Name ______________________________________ Date _____________ Period ___________

# Comparing Exponential and Logarithmic Rules

**Task 1: Looking closely at exponential and logarithmic patterns…**

1) In a prior lesson you graphed and then compared an exponential function with a logarithmic function and found that the functions are __________________________ functions.

2) When a function is the inverse of another function we know that if the _______________ of one function maps onto the *output* of another function then the inverse maps the _________________ to the ________________.

3) Given the function $f(x) = 2^x$ and a table with corresponding values

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $f(x)$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |

complete the table for the inverse function, $f^{-1}(x) = log_2 x$.

| $x$ | | | | | | |
|---|---|---|---|---|---|---|
| $f^{-1}(x)$ | | | | | | |

4) Using the tables, if $2^x = 8$, then $x$ = _______________. If $log_2 y = 3$, then $y$ = _______________.

5) Describe what you noticed about the relationship between the exponential and logarithmic equations in number 4.

_______________________________________________________________________________

_______________________________________________________________________________

6) Once more, if $2^x = 2$, then $x$ = _______________. If $log_2 y = 1$, then $y$ = _______________.

Let's make a rule for this relationship…..

**If $y = b^x$, then $log_b$_____ = _____.**

We read $\boldsymbol{log_b}$ ____ = _____ as _______________________________________.

7) Simplify the following:

a) $log_2 4 =$ ______ b) $log_3 27$ = ______ c) $log_4 16$ = ______

**Task 2: Using some examples to discover a log law…**

| | |
|---|---|
| First multiply 4 by 8, then find the log.<br>$\log_2(4 \cdot 8)$<br>$\log_2(\_\_\_\_\_\_)$<br>__________ | Compare this to finding the log of each factor separately:<br>$\log_2(4)$ = __________ and $\log_2(8)$ = __________ |
| Multiply 10 by 1,000,000, then find the log.<br>$\log_{10}(10 \cdot 1{,}000{,}000)$<br>$\log_{10}(\_\_\_\_\_\_\_\_\_\_\_\_)$<br>__________ | Compare this to finding the log of each factor separately:<br>$\log_{10}(10)$ = ______ and $\log_{10}(1{,}000{,}000)$ = ______ |
| Multiply $n^4$ by $n^6$, then find the log.<br>$\log_n(n^4 \cdot n^6)$<br>$\log_n(\_\_\_\_\_\_)$<br>__________ | Compare this to finding the log of each factor separately:<br>$\log_n(n^4)$ = ______ and $\log_n(n^6)$ = ______ |

| **Remember the exponent rule for multiplying with the same base:** | **Write the matching rule for logarithms:** |
|---|---|
| $\boldsymbol{n^x \cdot n^y = n^{x+y}}$ | $\boldsymbol{\log(a \cdot b) = \log a \;\_\_\_\_\; \log b}$ |

This is known as the ***Logarithmic Product Rule.***

8) Explain what each of these rules means using complete sentences.

____________________________________________________________

____________________________________________________________

____________________________________________________________

9) Make one example of your own for each of these laws.

____________________________ ____________________________

**Task 3: Using some other examples to discover a second log law…**

| | |
|---|---|
| Expand 4 to the 3rd power, then use the rule from the previous page to find the log.<br>$\log_2(4^3)$<br>$\log_2($________$)$<br>\_\_\_\_\_\_\_\_\_ | Compare this to finding the log of 4.<br>$\log_2(4)$ = __________ How can you use the 3? |
| Expand 1,000 to the 3rd power, then use the rule from the previous page to find the log.<br>$\log_{10}(1{,}000^3)$<br>$\log_{10}($____________________$)$<br>\_\_\_\_\_\_\_\_\_ | Compare this to finding the log of 1,000<br>$\log_{10}(1000)$ = ______ How can you use the 3? |
| Expand $n^4$ to the 5th power, then use the rule from the previous page to find the log.<br>$\log_n((n^4)^5)$<br>$\log_n($____________________$)$<br>_________ | Compare this to finding the log of $n^4$.<br>$\log_n(n^4)$ = ______ How can you use the 5? |

| | |
|---|---|
| **Remember the exponent rule for raising a power to a power:**<br>$(\boldsymbol{n^x})^y = \boldsymbol{n^{xy}}$ | **Write the matching rule for logarithms:**<br>$\mathbf{log}(\boldsymbol{a^b}) =$ ______________ |

This is known as the ***Logarithmic Power Rule.***

10) Explain what each of the rules means using complete sentences.

