



# Module #24

## Fatigue of Materials

#### **READING LIST**

▶ DIETER: Ch. 12, all

- Ch. 14 in Meyers & Chawla
- S. Suresh, <u>Fatigue of Materials</u>, 2<sup>nd</sup> Ed., Cambridge (1998)



# Topics

- Mechanisms for Fatigue
- Fatigue Crack Propagation

## Fatigue of Materials

- Many materials, when subjected to fluctuating stresses, fail.
- The <u>stresses</u> required to cause failure are <u>far below those</u> <u>needed to cause fracture on single application of load</u>.
- Fatigue failure is <u>failure under dynamic loading</u>.
- Fatigue is the cause for more than 90% of all service failures in structural materials. It is something that you would like to avoid.
- Fatigue failures generally occur with little or no warning (with catastrophic results).

## Fatigue Fractures

• They generally appear to be brittle.



Figure 14.1 Schematic representation of a fatigue fracture surface in a steel shaft, showing the initiation region (usually at the surface), the propagation of fatigue crack (evidenced by beach markings), and catastrophic rupture when the crack length exceeds a critical value at the applied stress.

[Meyers & Chawla]



Fatigue fracture markings. (a) Rotating steel shaft. Center of curvature of earlier "beach markings" locate crack origin at corner of keyway. (b) Clam shell markings (C) and ratchet lines (R) in aluminum. Arrows indicate crack propagation direction. [Figures adapted from R.W. Hertzberg, <u>Deformation & Fracture Mechanics of Engineering Materials, 4th ed.</u>, (John Wiley & Sons, New York, 1996) p. 523]





- Real stress cycles are far more complex and unpredictable than the ideal one showed on the prior viewgraph.
- Thus, fatigue failures are statistical in nature.
- Now we need to consider how fatigue parameters influence fatigue failure.

# Fatigue Results: the S-N curve

• Engineering fatigue data is generally presented on S-N (or  $\varepsilon$ -N) curves.



# cycles to failure (log scale)

 $S_L$  = fatigue/endurance limit

#### Fatigue Results: The S-NCurve



- In materials exhibiting a <u>fatigue limit</u>, cyclic loading at stresses below the fatigue limit cannot result in failure. In steels  $S_L/UTS = 0.4$  to 0.5).
- For non-ferrous materials we will generally define the <u>fatigue strength</u> as the stress that will cause fracture at the end of a specified number of cycles (usually 10<sup>7</sup>).

# **Categories of Fatigue**



[M.F. Ashby and D.R. Jones, <u>Engineering Materials 1, 2<sup>nd</sup> Edition</u>, Butterworth-Heinemann, Boston (1996), p. 146]





#### Stress-Life or Strain-Life?

- Most data is presented on *S*-*N* curves. This is typical of the stresslife method which is most suitable for high-cycle applications [i.e., in the high-cycle fatigue (HCF) regime].
- However, it is easier to implement strain-life experiments.
- The strain-life method is more applicable where there is measurable plastic deformation [i.e., in the low-cycle fatigue (LCF) regime].



O-A-B reflects initial loading in tension.

#### Strain-Life Method

• It's convenient to consider elastic and plastic strains separately.

 $\Delta \varepsilon = \Delta \varepsilon_{elastic} + \Delta \varepsilon_{plastic}$ 

• Elastic strain amplitude is determined from a combination of ,  $N_f$  and Hooke's law.

$$\frac{\Delta \varepsilon_e}{2} = \frac{\sigma_a}{E} = \left(\frac{\sigma_f'}{E}\right) \left(2N_f\right)^b$$

$$\frac{\Delta \varepsilon_e}{2} = \text{ elastic strain amplitude}$$
  

$$\sigma_a = \text{ true stress amplitude}$$
  

$$\sigma'_f = \text{ fatigue strength coefficient}$$
  

$$b = \text{ fatigue strength exponent}$$

#### Strain-Life Method (2)

The <u>plastic strain component</u> is described by the Manson-Coffin equation.

$$\frac{\Delta \varepsilon_p}{2} = \frac{\sigma_a}{E} = \varepsilon_f' \left( 2N_f \right)^c$$

$$\frac{\Delta \varepsilon_p}{2} = \text{plastic strain amplitude}$$

$$\sigma_a = \text{true stress amplitude}$$

$$\varepsilon'_f = \text{fatigue ductility coefficient}$$

$$c = \text{fatigue ductility exponent}$$

• The Manson-Coffin equation describes LCF.

