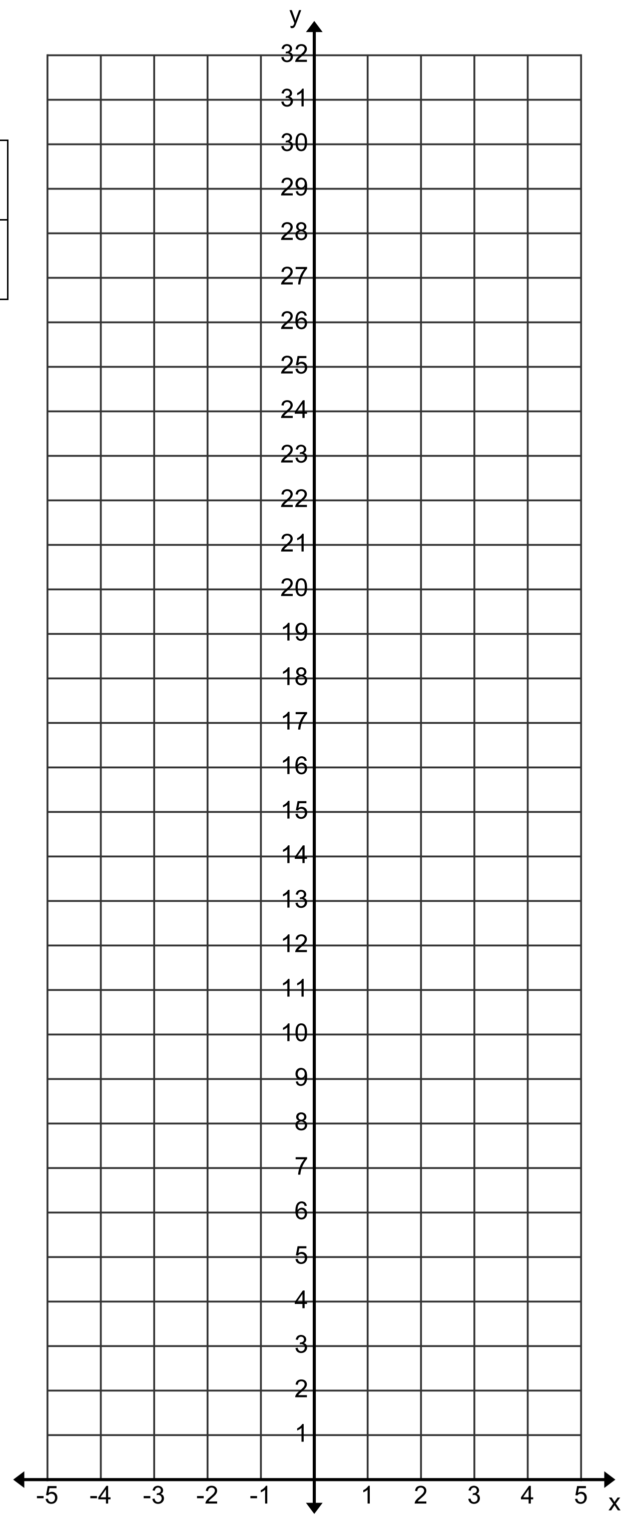
Name ______________________________________ Date ______________ Period ___________

Exponential and Logarithmic Graphs

1) Fill in the table below, make the graph, and identify the key features.

$$f(x) = 2^x$$

x	-3	-2	-1	0	1	2	3	4	5
f(x)									



a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ___________ y-intercept = ___________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

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g) Describe the end behaviors of this function. What happens to the function values as x gets very small (i.e. large negative)? What happens to the function values as x gets very large?

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2) Fill in the table below, make the graph, and identify the key features. $g(x) = \log_2 x$

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32
g(x)									

a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ________ y-intercept = ___________

d) For what intervals is this function increasing?

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e) For what intervals is this function decreasing?

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f) When are the function values positive? When negative?

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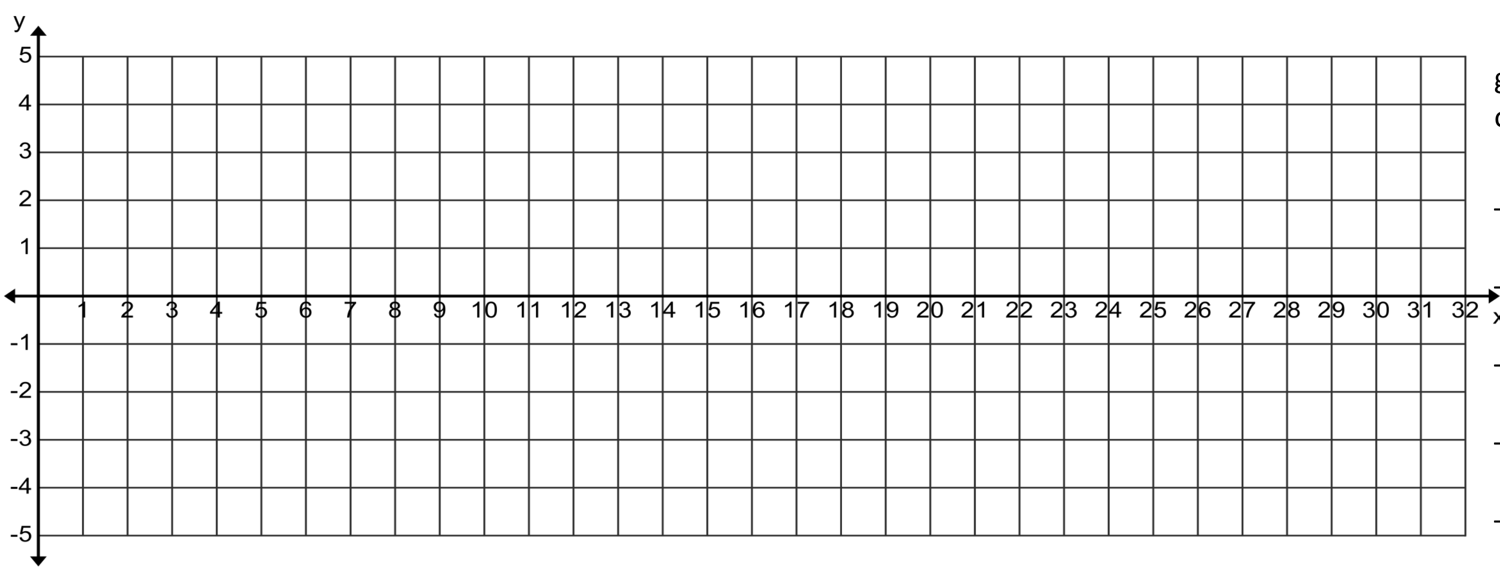

