\left[\mathbb{E}\left(S_i - rB^{-1}(F - A - CZ)\,|F\right)\right] \\
= \; & \mathbb{E}\left[F - rB^{-1}F + rB^{-1}A\right] \\
= \; & \bar{F}\left[I - rB^{-1}\right] + rB^{-1}A,
\end{aligned}$$

which is G defined in equation (15). Therefore, we have

$$\begin{aligned}
D_i = & \left(I + \frac{1}{\rho^2}U\Sigma\right)^{-1}W + \rho\left[I + \rho^2(U\Sigma)^{-1}\right]^{-1}V^{-1}(\bar{F} - rP) \\
& + \rho\Omega_i^{-1}G + \rho\Omega_i^{-1}(S_i - rP - G).
\end{aligned}$$

Q.E.D.

Proof of Proposition 4:

By equations (21), (22), and (23), we have

$$\frac{\frac{1}{P'M}\text{diag}(P)^{-1}CUCM}{\left(\frac{1}{P'M}\right)^2 M'CUCM}\frac{M'A}{P'M}$$
$$= \frac{\text{diag}(P)^{-1}CUCM}{M'CUCM}M'A.$$

This is the RHS of the Security Market Line relation. We want to show that this equals the difference between the risky assets' rates of return and the riskfree asset's rate of return, which is shown to be $\text{diag}(P)^{-1}A$ from equation (20).

Then, we have

$$\frac{\text{diag}(P)^{-1}CUCM}{M'CUCM}M'A = \text{diag}(P)^{-1}A$$
$$\Leftrightarrow \text{diag}(P)^{-1}CUCMM'A = \text{diag}(P)^{-1}AM'CUCM$$
$$\Leftrightarrow CUCMM'A = AM'CUCM.$$

The last equation holds because $M = \rho(CUC)^{-1}A$ and $(CUC)^{-1}$ is a symmetric matrix.

Q.E.D.

B Heterogeneous Risk Tolerances

In the model described in Section 2 in the paper, investors share a same risk aversion coefficient ρ. However, it is conceivably that differences in risk tolerances, and investor unawareness of other investors' risk tolerances, could resurrect the zero holdings conjecture. We therefore explore whether heterogeneous risk tolerances, together with the three justifications of the zero holding hypothesis discussed in the introduction, can justify the zero holding hypothesis and its asset pricing implications.

We extend the model in Section 2 by assuming that any investor i ($i \in [0,1]$) has the risk aversion coefficient ρ_i. Here, ρ_i is a continuous function of i. Let

$$\bar{\rho} = \int_0^1 \rho_i \mathrm{d}i \text{ and } \bar{\Sigma}^{-1} = \int_0^1 \rho_i \Omega_i^{-1} \mathrm{d}i.$$

Here, ρ is the average risk tolerance, and Σ^{-1} is the average precision of investors' private information that is weighted by their risk tolerances.

We again consider the linear pricing function as in equation (5),

$$F = A + BP + CZ, \quad \text{with } C \text{ nonsingular.}$$

Therefore, conditional on the price, assets' payoffs have the conditional distribution is

$$F|P \sim \mathcal{N}\left(A + BP, CUC'\right).$$

An investor i gleans such information from the price. She will also use the prior belief and her private information (if any) to derive her posterior belief about the assets' payoffs. Therefore, as in equation (9), an investor i's demand is

$$D_i = \rho_i \left[(CUC')^{-1}(B - rI) - r\Omega_i^{-1} - rV^{-1}\right] P + \rho_i \Omega_i^{-1} S_i + \rho_i[(CUC')^{-1}A + V^{-1}\bar{F}]. \quad (32)$$

Then, by integrating all investors' demands and equalizing the aggregate demand and the total supply (the aggregate endowments and the random supply shocks), we can derive Proposition 7 below, which is essentially same as Proposition 1.

