Module #17

Work/Strain Hardening

READING LIST

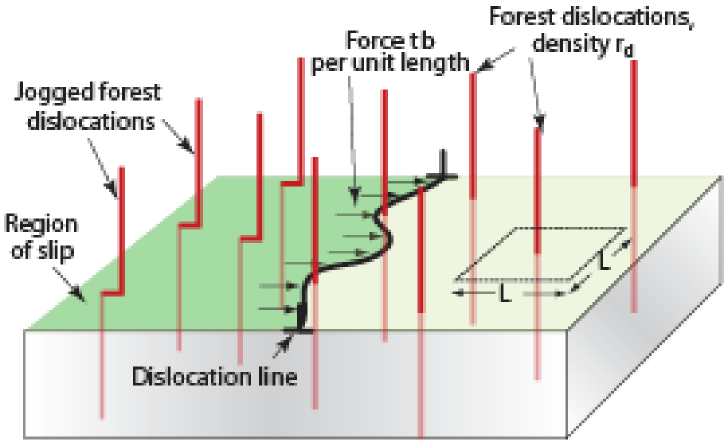
►DIETER: Ch. 4, pp. 138-143; Ch. 6 in Dieter

- D. Kuhlmann-Wilsdorf, *Trans. AIME*, v. 224 (1962) pp. 1047-1061

Work Hardening

- RECALL: During plastic deformation, *dislocation density increases*. We've addressed this earlier. It is this increase in dislocation density that ultimately leads to *work hardening*.

- Dislocations interact with each other and can assume configurations that restrict the movement of other dislocations.



[Ashby, Shercliff, & Cebon, p. 129]

- The situation gets more severe as the dislocation density increases leading to an increase in the flow stress.

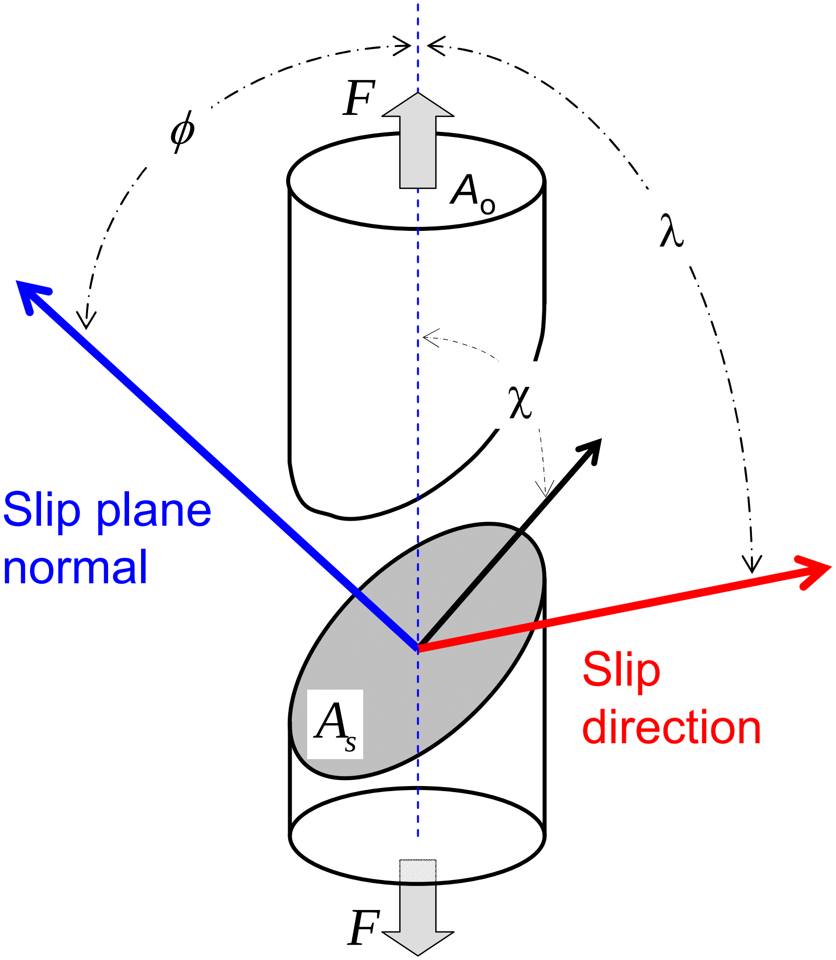
- The dislocations can be either "strong" or "weak" obstacles depending upon the types of interactions that occurs between moving dislocations.

Work Hardening (2)

- To properly discuss work hardening, let's first consider the plastic deformation of a single crystal.

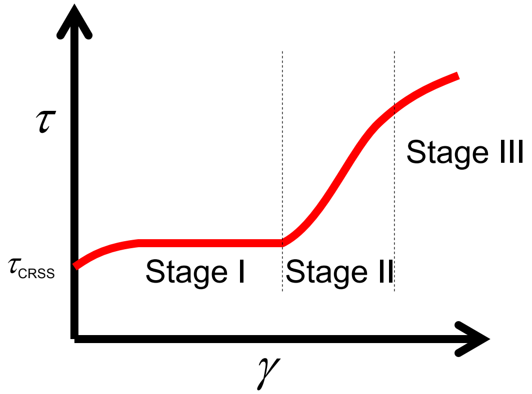
- Plastic deformation is initiated at a critical stress, the critical resolved shear stress (CRSS).

  Recall from our derivation of the Taylor-Orowan equation that this is the same stress at which dislocations begin to move on a specific slip system.



$$\tau_{CRSS} = \sigma_{ys} \cos\phi \cos\lambda$$

Single crystal deformation (1)

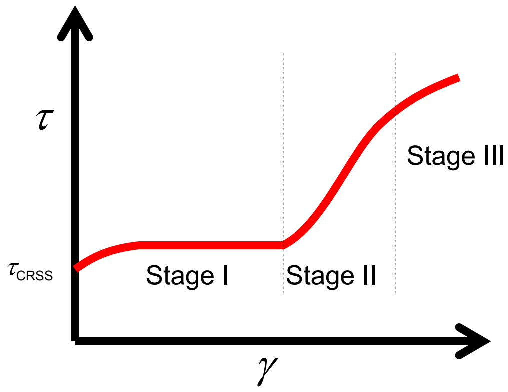


- The characteristic τ vs. γ curve for a single crystal oriented for slip on one slip system (i.e., *single slip*) is shown to the right.

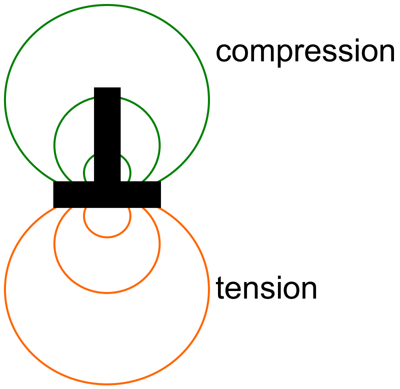
- It can be sub-divided into three stages based on work hardening behavior.

