Analytical Methods for Materials

Lesson 9
Stereographic Projections

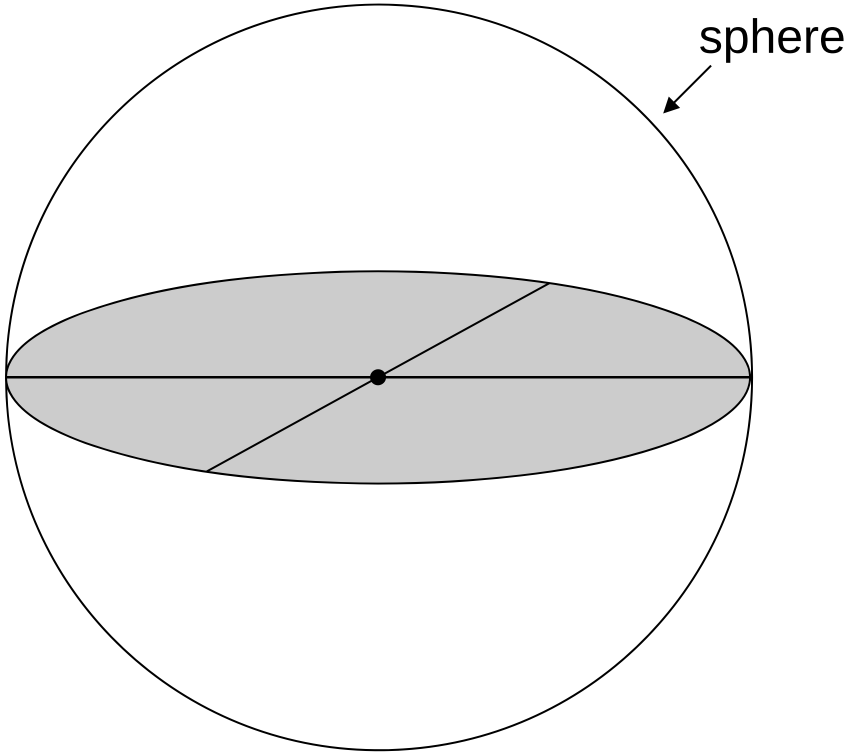
Suggested Reading

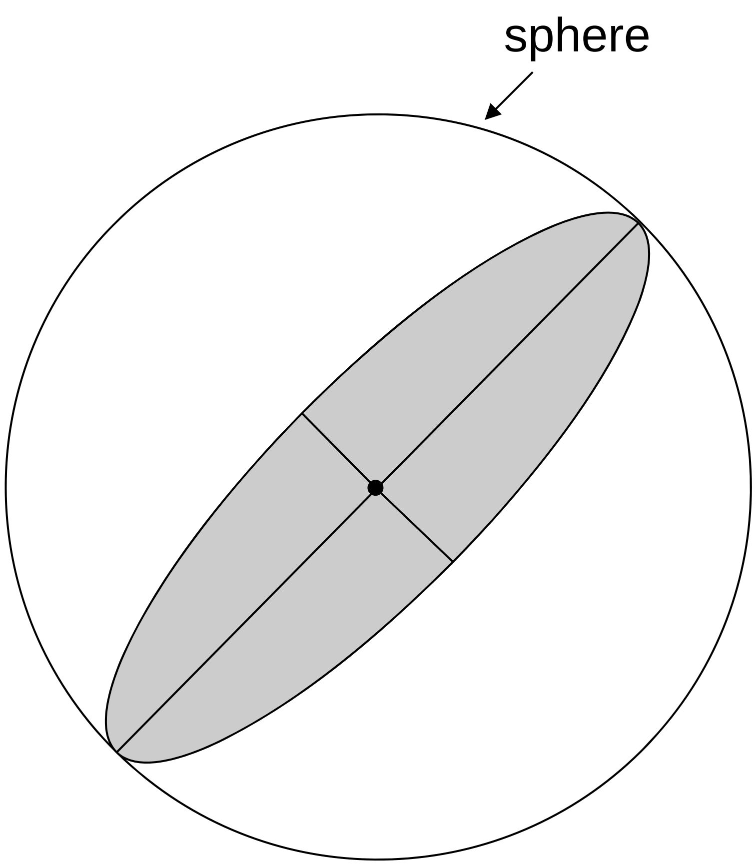
- Chapters 2 and 6 in Waseda
- Sections 1.13-2/29 in Abbaschian, Abbaschian, and Reed-Hill