g) Describe the end behaviors of the function.

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On the previous two pages you graphed and analyzed two functions,

$$f(x) = 2^x \qquad \text{and} \qquad g(x) = \log_2 x$$

Now you will compare the two functions to each other. Answer each prompt with complete sentences.

3) Compare the domains and ranges of the two functions. What do you notice?

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4) Compare the intercepts of the two functions. What do you notice?

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5) Compare the intervals on which the two functions are increasing.

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6) Compare the end behaviors of the two functions. What do you notice?

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7) What is the relationship between *f(*x) and *g*(x)?

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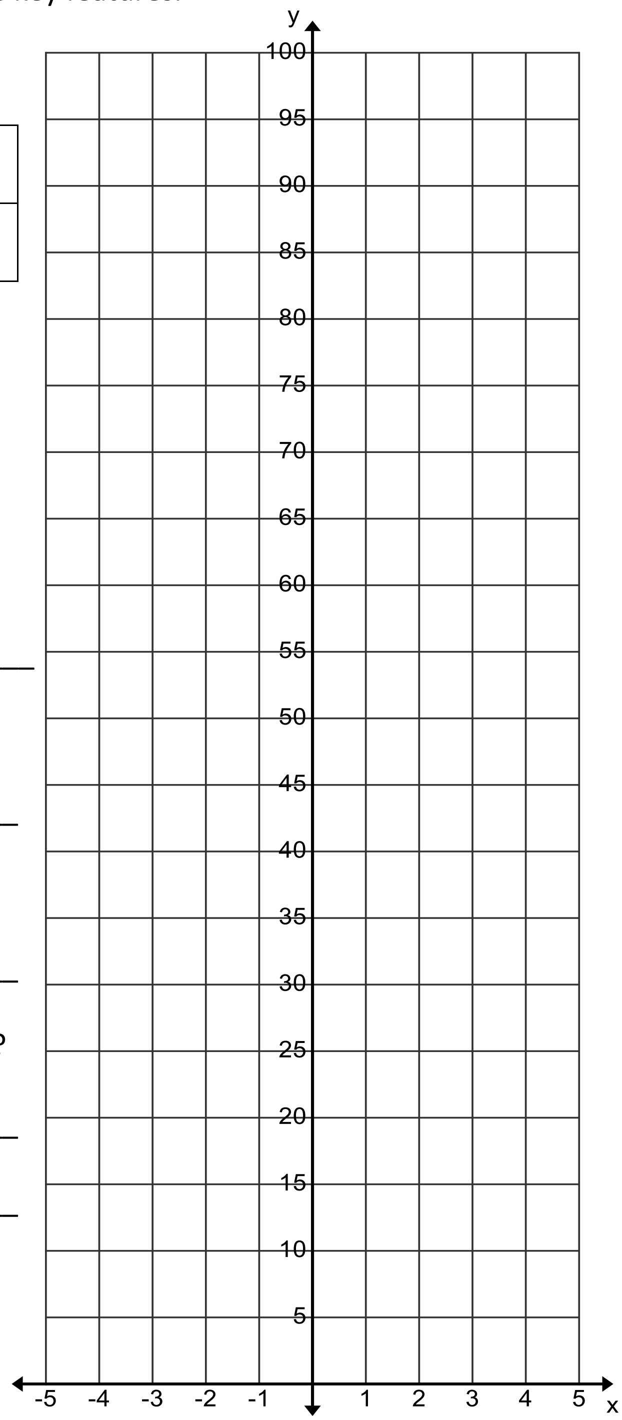
8) Fill in the table below, make the graph, and identify the key features.

$$j(x) = 10^x$$

x	-2	-1	0	1	2	3
j(x)						



a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ____________ y-intercept = ____________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

____

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g) Describe the end behaviors of this function. What happens to the function values as x gets very small? What happens to the function values as x gets very large?