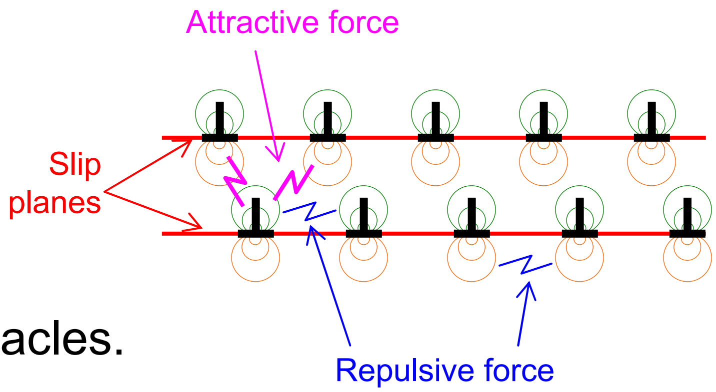
- Stage I: EASY GLIDE
  - After yielding, shear stress for plastic deformation is almost constant. There is little or no work hardening.
  - Typical of systems when single slip system is operative.
  - Very few dislocation interactions: "easy glide". Only interactions between the stress fields of dislocations to contend with. We mentioned this earlier.
  - The active slip system has a maximum Schmid factor.

Single crystal deformation (2)



- Stage I: Conceptual description of easy glide
  - Consider an array of dislocations moving on parallel slip planes. We'll use edge dislocations for schematic purposes. Edge dislocations have compressive stress fields above the slip plane and tensile stress fields below the slip plane.
  - These <u>stress fields interact</u> during the early stages of deformation resulting in "weak" drag effects. Think of it as a friction force that inhibits glide. This frictional effect will increase as dislocation density increases.
  - This is schematically illustrated to the right. You should consider how this relates to our prior discussions of dislocation configurations and the Peierls-Nabarro model for slip.
  - In stage I dislocations are "weak" obstacles.

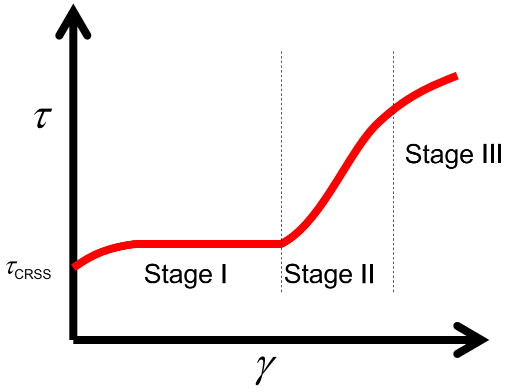




Single crystal deformation (3)

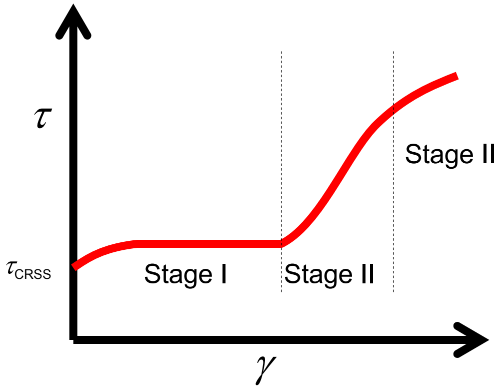
- Stage II: LINEAR HARDENING
  - τ needed to continue plastic deformation begins to increase. This increase is approximately linear. "Linear hardening".
  - This stage begins when slip occurs on multiple slip systems.
  - The work hardening rate increases due to interactions between dislocations moving on intersecting planes.
  - This results in the production of jogs and other sessile dislocation configurations such as Lomer-Cottrell Locks, etc.

- Stage III: PARABOLIC HARDENING
  - See a decreasing rate of work hardening due to an increase in the degree of cross slip. This is called "parabolic hardening". Have a parabolic shape to the curve.
  - NOTE: Polycrystals often go straight into stage III. Why?



More about Stage II on the next viewgraph

Single crystal deformation (4)

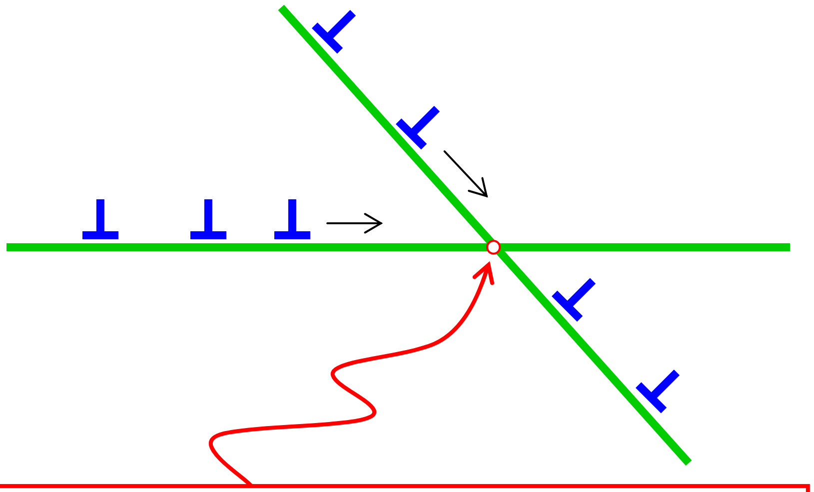


- Stage II:
  - Dislocation tangles ("forest dislocations") form strong obstacles to dislocation motion.
  - Tangles form when dislocation motion leads to the formation of immobile dislocation segments. Those segments will keep other dislocations from moving freely.
  - Work hardening depends strongly on dislocation density:

$$\rho_\perp \propto \frac{\perp \text{ line length}}{\text{unit volume}} = \frac{L}{L^3} = \frac{1}{L^2}$$

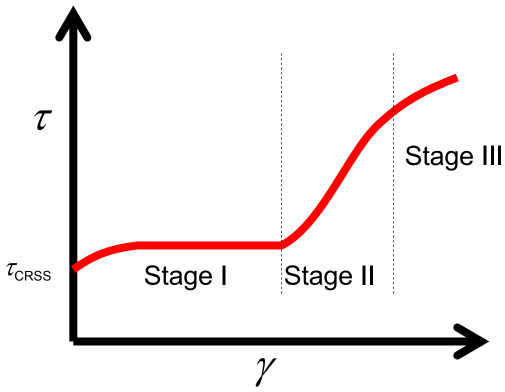
  - The average separation distance between dislocations is:

$$\bar{L} = \frac{\text{constant}}{\sqrt{\rho_\perp}}$$



Interactions can produce immobile dislocation configurations. Examples include Jogs and sessile dislocation locks (e.g., Lomer or Lomer-Cottrell locks).