Stereographic Projections

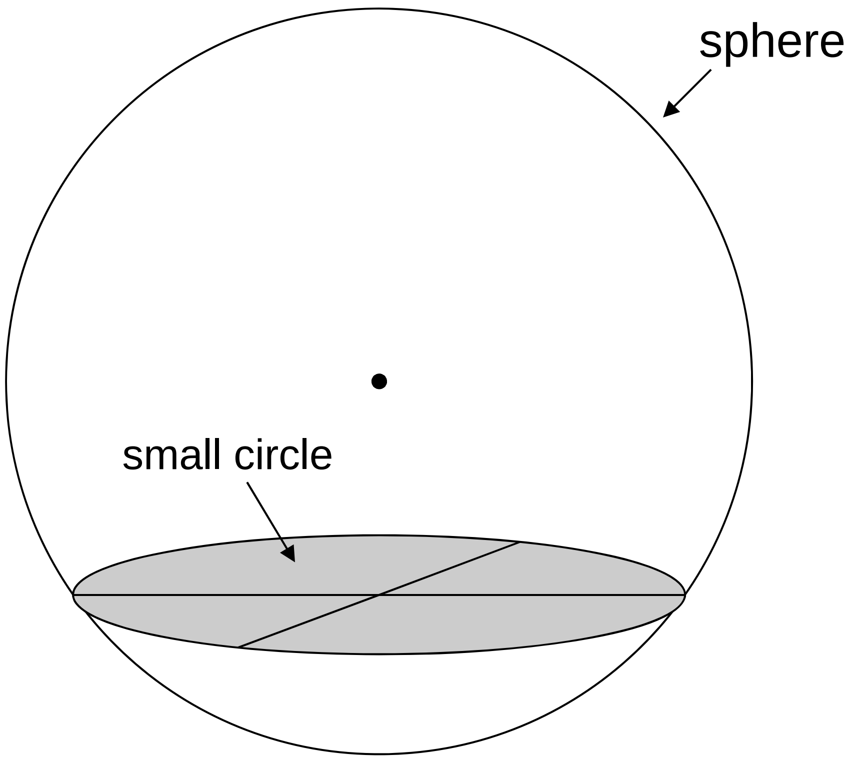
- Used to present 3-D orientation relationships on a 2-D figure.

- Easy to visualize crystallographic features
  - Slip planes and directions
  - Crystal planes and orientation relationships in electron microscopy
  - Crystal symmetry
  - Grain orientations (i.e., texture in polycrystals)
  - Etc…