#### Strain Life Method (3)

 $\Delta \varepsilon = \Delta \varepsilon_{elastic} + \Delta \varepsilon_{plastic}$ 

••••

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \left(\frac{\sigma_f'}{E}\right) \left(2N_f\right)^b + \varepsilon_f' \left(2N_f\right)^c$$

- This is the Manson-Coffin relationship between fatigue life and total strain. We can use it to determine the fatigue strength.
- At high temperatures, the Manson Coffin equation breaks down because N<sub>f</sub> decreases as temperature increases. In addition, N<sub>f</sub> depends on cyclic frequency.



#### Cyclic Strain-Controlled Fatigue

- Strain amplitude is held constant during cycling.
- Strain-controlled cyclic loading often occurs during thermal cycling (when a component expands and contracts due to fluctuations in temperature).
- This is particularly dangerous when a component is made from materials exhibiting different coefficients of thermal expansion.
- Also occurs during reversed bending between fixed displacements.
- Local plastic strains at notches subjected to cyclic loading can also result in strain-controlled conditions near the root of the notch. This is due to constraint placed on the material near the root by the surrounding mass of material.

#### **Trends for Engineering Metals**



**Figure 14.6** Trends in strain–life curves for strong, tough, and ductile metals. (Adapted from [Landgraf 70]; copyright © ASTM; reprinted with permission.)

- Constant strain-amplitude cycling:
  - High-strength materials are desirable for HCF.
  - High ductility materials are desirable for LCF.
  - High-strength materials have low values of  $\epsilon'_f$  and low ductility.
  - High-ductility materials have low values of  $\sigma'_{f}$  and low strength.

#### Schematic stress-strain hysteresis loop

- O-A-B reflects initial loading in tension.
- On unloading, curve B-C, yielding begins in compression at a lower value than was observed in tension. This is due to the Bauschinger effect.
- Re-loading in tension completes the hysteresis loop.
- A hysteresis loop is described by its width,  $\Delta \varepsilon$ , the total strain range, and its height  $\Delta \sigma$ , the stress range.



#### Response of Materials to Strain Cycles

 Metals can harden or soften during fatigue depending upon their initial state.











Effects of Mean Stress and Stress Ratio

> Variation of  $\sigma_m$ (and R) will cause the endurance limit to change





# Effects of Mean Stress and Stress Ratio

- As  $\sigma_m$  increases, the fatigue life decreases!
- As R increases, the fatigue life increases!

Figure 12-6 Two methods of plotting fatigue data when the mean stress is not zero.

#### Effects of Mean Stress and Stress Ratio

- There are several empirical equations to relate the alternating stress to the mean stress.
  - -Goodman



Most experimental data lies between the Goodman and Gerber values. The Goodman relation is more conservative and is safer for design purposes.

## Cumulative Damage and Life Exhaustion

• Most engineering structures are subjected to variable amplitude loading as is illustrated below.



- As you can see, a certain stress amplitude ( $\sigma_{a1}$ ) is applied for a number of cycles ( $N_1$ ). The number of cycles to failure for  $\sigma_{a1}$  is  $N_{f1}$ .
- The fraction of life used is  $N_1/N_{f1}$ . There will be additional expressions for regions of loading with different stress amplitudes.
- The total life used can be expressed as:

$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \dots + \frac{N_i}{N_{fi}}$$

## Cumulative Damage and Life Exhaustion

• The Palmgren-Miner rule can be used to determine whether or not the fatigue life is exhausted.



$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \dots = \sum_i \frac{N_i}{N_{fi}} = 1$$

• Fatigue failure is expected when the life fractions sum to unity (i.e., when 100% of the fatigue life is exhausted).