Single crystal deformation (6)



- Recall the general hardening law :

$$\tau_{\max} = \frac{Gb}{L}$$

- If we substitute our expression for $\bar{L}$ into the equation above, we can express the shear flow stress as:

$$\tau = \tau_o + \alpha G b \sqrt{\rho_\perp}$$

$$\boxed{\bar{L} = \frac{\text{constant}}{\sqrt{\rho_\perp}}}$$

where:

$\tau_o =$ intrinsic flow strength for $\perp$ free material

$\alpha =$ constant (0.2 for FCC, 0.4 for BCC)

- The strengthening increment derived from work hardening can now be approximated as:

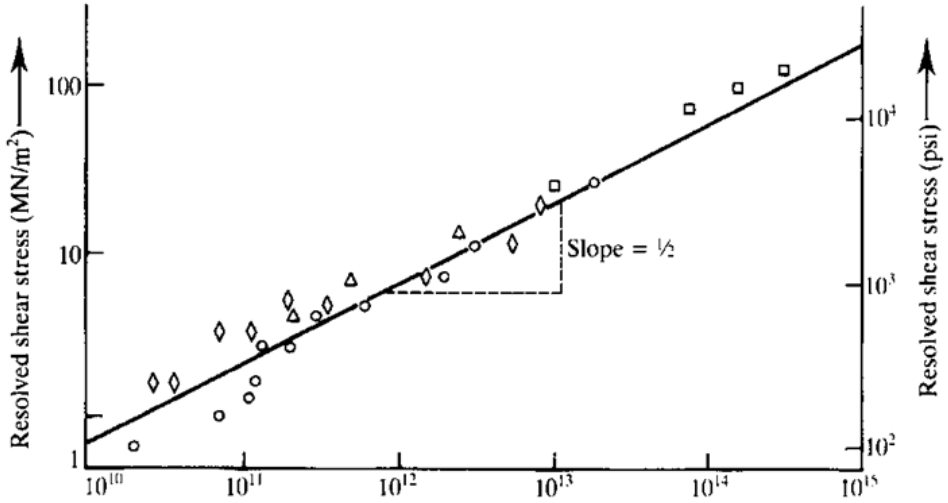
$$\boxed{\Delta\tau_\perp = \alpha G b \sqrt{\rho_\perp}} \quad \text{or} \quad \underbrace{\boxed{\Delta\sigma_\perp = M \alpha G b \sqrt{\rho_\perp} = k_\perp \sqrt{\rho_\perp}}}$$

In this equation, M is the Taylor factor.
G must be converted to E.

G.I. Taylor, *Proceedings of the Royal Society*, A145, pp. 362-388

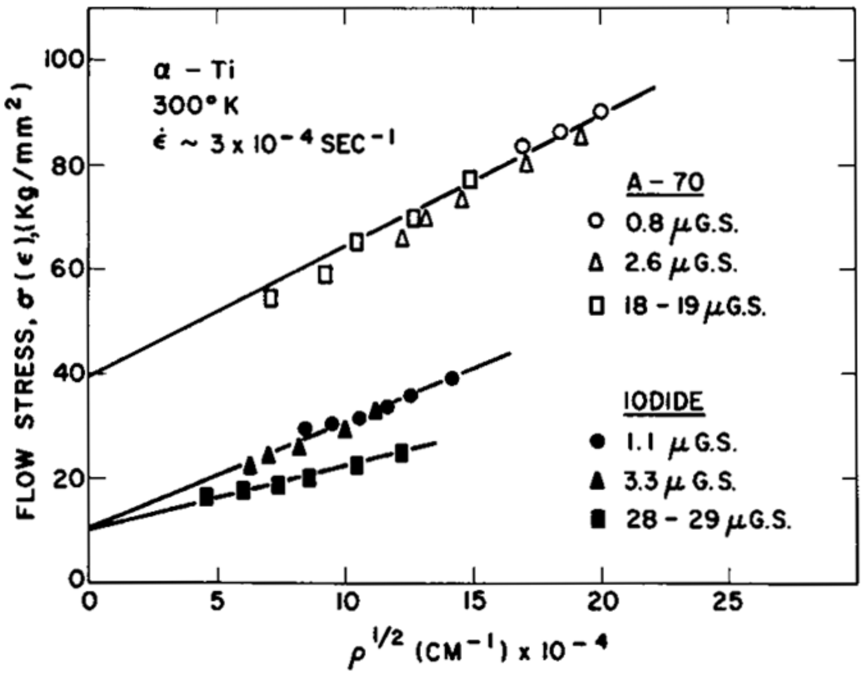
Single crystal deformation (7)

Note for both materials (in this case Ti and Cu) that the stress (σ and τ) exhibits a $(\rho_{\perp})^{1/2}$ dependence



FIGURE

CRSS versus $\sqrt{\rho_{\perp}}$ for Cu single crystals and polycrystals. □ - polycrystal; ○ - single crystal, one slip system; ◇ - single crystal, two slip systems; △ - single crystal, six slip systems. [Image scanned from Courtney's text. Originally from H. Weidersich, *J. Metals*, **16** (1964) 425]



FIGURE

Flow stress versus $\sqrt{\rho_{\perp}}$ for different grades of Ti deformed at room temperature. [R.L. Jones and H. Conrad, *Trans. AIME*, **245** (1969) 779 -789]

Strength $\propto \sqrt{\sigma_{\perp}}$

Dislocation microstructure as function of ε_p and $\rho_\perp$

- Low strain:
  somewhat *random*.

- High strain:
  dense tangles and "cellular" arrangements. Form near the end of Stage II in single crystals.
  - Cell boundaries have high $\rho_\perp$.
  - Cell interiors have low $\rho_\perp$.
  - Cells are called subgrains.