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9) Fill in the table below, make the graph, and identify the key features. $k(x) = \log x$

x	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000
k(x)						

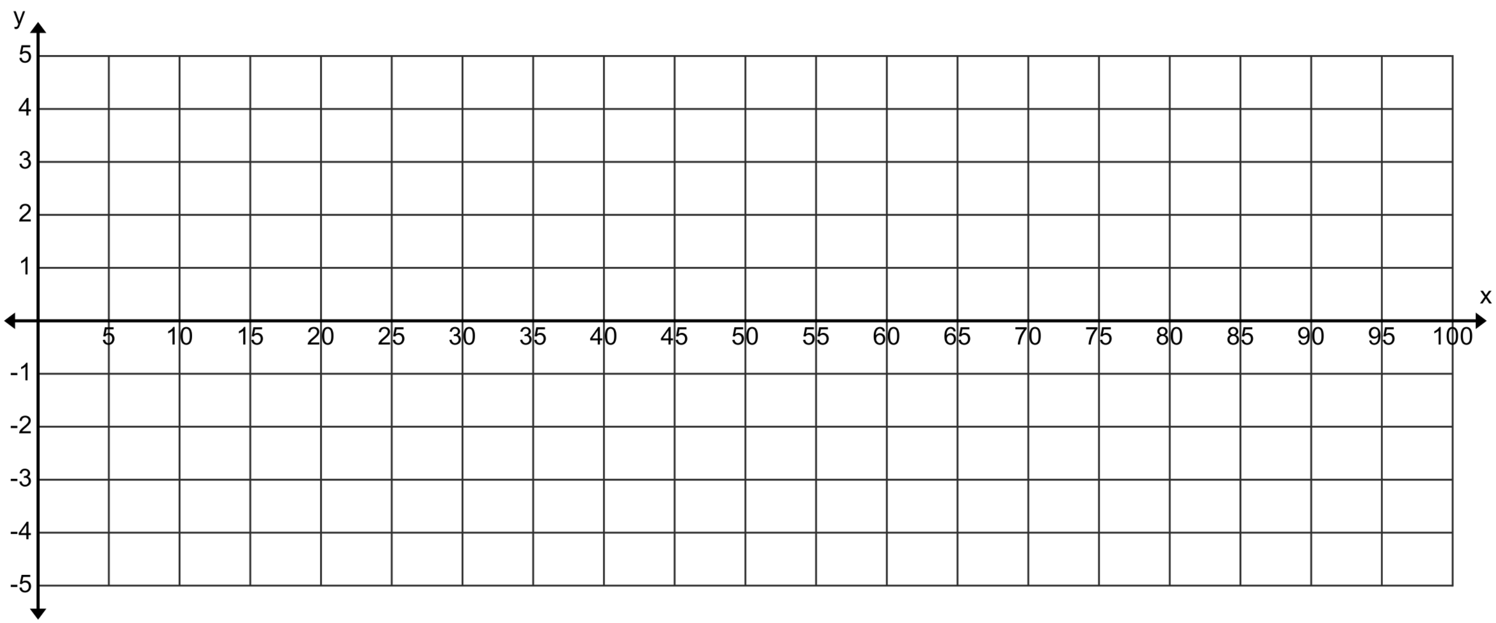
a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = __________ y-intercept = ___________



d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

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f) When are the function values positive? When negative?

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g) Describe the end behaviors of the function.

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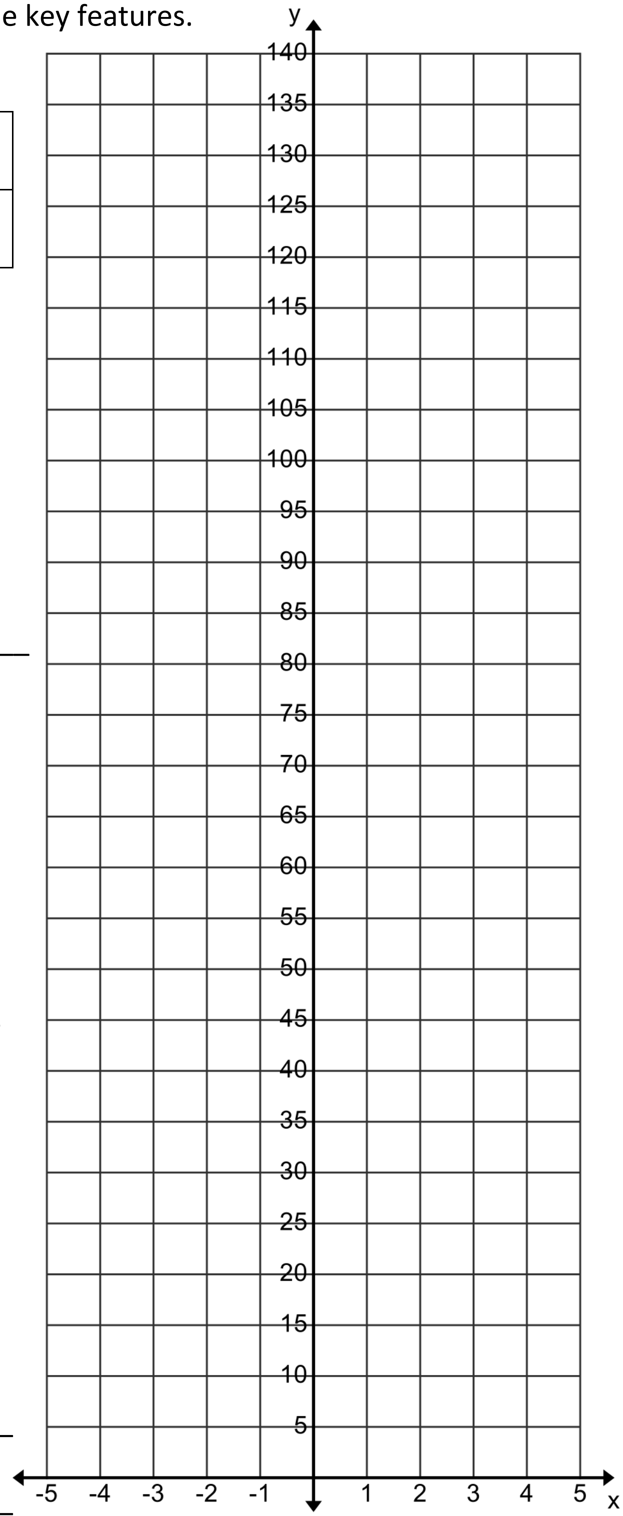
10) Fill in the table below, make the graph, and identify the key features.