## Cumulative Damage and Life Exhaustion

 If the variable amplitude loading cycle is repeated a number of times, it is convenient to sum cycle ratios over one repetition of the history and to then multiply that fraction by the number of repetitions required to reach unity.



 $B_f$  = number of repetitions to failure

## THE FATIGUE PROCESS

- There are 3 stages:
  - I. Crack initiation
  - II. Crack propagation or stable crack growth
  - III. Unstable crack growth or failure



## Fatigue Crack Growth (1)

- Regardless of the mechanism that controls fatigue at elevated temperatures, we are <u>concerned with how</u> <u>long it will take for a crack to grow to a critical length</u>.
- Everything starts with <u>crack initiation</u>!
- Crack initiation is not well understood. Most cracks initiate at free surfaces. However, in those instances where cracks initiate within a solid, some sort of interface is usually involved.

#### Fatigue Crack Growth (2) Where "slip" fits in.

- Crystalline solids generally deform by slip, which leaves slip bands on the surface.
- In unidirectional deformation, slip occurs uniformly through a grain.
- Slip planes are offset in one direction, the slip direction.



[Reed-Hill & Abbaschian, p. 756]



# Slip bands are observed long before the required number of cycles to fracture the material

[Figure scanned from Reed-Hill & Abbaschian, p. 756]

# Fatigue Crack Growth (3)

- (a) Under an alternating load, crystalline solids will still deform by slip leaving slip bands. Slip occurs in one direction.
- (b) However alternating loads produce slip in both directions. This can result in slip band intrusion.
- (c) This can also produce slip band extrusion.
- Intrusions and extrusions can act as stress concentrators (or notches) for cracks initiation.



[Figure adapted from Dieter, p. 396]

#### Fatigue Crack Growth (5) Where "slip" fits in.

In unidirectional deformation, slip occurs uniformly through a grain.



[Adapted from A.H. Cottrell and D. Hull, "Extrusion and Intrusion by Cyclic Slip in Copper," *Proc. Roy. Soc. A.*, v. 242 (1957) p.211-213]

- ► In fatigue, slip lines form on some grains but not on others.
- Additional deformation produces additional slip lines. Fewer slip lines are produced than the actual number of fatigue cycles.
- In many materials, slip rapidly reaches a saturation value, resulting in distorted regions of heavy slip.

## Fatigue Crack Growth (3)

- THUS, slip lines in fatigue tend to be grouped into distinct bands (i.e., regions of locally heavy deformation).
- Cracks are generally found to occur in these regions of heavy deformation.
- Some bands are more "persistent" than others. We call them persistent slip bands (PSBs).



[Reed-Hill & Abbaschian, p. 756]

#### Persistent Slip Bands and Crack Initiation

• PSBs are embryonic fatigue cracks that open when small tensile strains are applied.

PSB protrusions with extrusions and intrusions in a Cu single crystal fatigued at room temperature. 120,000 cycles at a plastic strain amplitude of 0.002. [From Ma and Laird, *Acta metall.*,**37** (1989) 325].





Section through a PSB containing intrusions at A and B and a crack at C. [Scanned from Reed-Hill and Abbaschian, p. 759].

- Once formed cracks will initially propagate along slip planes (Stage I).
- Later they assume directions perpendicular to the applied stress (Stage II).

## Stage I

(a) Surface intrusions and extrusions on a crystal subjected to an alternating stress. This surface morphology, due to the heterogeneous plastic deformation taking place on slip bands, is not observed during monotonic loading. At some point the extrusion assumes a cracklike nature, and a Stage I fatigue crack is considered nucleated. (b) The Stage I fatigue-crack-propagation direction is dictated by flow considerations. Thus, it is not normal to the stress axis. At some point it becomes normal, and Stage II slow crack growth commences. (Part (a) After R. Reed-Hill, Physical Metallurgy Principles, 2d edn., D. van Nostrand, New York 1973; (b) after A. S. Tetelman and A. J. McEvily, Jr., Fracture of Structural Materials, Wiley, New York, 1967.)



- Cracks propagate
   via crystallographic
   shear modes.
- A few cracks nucleate along crystallographic slip planes.
- The rate of crack propagation is low (on the order of a few Å/cycle). The fracture surface in Stage I is nearly featureless.