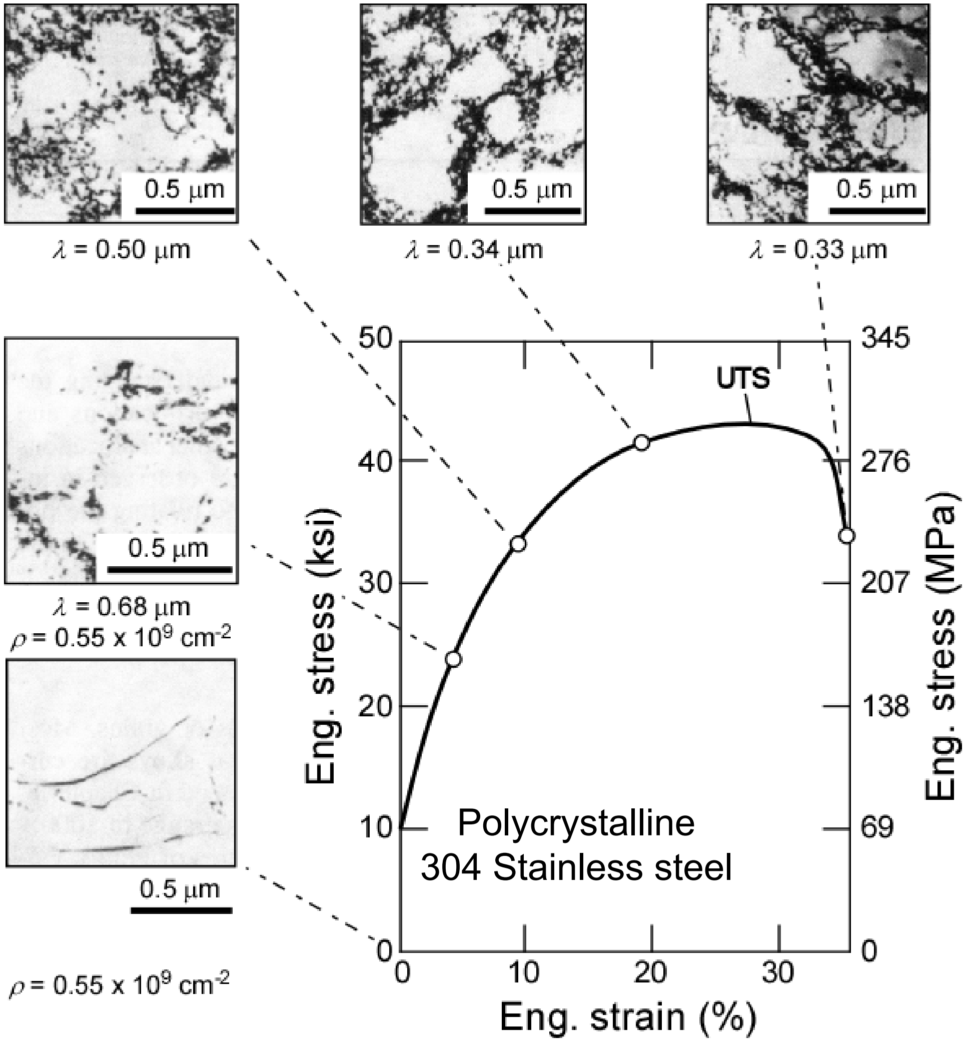
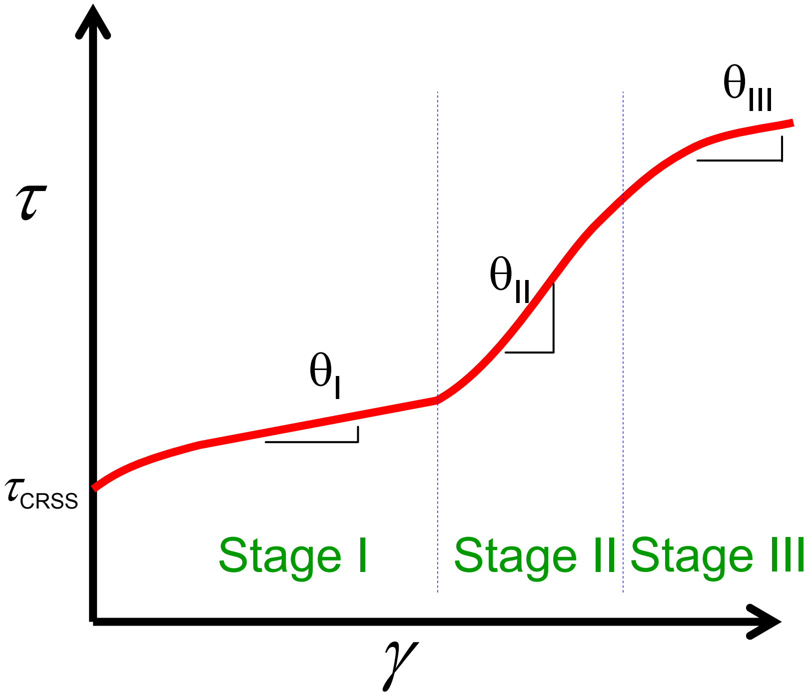


Figure adapted from J.R. Foulds, A.M. Ermi, and J. Moteff, *Materials Science and Engineering*, **45** (1980) 137-141. This journal paper provides a nice description of how dislocation microstructures evolve.

- Why do they form?

  Is strength still proportional to $\sqrt{\rho_\perp}$?



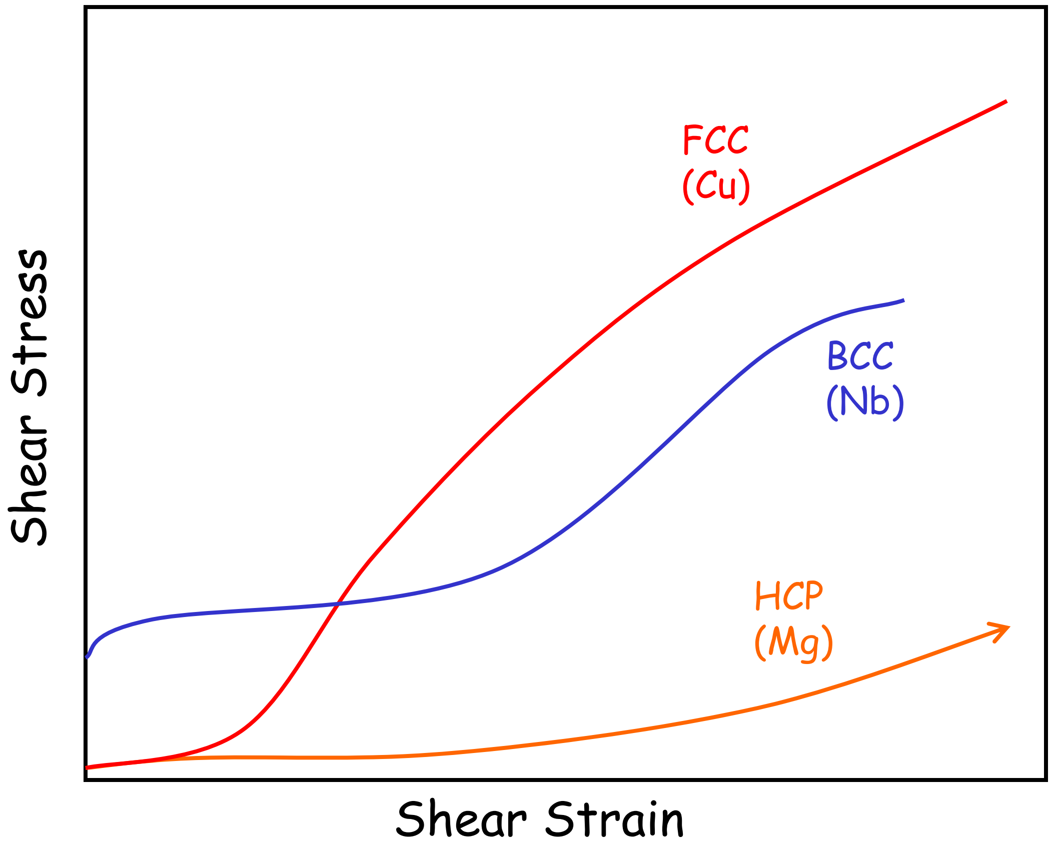
$\theta_I \sim G/10,000$

$\theta_{II} \sim G/300$

← Typical work hardening rates

This stage is often absent when multiple slip systems are operative

Influence of crystal structure on single crystal stress-strain curves



Stress-strain curves for three typical metallic crystal structures.