$$m(x) = 5 \cdot 3^x$$



x	-2	-1	0	1	2	3
$m(x)$						

a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ____________ y-intercept = ______________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

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g) Describe the end behaviors of this function. What happens to the function values as x gets very small? What happens to the function values as x gets very large?

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11) Fill in the table below, make the graph, and identify the key features. $n(x) = \log_3 \frac{x}{5}$

x	$\frac{5}{9}$	$\frac{5}{3}$	5	15	45	135
n(x)						

a) What is the domain of this function?

____

b) What is the range of this function?

____

c) x-intercept = ________ y-intercept = __________

d) For what intervals is this function increasing?

____

e) For what intervals is this function decreasing?

____

f) When are the function values positive? When negative?

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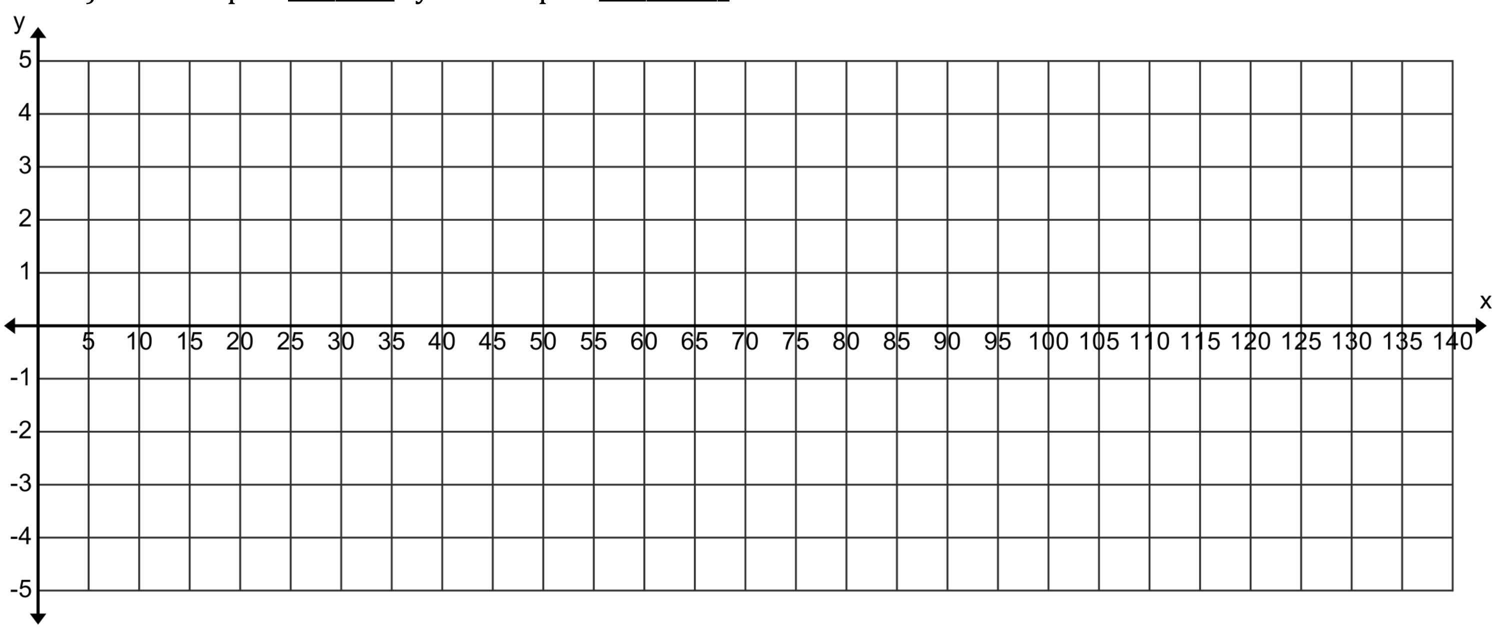

g) Describe the end behaviors of the function.

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12) You have now graphed and analyzed three exponential functions and the three logarithmic functions that are their inverses. Look at the exponential functions and compare them. Look at the logarithmic functions and compare them.

What do the exponential functions have in common? Be sure to mention all of the key features of the graphs.

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13) What do the logarithmic functions have in common? Be sure to mention all of the key features of the graphs.

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Teacher Directions:
Exponential and Logarithmic Graphs

Materials
Optional: Desmos or other graphing software (1 device per pair or small group of students)
Or
Teacher displays logarithmic graphs using Desmos or other graphing software

Objective
Students will analyze the features of pairs of exponential and logarithmic graphs, which will further develop the concept of inverse and the relationship between exponential and logarithmic functions.