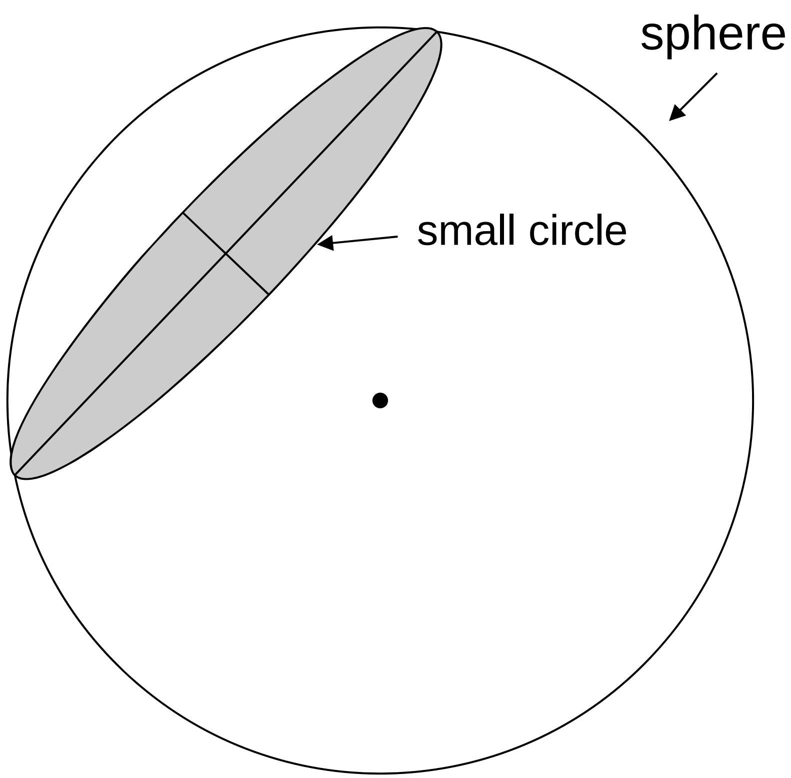




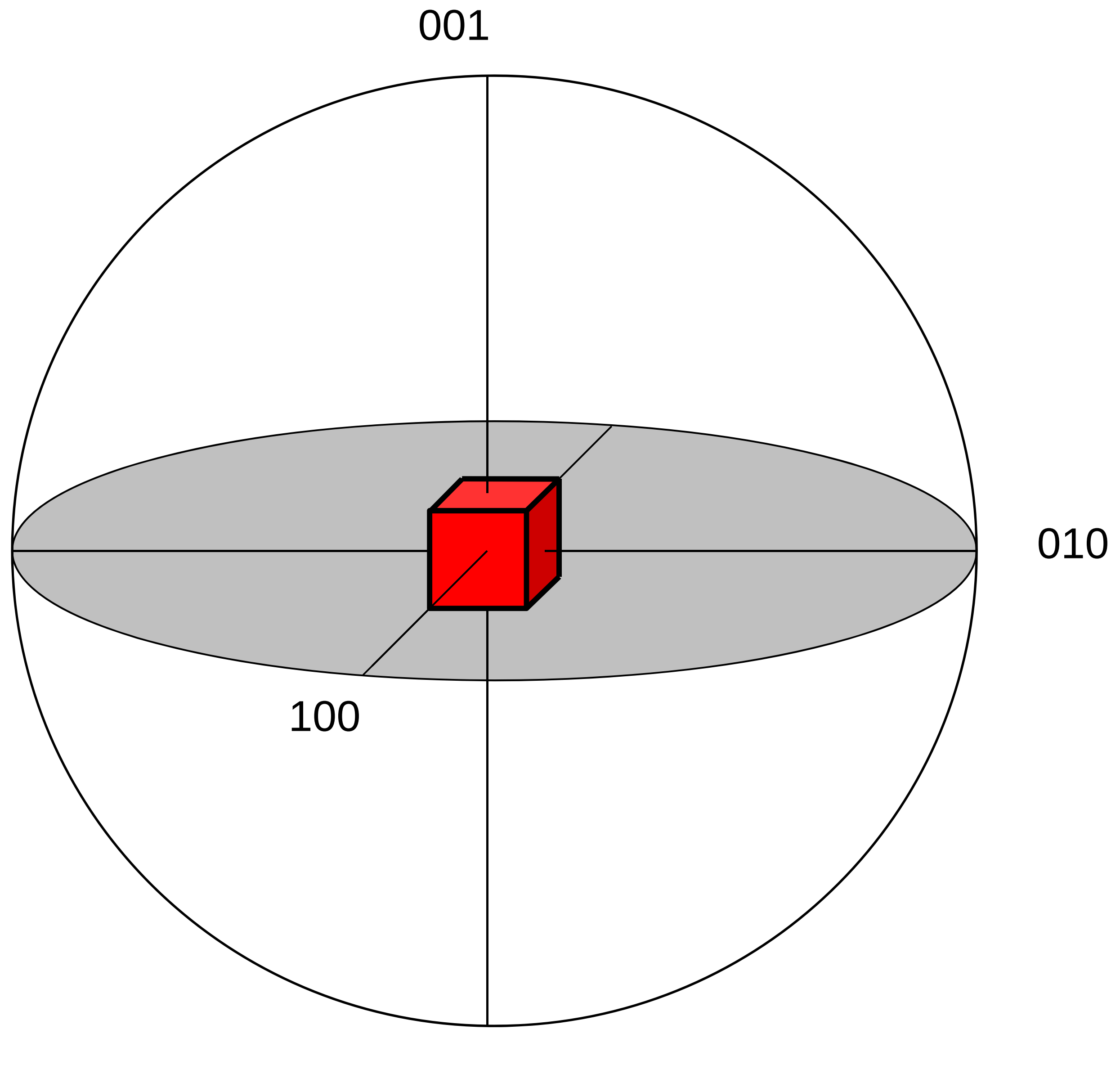
Great circles:
diameter equal to that of sphere