Proposition 7 (Equilibrium with Heterogeneous Risk Tolerances) *In the model with supply shocks and heterogeneous risk tolerances, there exists an equilibrium with pricing function*

$$P = B^{-1}\left[F - A - CZ\right], \tag{33}$$

where

$$A = \left[\bar{\rho}(\bar{\Sigma}U\bar{\Sigma})^{-1} + \bar{\Sigma}^{-1}\right]^{-1}\left(W - \bar{\rho}V^{-1}\bar{F}\right) \tag{34}$$

$$B = rI + r[\bar{\rho}(\bar{\Sigma}U\bar{\Sigma})^{-1} + \bar{\Sigma}^{-1}]^{-1}\bar{\rho}V^{-1} \tag{35}$$

$$C = \bar{\Sigma}. \tag{36}$$

Any investor i's risky asset holding is

$$D_i = \rho_i\left(\bar{\rho} + U\bar{\Sigma}\right)^{-1}W + \rho_i\left[I - \bar{\rho}\left(\bar{\rho} + U\bar{\Sigma}\right)\right]V^{-1}\left(\bar{F} - rP\right) + \rho_i\Omega_i^{-1}\left(S_i - rP\right). \tag{37}$$

Proposition 7, especially investor i's equilibrium demand function equation (37), implies that when investors have heterogeneous risk tolerances, the zero holding hypothesis and its asset pricing implications are still not valid.

First, a separation theorem holds. In the first step, any investor i can hold the position as the first two terms in equation (37) directly, or she can buy ρ_i shares of an index fund that provides the "informationally passive market portfolio" $\left(\bar{\rho} + U\bar{\Sigma}\right)^{-1}W + \left[I - \bar{\rho}\left(\bar{\rho} + U\bar{\Sigma}\right)\right]V^{-1}\left(\bar{F} - rP\right)$. Second, investor i use her own private information to form the information-based trading portfolio $\rho_i\Omega_i^{-1}\left(S_i - rP\right)$. Finally, investor i invests the rest of her endowments in the risk-free asset.

Then because the informationally passive market portfolio is on average positive, investors hold strictly positive positions of all risky assets. Furthermore, if investors hold diffuse prior beliefs, the informationally passive market portfolio shrinks to the risk-adjusted market portfolio, which is deterministic and strictly positive. Consequently, though there is information asymmetry in the equilibrium, all investors, including those without any private information, will participate the markets of all assets. In addition, mild transaction costs will not prevent investors from participating.

If investors are unaware of different subsets of assets, as well as other investors' risk tolerances, the separation theorem also implies that there is an equilibrium, in which all investors will buy the informationally passive market portfolio provided by an index fund that knows all traded assets (and the average risk tolerance and the weighted average precision of investors' private information). Therefore, heterogeneous risk tolerances and unawareness cannot justify the zero holdings hypothesis.

Finally, if we use the portfolio

$$\bar{\rho}\left[\left(\bar{\rho}+U\bar{\Sigma}\right)^{-1}W+\left[I-\bar{\rho}\left(\bar{\rho}+U\bar{\Sigma}\right)\right]V^{-1}\left(\bar{F}-rP\right)\right] \tag{38}$$

for pricing, assets have no extra risk premia beyond those predicted by the CAPM. This is intuitive. Since ρ_i is continuous in i, there must be some an uninformed investor j with this risk tolerance, $\rho_j=\bar{\rho}$. Then the portfolio in (38) is just investor j's equilibrium demand. Therefore, such a portfolio must be mean-variance efficient.

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Table 1: Parameter Values for Calibration

This table presents parameter values that are used to compute the critical transaction cost (C^*) below which uninformed investor i participates in asset 1's market.

Prior Distribution of Any Asset i's Payoff		
$\bar{F}_i$:	Ex-ante expected payoff	1
$(v^2)^{-1}$:	Precision of prior distribution	25
An informed investor's private signal		
$\mathbb{E}(\epsilon)$:	Mean of the noise term	0
$(\sigma^2)^{-1}$:	Precision of the noise term	5
Liquidity Trading of Asset i		
$\mathbb{E}(z_i)$:	Mean of liquidity trading	0
$(u^2)^{-1}$:	Precision of liquidity trading	4
Endowment and Preferences		
ρ:	Risk tolerance	0.5
W_i:	Total endowment of asset i	1

Table 2: Critical Transaction Costs

This table shows how the critical transaction cost (C^*) varies when the fraction of investors who receive private signals about asset 1 (λ_1) changes. At the critical transaction cost C^*, a small uninformed investor i is indifferent between participating in asset 1's market and refraining from participating.

λ_1	0	0.1	0.4	0.5	0.7	0.8
Critical transaction cost C^*	0.1269	0.0684	0.0907	0.0581	0.0503	0.0315