[Adapted from Courtney, p. 572]

## Stage II

 Stage II crack propagation shows ripples/striations.



From R.M.N. Pelloux, "Fractography," in <u>Atomistics</u> <u>of Fracture</u>, edited by R.M. Latanision and J.R. Pickens, Plenum Press (1981) pp. 241-251

- Each striation represents the position of the advancing crack. The crack propagation rate is high (on the order of a µm/cycle).
- Striations are the result of a combination of crack propagation and blunting.



## Stage III

- The fatigue crack becomes too large.
- The  $K_c$  of the material is exceeded resulting in fast fracture.



Variation in fatigue crack length with # cycles to failure

## Fatigue Crack Propagation (1)

• The crack growth rate can be expressed as:

$$\frac{da}{dN} = C\sigma_a^m a^n$$

C = constant  $\sigma_a = \text{ alternating stress}$  a = crack length m = a constant ranging from 2 to 4n = a constant ranging from 1 to 2

• Is equation can be re-written in terms of the total accumulated strain:

$$\frac{da}{dN} = C_1 \varepsilon^{m_1}$$

 $C_1$  = constant  $\varepsilon$  = total strain  $m_1$  = a constant ranging from 2 to 4

• This is known as the Paris equation. It predicts the crack growth rate in the region of stable crack growth (Stage II).

- We can make fatigue crack propagation more useful by relating it to linear elastic fracture mechanics (LEFM).
- Consider a thin sheet specimen of width w with a crack already in it. The crack could result from the presence of a manufacturing defect.



• The stress near the crack tip is:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} F_{ij}(\theta) + \dots$$

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} F_{ij}(\theta) + \dots$$

• The K is the stress intensity factor that we defined previously as fracture toughness.



- Values of K have been tabulated for materials with different crack geometries.
- Of course, we are interested in the critical K values (i.e.,  $K_c$ ).

- At  $K_c$  an incremental increase in the crack length (*da*) results in a small change in the elastic strain energy release rate (i.e.,  $\pi\sigma^2 a/E$ ) which is equaled by the energy required to extend a crack.
- If you exceed  $K_c$  the crack opens up. If you are below  $K_c$  it does not.

$$\Delta \sigma = \sigma_{\rm max} - \sigma_{\rm min}$$

$$K = Y\sigma\sqrt{\pi a}$$
$$\therefore$$
$$\Delta K = K_{\text{max}} - K_{\text{min}} = Y\Delta\sigma\sqrt{\pi a}$$

• For constant  $\Delta \sigma$ ,  $\Delta K$  correlates with the fatigue crack growth rate. This is illustrated on the next page.



• Obviously 
$$\frac{da}{dN} = F(\Delta K)$$

- The variation of the fatigue crack growth rate with  $\Delta K$  is shown on the next page.



- I. Crack initiation little or no crack growth
- II. Crack propagation Paris law region
- III. Unstable crack growth accelerated crack growth

 Paris, Gomez and Anderson (1961) showed that the fatigue crack growth rate could be related to the stress intensity factor range by the relationship:

$$\frac{da}{dN} = A \left(\Delta K\right)^p$$

where A and p are constants that depend upon material, environment, and test conditions.

• The influence of the mean stress, written in terms of the stress ratio *R*, is given by:

$$\frac{da}{dN} = \frac{A(\Delta K)^p}{(1-R)K_c - \Delta K}$$

 An increase tends to increase crack growth rates in all portions of the crack growth curve.

Region	Comments
I.	Growth rates are controlled by microstructure, $\sigma_{\rm m},$ and environment.
II.	Growth rates are controlled by microstructure, environment, and frequency. In this region, p = 3 for steels and 3-4 for Al alloys.
III.	Growth rates are controlled by microstructure, $\sigma_{m}$ , and thickness.