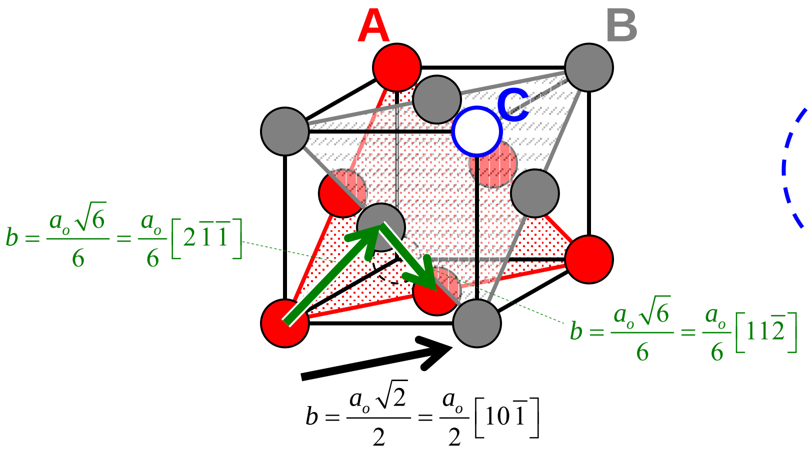
Adapted from T. Suzuki, S. Takeuchi, and H. Yoshinaga, Dislocation Dynamics and Plasticity, Springer-Verlag, Berlin, 1991, p. 15.

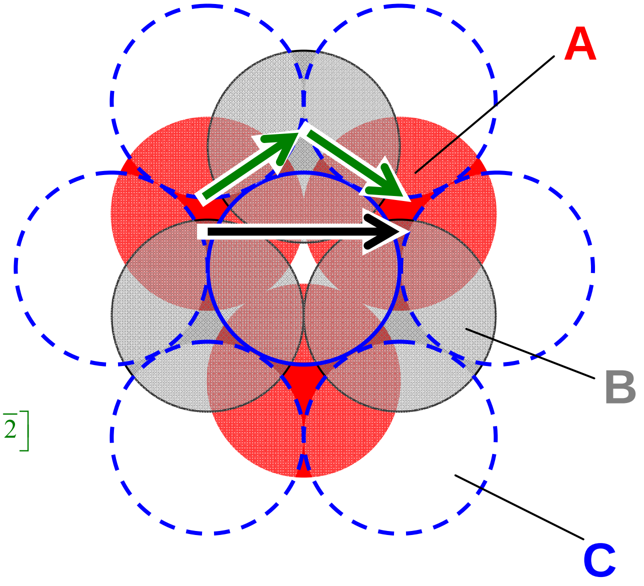
In some crystal structures,
dislocations dissociate to slip

DOES THIS HAVE ANY IMPACT ON
WORK HARDENING?

RECALL: Dissociation of unit dislocations in an FCC crystal

- In this example, the separation into partial dislocations is energetically favorable. There is a decrease in strain energy.



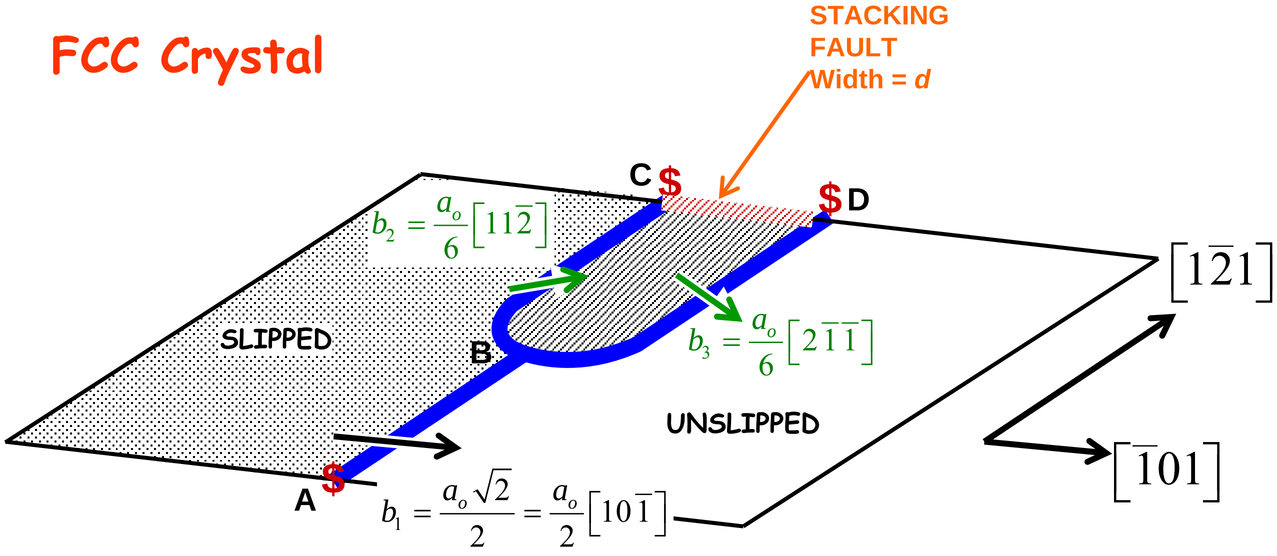


$$\frac{a_o}{2}[10\bar{1}] \rightarrow \frac{a_o}{6}[2\bar{1}\bar{1}] + \frac{a_o}{6}[11\bar{2}]$$

$$\frac{a_o^2}{2} > \frac{a_o^2}{6} + \frac{a_o^2}{6}$$

- Separation produces a *stacking fault* between the partials.