Directions
Have students complete the table, graph and questions on page 1 in partners or groups. After students have completed the page, select a group to come up and share their answers. Be sure to emphasize each of the key features for the exponential function. Remind students about what they learned in the prior lesson and then have them complete page 2. Once again, select a group to come share their answers. To reinforce the relationship between exponential and logarithmic functions, it may be useful to display the graphs to the class using Desmos Graphing Calculator. You may want to graph $y = n^x$ and $y = \log_n x$ with a slider for n. Or, you can have students try this out if they have access to Desmos. Have groups complete the analysis questions on page 3 and have a class discussion about each of the questions, randomly selecting groups to share their answers.

Have students move on to the next two pairs of graphs.

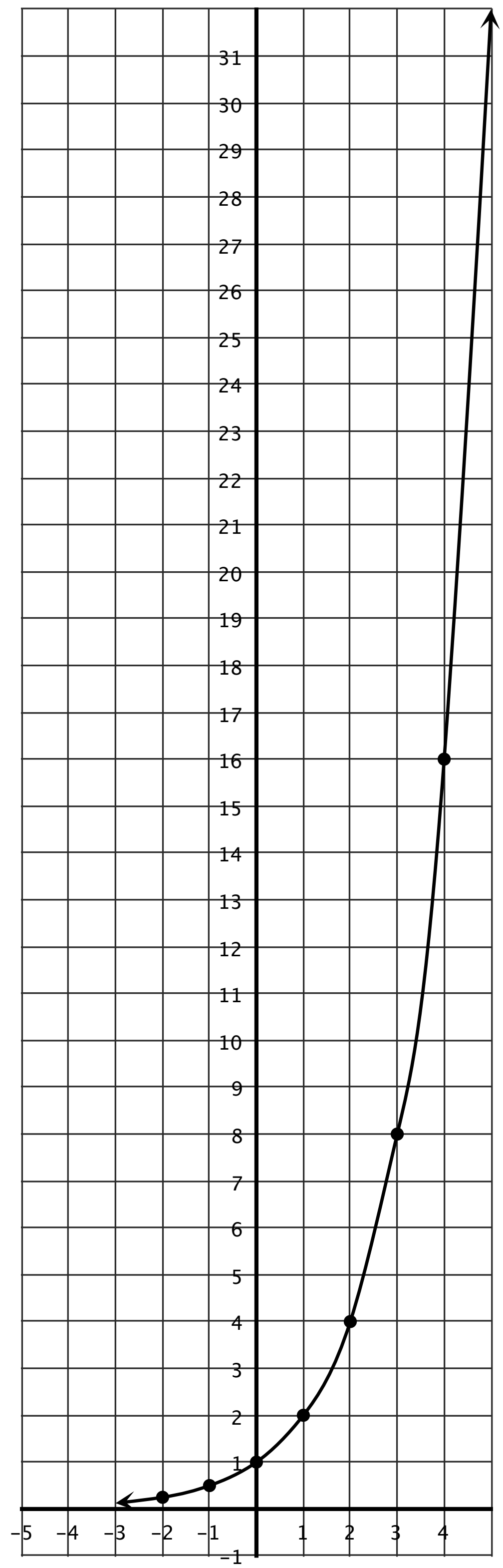
Note: If you feel doing all three pairs of graphs will be too repetitive, you can have half the class do the second pair while the other half does the third pair. Students can then get up and find a partner from the opposite side to share their results using inside-outside lines. Have the line shift at least one more time to be sure that students have all of their questions answered by the students that did the other pair of graphs.

Once students have completed all of the pairs of graphs have them list all of the commonalities of the exponential functions, and then the logarithmic functions with specific attention to key features of each function.

1) Fill in the table below, make the graph, and identify the key features.

$$f(x) = 2^x$$

x	-3	-2	-1	0	1	2	3	4	5
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32



a) What is the domain of this function? All real numbers.

b) What is the range of this function? $(0, \infty)$

c) x-intercept = There is no *x*-intercept   y-intercept = (0, 1)

d) For what intervals is this function increasing?
$(-\infty, \infty)$

e) For what intervals is this function decreasing?
The function does not have any intervals over which it is decreasing.

f) When are the function values positive? When negative?
The function values are always positive because the range is restricted to all numbers greater than zero.

g) Describe the end behaviors of this function. What happens to the function values as x gets very small? What happens to the function values as x gets very large?

As *x-values* become very small the function values approach zero. As the *x-values* increase the function values approach ∞.

2) Fill in the table below, make the graph, and identify the key features. $g(x) = \log_2 x$