#### FATIGUE CRACK GROWTH RATES ARE STRUCTURE INSENSITIVE

(a) dc/dN vs.  $\Delta K$  for several Ti, Al, and steel alloys. (b) The same data replotted as dc/dN vs.  $\Delta K/E$ . Normalizing  $\Delta K$  by dividing by the modulus (*E*) produces a curve (with some scatter about it) in which crack-growth rates in several materials cannot be as clearly differentiated as they are in (a). The results indicate that fatigue-crack growth rates are not structure-sensitive; this is in contrast to most mechanical properties. This comes from Bates and Clark, *Trans. ASM*, **62**, 380 (1969).

[Courtney, p. 590]

## **Creep-Fatigue Interactions**

- Materials are generally used in "hostile" environments where diffusional processes can operate and/or where they are subject to environmental attack.
  - Ex. High-temperature turbine blade in an aircraft engine.
     Temperatures are in the creep regime. The environment is "toxic." Rotational stresses and differences in thermal expansion result in cyclic loading.

#### ★ How do we correlate everything that is going on?

- Are we talking about creep enhanced by fatigue environment?
- Are we talking about fatigue enhanced by creep?
- What is the answer?

## **Creep-Fatigue Interactions**

- When the cyclic stress or strain amplitude is small compared to the mean stress (i.e.,  $\sigma_a << \sigma_m$ ) creep accelerated by fatigue.
  - This definition also applies when temperature is high and the applied frequency is low.
- Under opposite circumstances we have fatigue failure accelerated by diffusional processes (i.e., creep).

- At high temperatures fracture is caused by grainboundary cavitation, which is followed by cavity growth and cavity coalescence resulting in a flaw of critical size.
- Several modes of high temperature fracture are illustrated on the next viewgraph.

## Modes of High-Temperature Fracture



Intergranular creep-controlled fracture. (a,b) Grain boundary sliding stimulates nucleation of grain boundary voids. (c), the voids grow by diffusion, but diffusion fields of neighboring voids do not overlap, so that each void is contained within a cage of powerlaw creeping material. [Fig. 21]



Diffusional void growth. Voids that lie on boundaries which carry a tensile stress can grow by diffusion. This mechanism is the limiting case of that shown in the Figure to the left when the diffusion fields of the growing voids overlap. [Fig. 22]



Rupture at high temperatures. Generally associated with dynamic recovery or recrystallization. [Fig. 23]

[Figures from H.J. Frost and M.F. Ashby, Deformation-Mechanism Maps: The Plasticity and Creep of Metals and Ceramics, (Pergamon Press, Oxford, 1992)]

(a)

## **Creep-Fatigue Interaction**

- Recall from fracture mechanics that tensile stresses tend to open up cracks whereas compressive stresses tend to heal cracks.
- How can cyclic loading accelerate creep?



#### GBS balanced by diffusion

**Figure** Schematic illustrating how a cylindrical strain can accelerate void growth in a creeping material. (a) A cavity is situated on a grain boundary. (b) during boundary sliding the respective halves of the cavity are displaced, and (c) a diffusive flux results so as to maintain the equilibrium dihedral angle at the boundary-cavity junction. (d) This results in cavity growth and (e) the process si repeated on stress reversal. (From C. Wigmore and G.C. Smith, *Metal Science Journal*, v. 5 (1971) p. 58.)

## Fracture Criterion for Creep-Fatigue

[Courtney, 1<sup>st</sup> Ed., p. 595]

$$\sum \frac{N_i}{N_{fi}} + \sum \frac{t_i}{t_{fi}} = 1$$

 $N_i = \#$  cycles at stress amplitude  $\sigma_{ai}$  $N_{fi} = \#$  cycles to failure at stress amplitude  $\sigma_{ai}$  $t_i =$  time spent at stress-temperature combination  $t_{fi} =$  creep fracture life

Comparison of experimental results on life in creep-fatigue environments to those predicted by the combined life-fraction rule (Eq. (12.10), which is denoted by the line marked linear model). Cyclical-hardening materials exhibit lifetimes greater than those predicted by the linear model, whereas cyclical-softening materials fail prematurely in comparison to lifetimes predicted by Eq. (12.10). It is of special interest that a small cyclic stress can enhance creep life (for cyclichardening materials), as evidenced by the positive slope of the experimental curve at small values of  $\sum (N_i/N_f)$ . (D. M. R. Taplin and A. L. Collins. Reproduced with permission, from the Ann. Rev. Matls. Sc., 8, 235, © 1978 by Annual Reviews Inc.)