FCC Crystal



$$SFE = \gamma_{SFE} = \frac{Gb_2b_3}{2\pi d}$$

d = partial dislocation separation

G = shear modulus

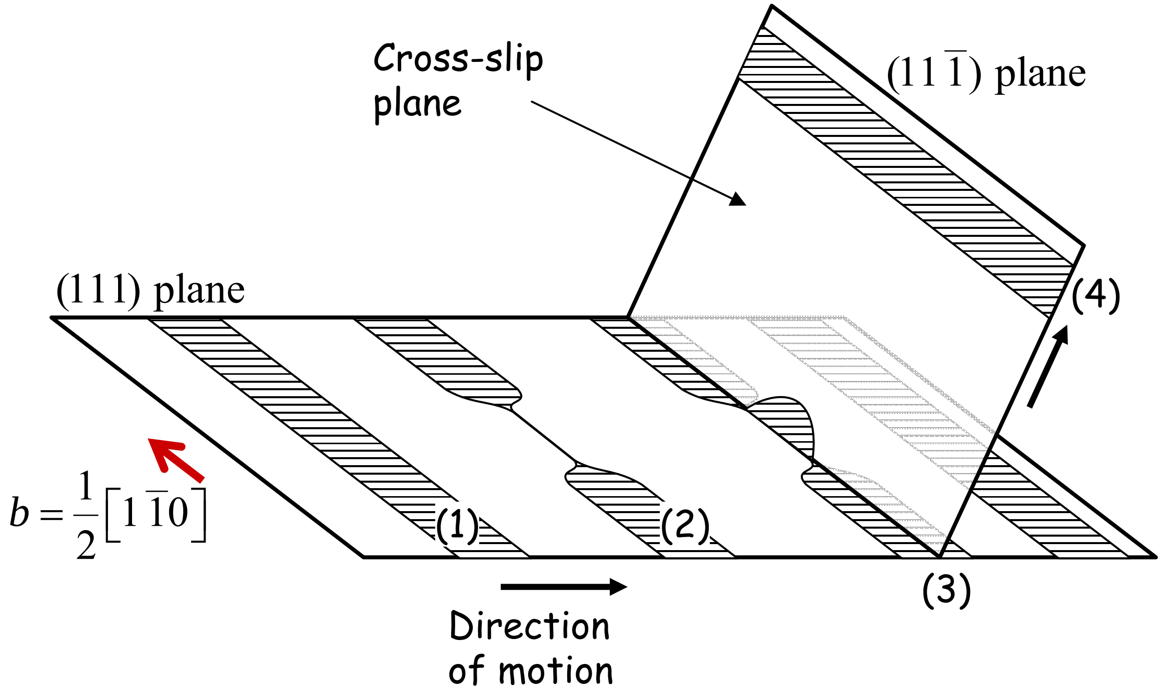
- AB represents a regular (un-extended) dislocation.
- **BC** and **BD** represent partial dislocations.
- The region between BC and BC represents the stacking fault. In this region, the crystal has undergone “intermediate” slip.
- BC + stacking fault + BD represents an *extended dislocation*.
- Extended dislocations (in particular screw dislocations) define a specific slip plane. Thus, *extended screw dislocations can only cross-slip when the partial dislocations recombine*. See the illustration on the next page.
- This process requires some energy.

Extended Dislocation

[Partial Dislocation + SF + Partial Dislocation]

- An extended screw dislocation must constrict before it can cross slip.

Schematic illustration



(1) Extended ⊥

(2) Formation of constricted segment

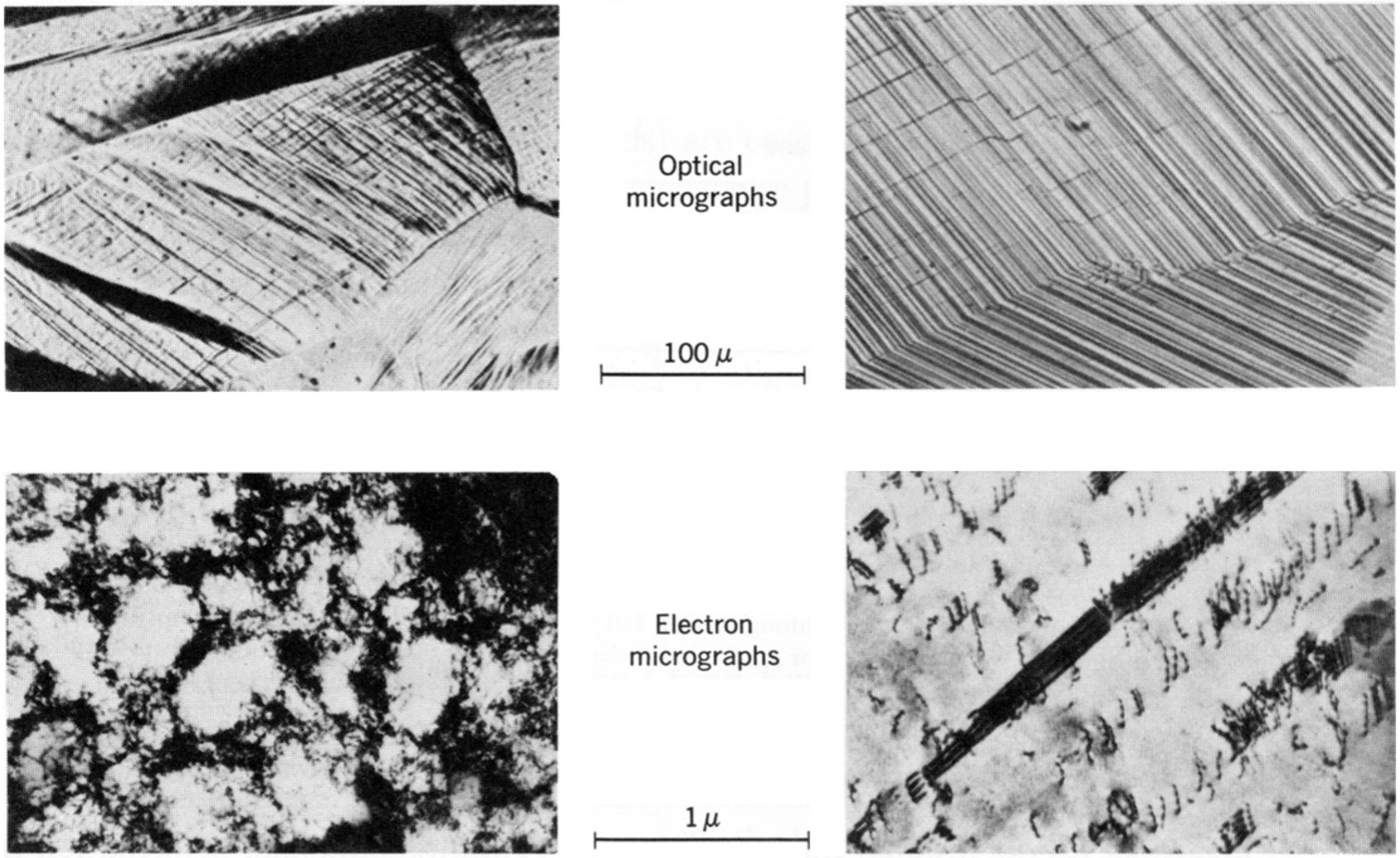
(3) Cross-slip of constricted segment and separation into extended ⊥

(4) Slip of extended ⊥ on cross-slip plane

Influence of partial dislocations and SFE on deformation

High SFE
higher strain hardening rate

Low SFE
lower strain hardening rate



Wavy slip lines

Straight slip lines

Dislocation cells and tangles

Planar dislocation arrangement

(a) Cu having a high SFE and therefore essentially un-dissociated dislocations. Cross-slip leads to the wavy slip bands and tangled dislocations

(b) Cu-7.5 wt.% Al alloy, having a low SFE and dissociated dislocations. The absence of cross-slip leads to straight slip traces and uniform dislocation lines.