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32
g(x)	-3	-2	-1	0	1	2	3	4	5

a) What is the domain of this function? $x > 0$

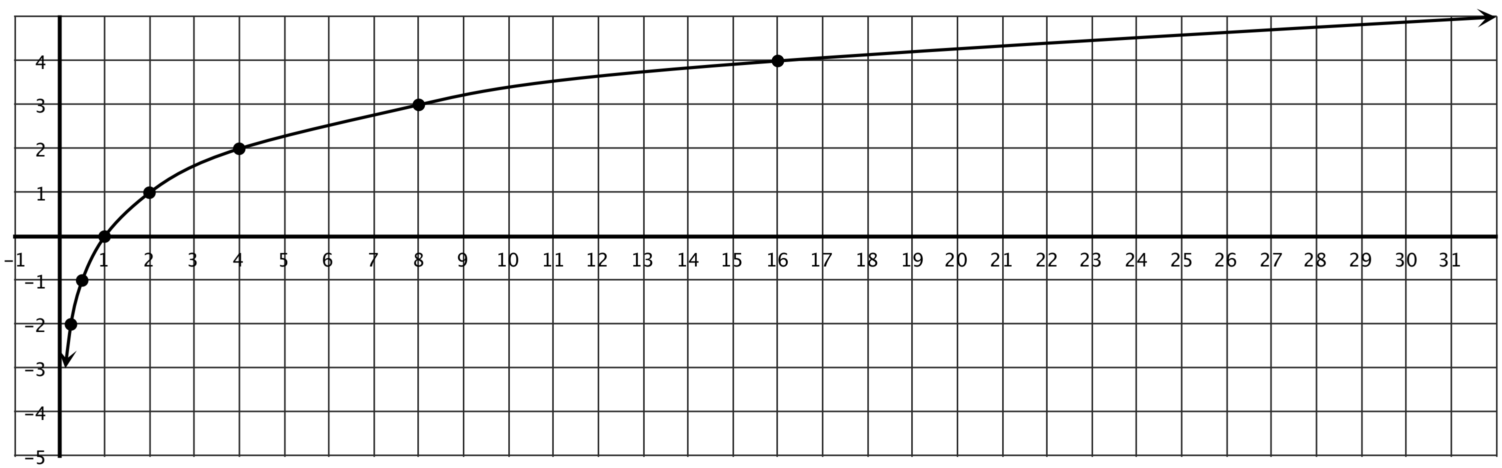
b) What is the range of this function? All real numbers.

c) x-intercept = (1, 0) y-intercept = There is no *y*-intercept.

d) For what intervals is this function increasing? $(0, \infty)$

e) For what intervals is this function decreasing? The function does not have any intervals over which it is decreasing.

f) When are the function values positive? When negative? The function values are positive when $x > 1$. The function values are negative when $x < 1$.



g) Describe the end behaviors of the function.
As x approaches zero, the function values approach $-\infty$.

As x approaches ∞, the function values approach ∞.

On the previous two pages you graphed and analyzed two functions,

$$f(x) = 2^x \qquad \text{and} \qquad g(x) = \log_2 x$$

Now you will compare the two functions to each other. Answer each prompt with complete sentences.

3) Compare the domains and ranges of the two functions. What do you notice?
The domain and range have switched between the two functions. The domain of $f(x)$ is *all real numbers*, which is the range of $g(x)$.

4) Compare the intercepts of the two functions. What do you notice?
Answers may vary, but students might state that the functions look very similar, one is just a transformation of the other; the graphs are reflections of one another over $y = x$.

5) Compare the intervals on which the two functions are increasing.
For $f(x)$ the functions is increasing over $(-\infty, \infty)$, while for $g(x)$ the function is increasing over $(0, \infty)$. Both functions are increasing over their entire domain.

6) Compare the end behaviors of the two functions. What do you notice?
For $f(x)$, as the *x-values* become very small the function values approach zero. As the *x-values* increase the functions values approach ∞.

For $g(x)$ as x approaches zero, the function values approach $-\infty$. As x approaches ∞, the function values approach ∞.

The end behaviors appear to be inverses of each other.

7) What is the relationship between $f(x)$ and $g(x)$?
The two functions are *inverse functions*.

8) Fill in the table below, make the graph, and identify the key features.

$$j(x) = 10^x$$

x	-2	-1	0	1	2	3
j(x)	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000

a) What is the domain of this function? All real numbers.

b) What is the range of this function? $f(x) > 0$

c) x-intercept = There is no *x*-intercept y-intercept = (0, 1)

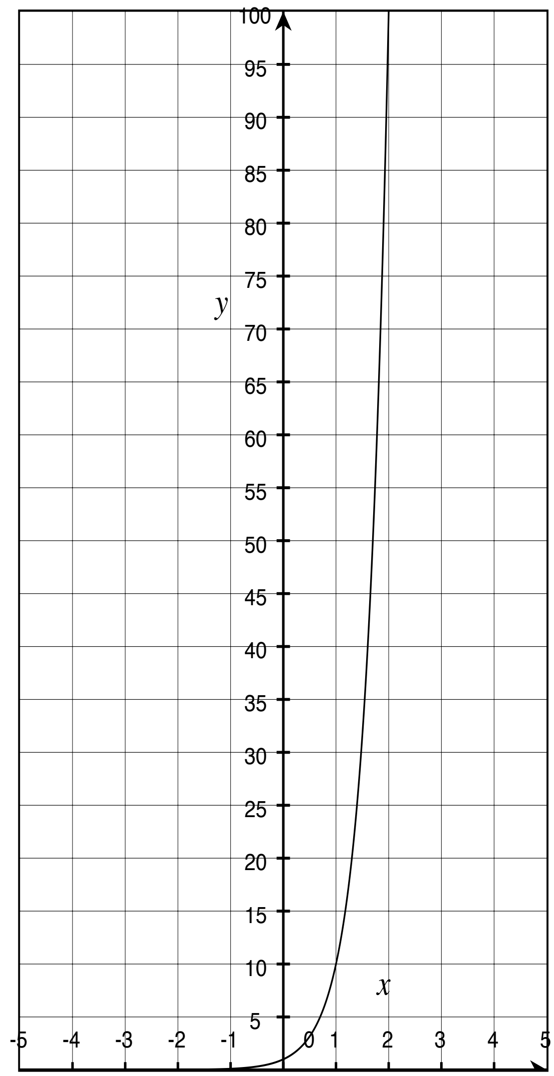
d) For what intervals is this function increasing? $(-\infty, \infty)$

e) For what intervals is this function decreasing?
The function does not have any intervals over which it
is decreasing.

f) When are the function values positive? When negative?
The function values are always positive because the range
is restricted to all numbers greater than zero.

g) Describe the end behaviors of this function.
What happens to the function values as x gets very small?
What happens to the function values as x gets very large?