- Model predicts linear relationship.
- Behavior is different in actual materials,



## Influence of Environment (1)

- Exposure to high temperatures generally reduces  $N_f$  at a given  $\Delta \varepsilon_{pf}$ .
- Oxygen in air can also have an "embrittling" effect on the fatigue life of materials.
   Elevated temperatures can enhance this effect.
   The next pages shows this.
- The frequency of cyclic loading (v) also has an effect. When v is lower, sample lifetimes are smaller for a given plastic strain range.



(a) Effect of temperature on the plastic strain amplitude-cycles to failure relationship. Except at very high plastic strain amplitudes (and correspondingly low  $N_f$ ), temperature generally diminishes a material's fatigue life. The graph is constructed for a constant frequency. (b) The effect of frequency and environment on fatigue life of stainless steel, whose fatigue-creep behavior is typical of many engineering alloys. Air environments decrease fatigue life, and the effect is greatest at low frequencies; it is clear that elevated-temperature fatigue of stainless steel in air also involves environmental "embrittling" effects. The curve is constructed for a fixed temperature. (*Part (b) After L. F. Coffin, Adv. in Rsch. on the Strength and Fracture of Materials, ed. D. M. R. Taplin, Vol. 1, Pergamon Press, New York, 1977, p. 263.*)

[Courtney, p. 604]

#### Influence of Environment (2)



Frequency-modified fatigue life,  $N_f v^{k-1}$ 

Plastic strain range vs. frequency modified fatigue life for elevated-temperature fatigue of stainless steel. Data were obtained over a range of frequencies. At a given temperature, all of the data cluster about the same line. Note that, for low-cycle fatigue, temperature, and environment have little effect on fatigue life (cf. also Figs. 12.27). Moreover, it is clear the elevated-temperature fatigue life of stainless steel is much greater in vacuum than in air. (*Data from J. T. Berling and T. Slot, ASTM STP 459, Philadelphia, Pa., 1969, p. 3.*)

[Courtney]



[Courtney, p. 605]<sup>12.29</sup>

Fatigue fracture maps for two copper alloys: (a) copper alloy 175 and (b) AMAX MZC. At low homologous temperatures, low-cycle fatigue is supplanted by tensile failure as the plastic strain range is increased. For copper alloy 175, two types of high-temperature fracture are found: cavitation for  $0.4 \leq T/T_m \leq 0.6$  and oxidation-instigated failure at higher temperatures. The several regions on the map indicate the type of failure expected at different strain range-temperature combinations. Contours of constant  $N_f$  values can be superimposed on such a map, as done here. Cavitation in Cu alloy 175 is accompanied by a notable diminution in fracture strain; this is also true for AMAX MZC (see curves for  $N_f = 10^2$  and  $10^3$  at  $T/T_m = 0.5$ ), but to a lesser extent. The line at the lowest values of  $\Delta \varepsilon_{pl}$  represents the transition at which the elastic and plastic strain ranges are equal. (D. M. R. Taplin and A. L. Collins. Reproduced with permission, from the Ann. Rev. Matls. Sc., **8**, 235, © 1978 by Annual Reviews Inc.)

## Some Variables That Affect Fatigue Life

- Surface finish
  - Most fatigue cracks initiate from preexisting defects on the component's surface. Improving surface finish leads to longer fatigue lives.
- Grain size
  - Fatigue strength increases as grain size decreases. Grain boundaries are good obstacles to fatigue crack propagation. This is identical to crack propagation in brittle solids during fracture.
- Residual stresses
  - Compressive residual stresses inhibit cracks from opening up. Compressive stresses close cracks. Shot peening is a method to induce compressive residual stresses on component surfaces.
- Temperature
  - Fatigue strength increases as temperature decreases. Materials are generally stronger.
- Environment
  - Corrosion reduces fatigue strength by reducing the amount of material present to carry the applied load. This actually concentrates stress. Limit contact to corrosive environments.