Figure 9-11 Comparison of the deformation of two types of uniform polycrystalline materials in stage II deformation. From Guy, Introduction to Materials, McGraw-Hill, New York (1972) p. 419 [originally in Johnston and Feltner, *Metallurgical Transactions*, v. 1, n. 5 (1970) pp. 1161-1167].

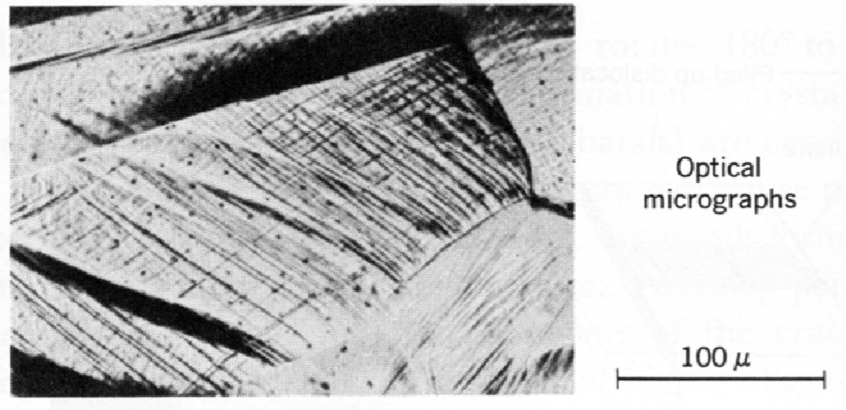
Effect of γ_{SFE} on Deformation Mechanism-1

(for FCC crystals)

$$SFE = \gamma_{SFE} = \frac{Gb_2b_3}{2\pi d}$$

High γ_{SFE}

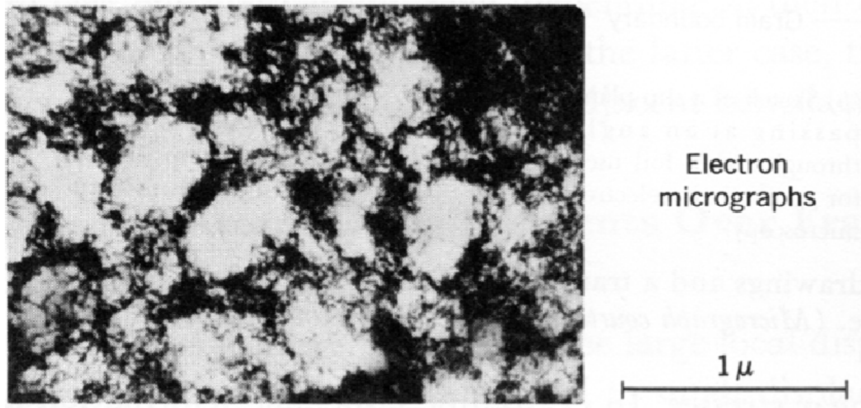
Wavy slip steps observed



Pure Cu

Few partial dislocations or small *d*. Thus, it is easier for cross-slip to occur.

Dislocation cells and tangles



Pure Cu

Random distribution of dislocations is observed because dislocations are not confined to specific slip planes. They can cross slip to overcome obstacles.

Figure 9-11 Comparison of the deformation of two types of uniform polycrystalline materials in stage II deformation. From Guy, Introduction to Materials, McGraw-Hill, New York (1972) p. 419 [originally in Johnston and Feltner, *Metallurgical Transactions*, v. 1, n. 5 (1970) pp. 1161-1167].

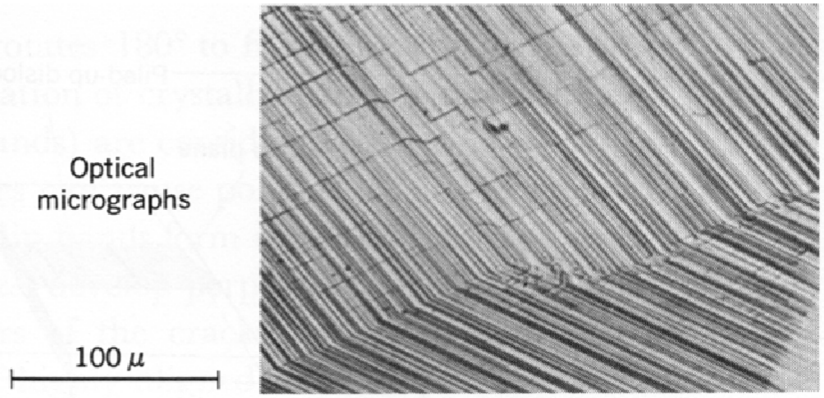
Effect of γ_{SFE} on Deformation Mechanism-2

(for FCC crystals)

$$SFE = \gamma_{SFE} = \frac{Gb_2b_3}{2\pi d}$$

Low γ_{SFE}

Straight slip steps observed

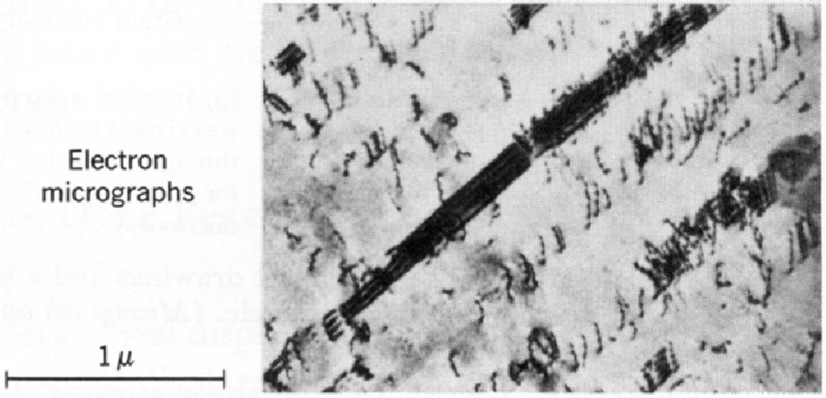


Cu – 7.5 wt.% Al

Slip is confined to specific planes. Leads to straight slip lines/steps on surface.