______________________________________________________________________

______________________________________________________________________

______________________________________________________________________

______________________________________________________________________________

______________________________________________________________________________

11) Make one example of your own for each of these laws.

____________________________________    ____________________________________

12) Using what you have learned about the *product rule* for logs, if $\log(a \cdot b) = \log a + \log b$, then how might we be able to rewrite $\log \frac{a}{b}$?

________________________________

This is known as the ***Logarithmic Quotient Rule.***

# Teacher Directions:
# Comparing Exponential and Logarithmic Rules

**Materials**
Calculator

**Objective**
Students will examine specific examples of log computations to discover and describe two basic log rules and compare them to related exponential rules.

**Directions**
Have students get into groups and pass out the activity sheet. Remind students about what they learned about the graphs of logarithmic and exponential functions, that they are inverse functions. Let students know that they will now be looking for patterns between logarithmic expressions and expressions with exponents in order to come to some general rules. In this lesson, whole class discussion should come after student exploration and effort at seeing the pattern and writing the rule. Some pairs or teams of students will recognize the pattern faster than others. You will need to decide when to have a whole class discussion about the pattern and the rule. This should not be immediately after the fastest team makes the discovery, but it may not be practical to wait until every team has.

**Task 1: Looking closely at exponential and logarithmic patterns…**
For Task 1, it may be helpful for students to have out the *Exponential and Logarithmic Graphs* lesson so that they can look more closely at the patterns between the tables of an exponential function and its inverse, the log function. Have students complete questions 1 and 2 with their group and then randomly select a group to share their answers.

> 1) In a prior lesson you graphed and then compared an exponential function with a logarithmic function and found that the functions are inverse functions.
>
> 2) When a function is the inverse of another function we know that if the input of one function maps onto the output of another function then the inverse maps the output to the input.

Next have students complete questions 3, 4 and 5 with their groups and once again, randomly select a group to share their answers and the relationship or pattern they saw. Record answers to number 5 on the board; you may need to call on a few groups until you get the pattern we are looking for.

3) Given the function $f(x) = 2^x$ and a table with corresponding values

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $f(x)$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |

complete the table for the inverse function, $f^{-1}(x) = log_2 x$.

| $x$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |
|---|---|---|---|---|---|---|
| $f^{-1}(x)$ | -2 | -1 | 0 | 1 | 2 | 3 |

4) Using the tables, if $2^x = 8$, then $x$ = 3 . If $log_2 y = 3$, then $y$ = 8.

5) Describe what you noticed about the relationship between the exponential and logarithmic equations in number 4.
Student answers may vary, but should be something such as: I noticed that the *base* of the exponential equation is the *base* of the logarithmic equation. In the exponential equation if I want to know what *x* is think "two to what power gives me eight?".

Have students work on problem 6 and then write in the rule as a class.

6) Once more, if $2^x = 2$, then $x$ = 1. If $log_2 y = 1$, then $y$ = 2.

Let's make a rule for this relationship.....

**If $y = b^x$, then $log_b y = x$.**

We read $\boldsymbol{log_b}$ **y = x** as log base *b* of *y* equals *x*.

Have students complete number 7 in preparation for Task 2. If students need a few more examples before moving on, provide them with a few more, using numbers from the tables above.

7) Simplify the following:

a) $log_2 4 =$ 2 b) $log_3 27$ = 3 c) $log_4 16$ = 2

**Task 2: Using some examples to discover a log law…**

Once again, let students know that they will now be looking for in order to come to some general rules. In this lesson, whole class discussion should come after student exploration and effort at seeing the pattern and writing the rule. Some pairs or teams of students will recognize the pattern faster than others. You will need to decide when to have a whole class discussion about the pattern and the rule. This should not be immediately after the fastest team makes the discovery, but it may not be practical to wait until every team has.