As *x-values* become very small the function values approach zero. As the *x-values* increase the function values approach ∞.



9) Fill in the table below, make the graph, and identify the key features. $k(x) = \log x$

x	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000
k(x)	-2	-1	0	1	2	3

a) What is the domain of this function? $x > 0$

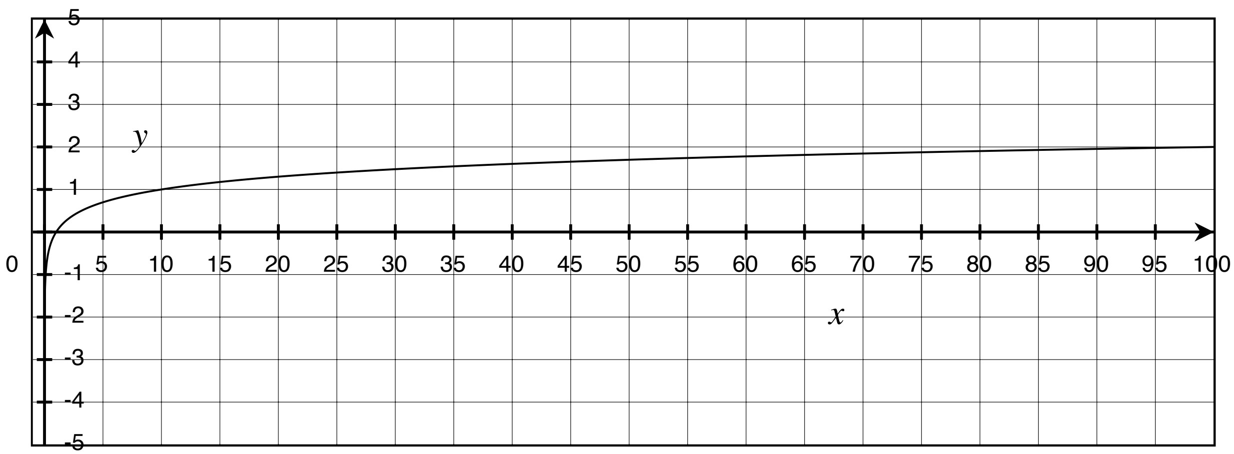
b) What is the range of this function? All real numbers.

c) x-intercept = (1, 0) y-intercept = There is no *y*-intercept.

d) For what intervals is this function increasing? $(0, \infty)$

e) For what intervals is this function decreasing? The function does not have any intervals over which it is decreasing.

f) When are the function values positive? When negative? The function values are positive when $x > 1$. The function values are negative when $x < 1$.



g) Describe the end behaviors of the function. As x approaches zero, the function values approach $-\infty$.

As x approaches ∞, the function values approach ∞.

10) Fill in the table below, make the graph, and identify the key features.

$$m(x) = 5 \cdot 3^x$$

x	-2	-1	0	1	2	3
$m(x)$	$\frac{5}{9}$	$\frac{5}{3}$	5	15	45	135

a) What is the domain of this function? All real numbers.

b) What is the range of this function? $f(x) > 0$

c) x-intercept = There is no *x*-intercept   y-intercept = (0, 5)

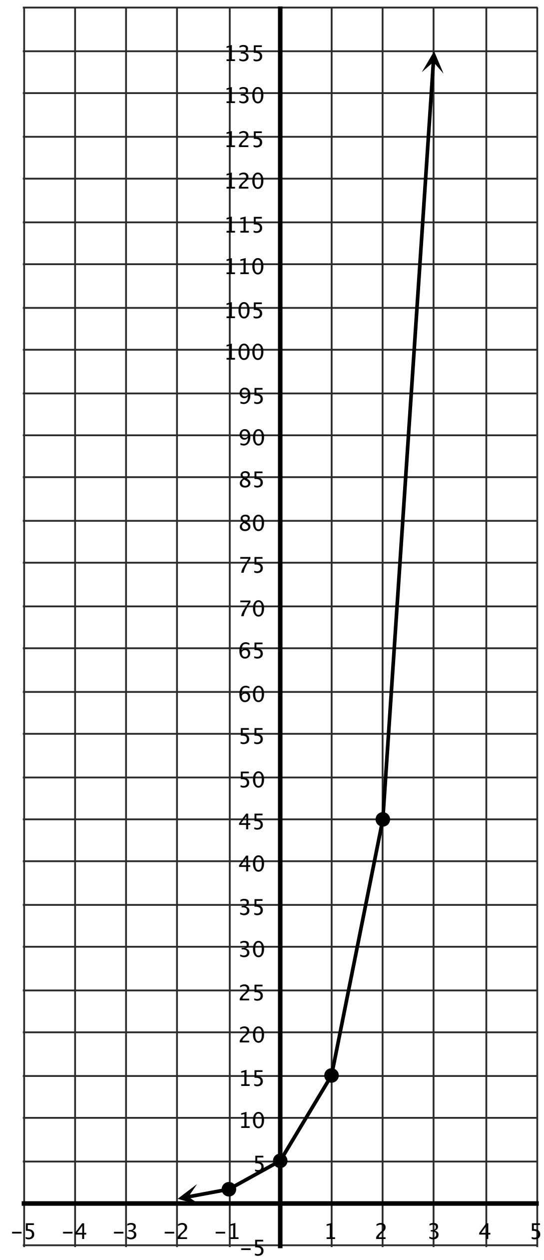
d) For what intervals is this function increasing?
$(-\infty, \infty)$