Lots of partials, large *d* makes cross slip difficult.

Dislocations seem to be confined to specific bands/planes



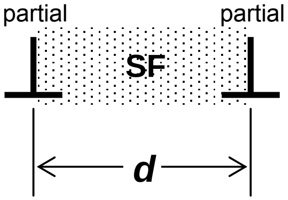
Cu – 7.5 wt.% Al

Dislocation movement is restricted due to because cross slip is inhibited.

RECALL: partials must recombine to cross slip.

[Images from Guy, Introduction to Materials, McGraw-Hill, New York (1972) p.419]

Partial to Partial Dislocation separation distance vs. γ_{SFE}



$$\gamma_{SFE} = \frac{Gb_p^2}{8\pi d}\left[\frac{2-\nu}{1-\nu}\left(1-\frac{2\nu\cos 2\theta}{2-\upsilon}\right)\right]$$

If θ = 0, v = 1/3

$$d \cong \frac{Gb_p^2}{5\pi\gamma_{SFE}} \quad or \quad \boxed{d \propto \frac{1}{\gamma_{SFE}}}$$

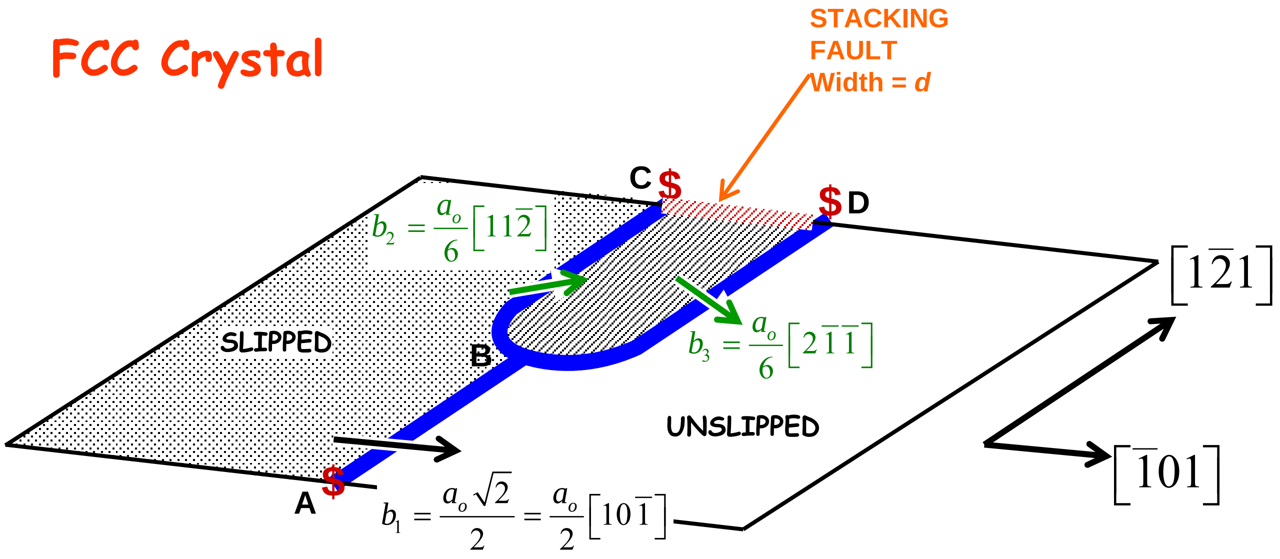
Table: Stacking-fault free energies and separation between Shockley partial dislocations for metals (θ = 30°).

Metal	**γ (mJ/m^3)**	**a_o (nm)**	***b* (nm)**	***G* (GPa)**	***d* (nm)**
Al	166	0.410	0.286	26.1	1.0
Cu	78	0.367	0.255	48.3	3.2
Au	45	0.408	0.288	27.0	4.0
Ni	128	0.352	0.249	76.0	2.9
Ag	22	0.409	0.289	30.3	9.0

Notice the big change in *d* with γ_{SFE}

Cross-slip easier when *d* is small γ_{SFE} is high

FCC Crystal



$$SFE = \gamma_{SFE} = \frac{Gb_2b_3}{2\pi d}$$

d = partial dislocation separation

G = shear modulus

- It is more difficult to re-combine wide stacking faults (i.e., those with large *d*).
- Cross-slip is more difficult in materials with low SFE. Thus high SFE materials will work harden more rapidly.

Material	**SFE (mJ/m^2)**	**Fault width**	**Strain Hardening rate**	**REASONS**
Stainless Steel	<10	~0.45	High	Cross slip is more difficult
Copper	~90	~0.30	Med	
Aluminum	~250	~0.15	Low	Cross slip is easier

See pages 74-79 in Hertzberg

Summary of hardening/strengthening mechanisms for crystalline solids

Hardening Mechanism	**Nature of Obstacle**	**Strong or Weak**	**Hardening Law**
Work hardening	Other dislocations	Strong	$\Delta\tau = \alpha G b \sqrt{\rho}$ (see [1])
Grain size / Hall-Petch	Grain boundaries	Strong	$\Delta\tau = k'_y \big/ \sqrt{d}$ (see [2])
Solid solution	Solute atoms	Weak (see [3])	$\Delta\tau = G\varepsilon_s^{3/2} c^{1/2} / 700$ (see [4])
Deforming particles	Small, coherent particles	Weak (see [5])	$\Delta\tau = CG\varepsilon^{3/2}\sqrt{\frac{fr}{b}}$ (see [6])
Non-deforming particles	Large, incoherent particles	Strong (see [7])	$\Delta\tau = \frac{Gb}{(L-2r)}$

[1] α equals about 0.2 for FCC metals and about 0.4 for BCC metals.

[2] k'_y scales with inherent flow stress and/or shear modulus; therefore k'_y is generally greater for BCC metals than for FCC metals.

[3] Exception to weak hardening occurs for interstitials in BCC metals; the shear distortion interacts with screw dislocations leading to strong hardening.

[4] Equation apropos to substitutional atoms; parameter ε_s is empirical, reflecting a combination of size and modulus hardening.

[5] Coherent particles can be "strong" in optimally aged materials.

[6] Constant C depends on specific mechanism of hardening; parameter ε relates to hardening mechanism(s). Equation shown applies to early stage precipitation. Late stage precipitation results in saturation hardening.

[7] Highly overaged alloys can represent "weak" hardening.

SYMBOLS : $G =$ shear modulus; $b =$ Burgers vector; $\rho =$ dislocation density ; $d =$ grain size; $c =$ solute atom concentration (at.%); $f =$ precipitate volume fraction; $r =$ precipitate radius; $L =$ spacing between precipitates on slip plane.

Table adapted from Courtney, *Mechanical Behavior of Materials*, 2nd edition, p. 232.