| | |
|---|---|
| First multiply 4 by 8, then find the log.<br>$\log_2(4 \cdot 8)$<br>$\log_2(32)$<br>5 | Compare this to finding the log of each factor separately:<br>$\log_2(4) = 2$ and $\log_2(8) = 3$ |
| Multiply 10 by 1,000,000, then find the log.<br>$\log_{10}(10 \cdot 1{,}000{,}000)$<br>$\log_{10}(10{,}000{,}000)$<br>7 | Compare this to finding the log of each factor separately:<br>$\log_{10}(10) = 1$ and<br>$\log_{10}(1{,}000{,}000) = 6$ |
| Multiply $n^4$ by $n^6$, then find the log.<br>$\log_n(n^4 \cdot n^6)$<br>$\log_n(n^{10})$<br>10 | Compare this to finding the log of each factor separately:<br>$\log_n(n^4) = 4$ and<br>$\log_n(n^6) = 6$ |

After completing the table, have students move on to writing the general rule.

| | |
|---|---|
| **Remember the exponent rule for multiplying with the same base:**<br>$\boldsymbol{n^x \cdot n^y = n^{x+y}}$ | **Write the matching rule for logarithms:**<br>$\boldsymbol{\log(a \cdot b) = \log a + \log b}$ |

8) Explain what each of these rules means using complete sentences.
When multiplying two powers, if the base is the same, the base remains the same and we add the exponents. Multiplication inside the log can be turned into addition outside the log.

Note: You may want to ask students if the opposite is true, if we are adding two logarithmic expressions, can we rewrite them as the product of a log? (Yes) In fact, the reason logarithms were invented (Napier, https://en.wikipedia.org/wiki/History_of_logarithms#Tables_of_logarithms) was to make multiplication easier!]

9) Make one example of your own for each of these laws.

Answers may vary. Have several students bring up their examples and ask the class to vote as to whether the examples accurately represent each of the rules.

**Task 3: Using some other examples to discover a second log law…**

Again, you will need to have a whole class discussion about the pattern and rule, and decide when your class is ready for that discussion. The explanations that students write can be used as formative assessment by either selecting students at random to read what they have written or by collecting it.

| | |
|---|---|
| Expand 4 to the 3rd power, then use the rule from the previous page to find the log.<br>$\log_2(4^3)$<br>$\log_2(64)$<br>6 | Compare this to finding the log of 4.<br>$\log_2(4)$ = 2 How can you use the 3?<br>We can use the 3 and multiply it by 2 to find the solution, 6. |
| Expand 1,000 to the 3rd power, then use the rule from the previous page to find the log.<br>$\log_{10}(1{,}000^3)$<br>$\log_{10}(1{,}000{,}000{,}000)$<br>9 | Compare this to finding the log of 1,000<br>$\log_{10}(1000)$ = 3 How can you use the 3?<br>We can use the 3 and multiply it by 3 to find the solution, 9. |
| Expand $n^4$ to the 5th power, then use the rule from the previous page to find the log.<br>$\log_n((n^4)^5)$<br>$\log_n(n^{20})$ | Compare this to finding the log of $n^4$.<br>$\log_n(n^4)$ = 4 How can you use the 5?<br>We can use the 5 and multiply it by 4 to find the solution, 20. |

| **Remember the exponent rule for raising a power to a power:** $(\boldsymbol{n^x})^y = \boldsymbol{n^{xy}}$ | **Write the matching rule for logarithms:** $\mathbf{log}(\boldsymbol{a^b}) = \boldsymbol{b} \cdot \boldsymbol{log\ a}$ |
|---|---|

10) Explain what each of the rules at the bottom of the previous page means using complete sentences.
When a power is raised to a power, the base remains the same and we multiply the powers. An exponent inside a log can be moved out front as a multiplier.

11) Make one example of your own for each of these laws.
Answers may vary. Have several students bring up their examples and ask the class to vote as to whether the examples accurately represent each of the rules.

12) Using what you have learned about the *product rule* for logs, if $\log(a \cdot b) = \log a + \log b$, then how might we be able to rewrite $\log \frac{a}{b}$?

Students should be able to reason that multiplication became addition, so division would be subtraction. $\log \frac{a}{b} = \log a - \log b$

You might also want students to try a few examples.