e) For what intervals is this function decreasing?
The function does not have any intervals over which it is decreasing.

f) When are the function values positive? When negative?
The function values are always positive because the range is restricted to all numbers greater than zero.

g) Describe the end behaviors of this function.
What happens to the function values as x gets very small?
What happens to the function values as x gets very large?

As *x-values* become very small the function values approach zero. As the *x-values* increase the function values approach ∞.



11) Fill in the table below, make the graph, and identify the key features. $n(x) = \log_3 \frac{x}{5}$

x	$\frac{5}{9}$	$\frac{5}{3}$	5	15	45	135
n(x)	-2	-1	0	1	2	3

a) What is the domain of this function? $x > 0$

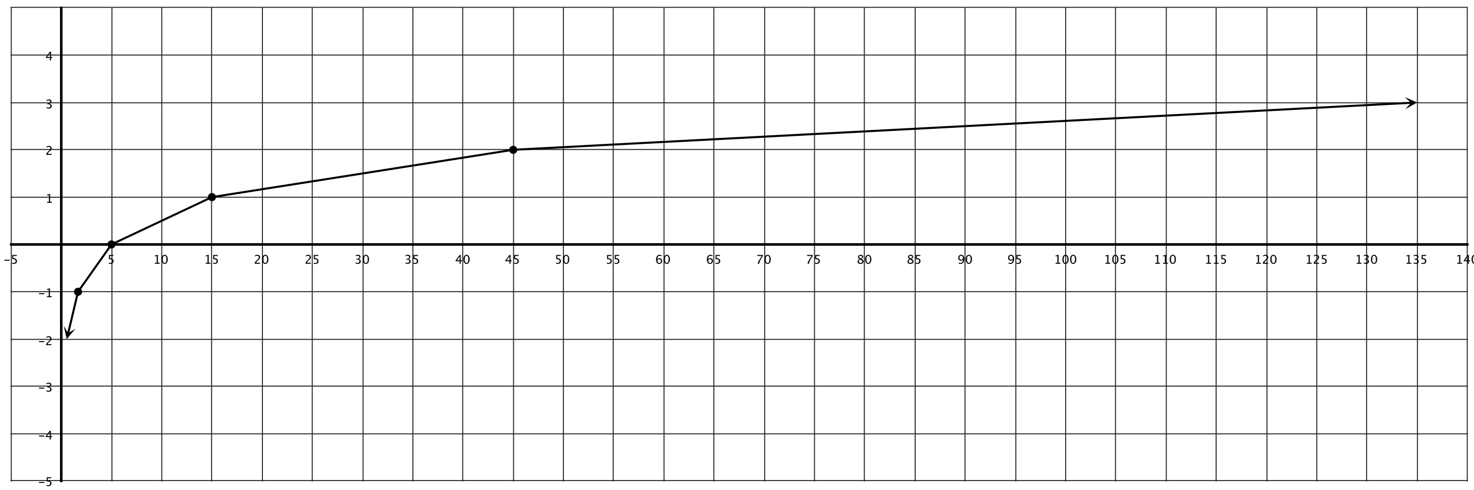
b) What is the range of this function? All real numbers.

c) x-intercept = (5, 0) y-intercept = There is no *y*-intercept.

d) For what intervals is this function increasing? $(0, \infty)$

e) For what intervals is this function decreasing? The function does not have any intervals over which it is decreasing.

f) When are the function values positive? When negative? The function values are positive when $x > 5$. The function values are negative when $x < 5$.



g) Describe the end behaviors of the function. As x approaches zero, the function values approach $-\infty$.

As x approaches ∞, the function values approach ∞.

12) You have now graphed and analyzed three exponential functions and the three logarithmic functions that are their inverses. Look at the exponential functions and compare them. Look at the logarithmic functions and compare them.

What do the exponential functions have in common? Be sure to mention all of the key features of the graphs.

Answers may vary but should include:

- A domain of *all real numbers*
- A range of $f(x) > 0$
- A *y*-intercept, but no *x*-intercept
- Increasing over $(-\infty, \infty)$
- The function values are always positive because the range is restricted to all numbers greater than zero.
- As *x-values* become very small the function values approach zero. As the *x-values* increase the functions values approach ∞.

13) What do the logarithmic functions have in common? Be sure to mention all of the key features of the graphs.

Answers may vary but should include:

- A domain of $x > 0$
- A range of *all real numbers*
- An *x*-intercept, but no *y*-intercept
- Increasing over (0, ∞)
- The function does not have any intervals over which it is decreasing.
- The function values are positive when x is approaching infinity. The function values are negative when $x$is approaching negative infinity.