Small circles:
diameter less than that of sphere

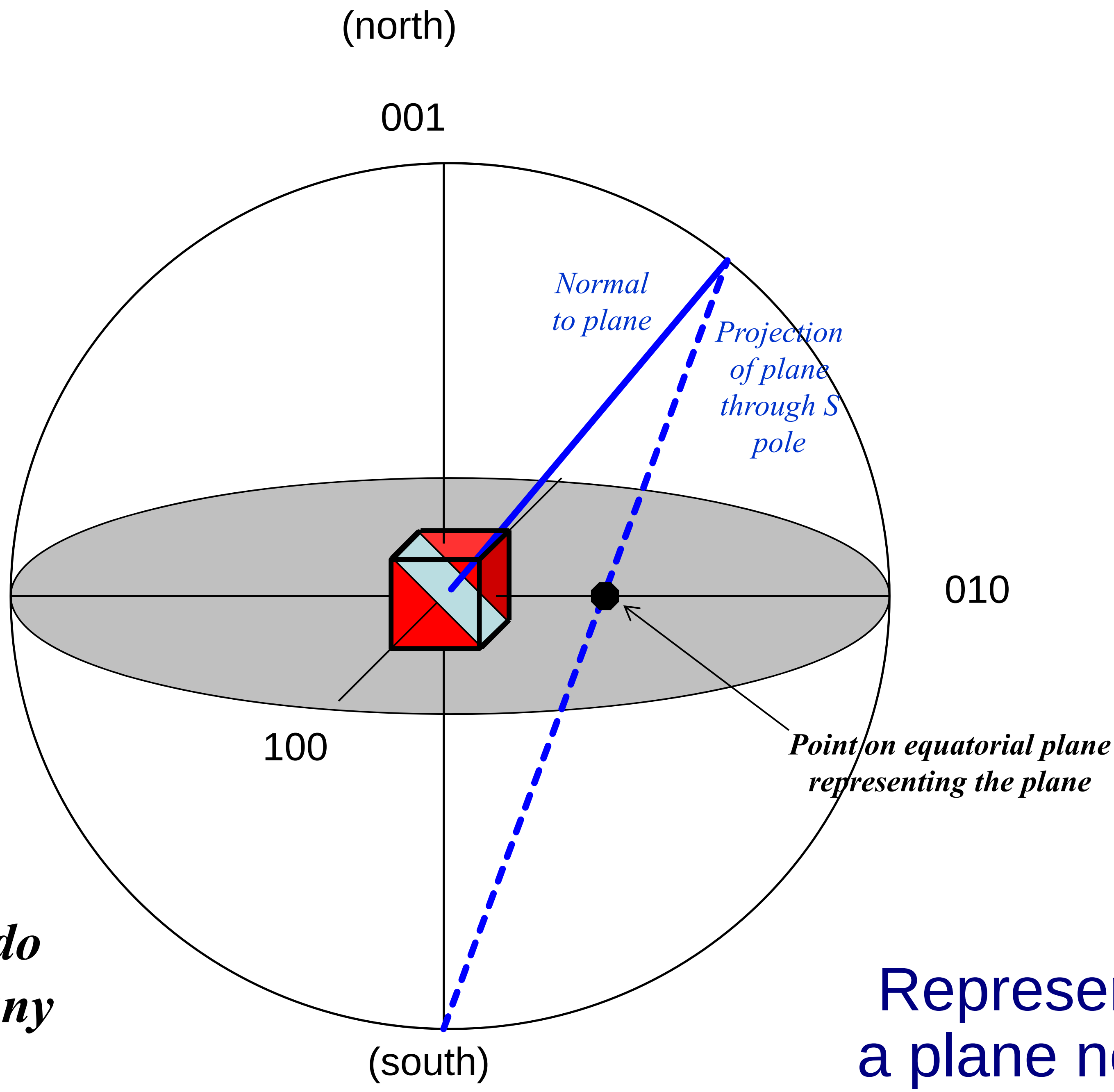


To represent angles and planes

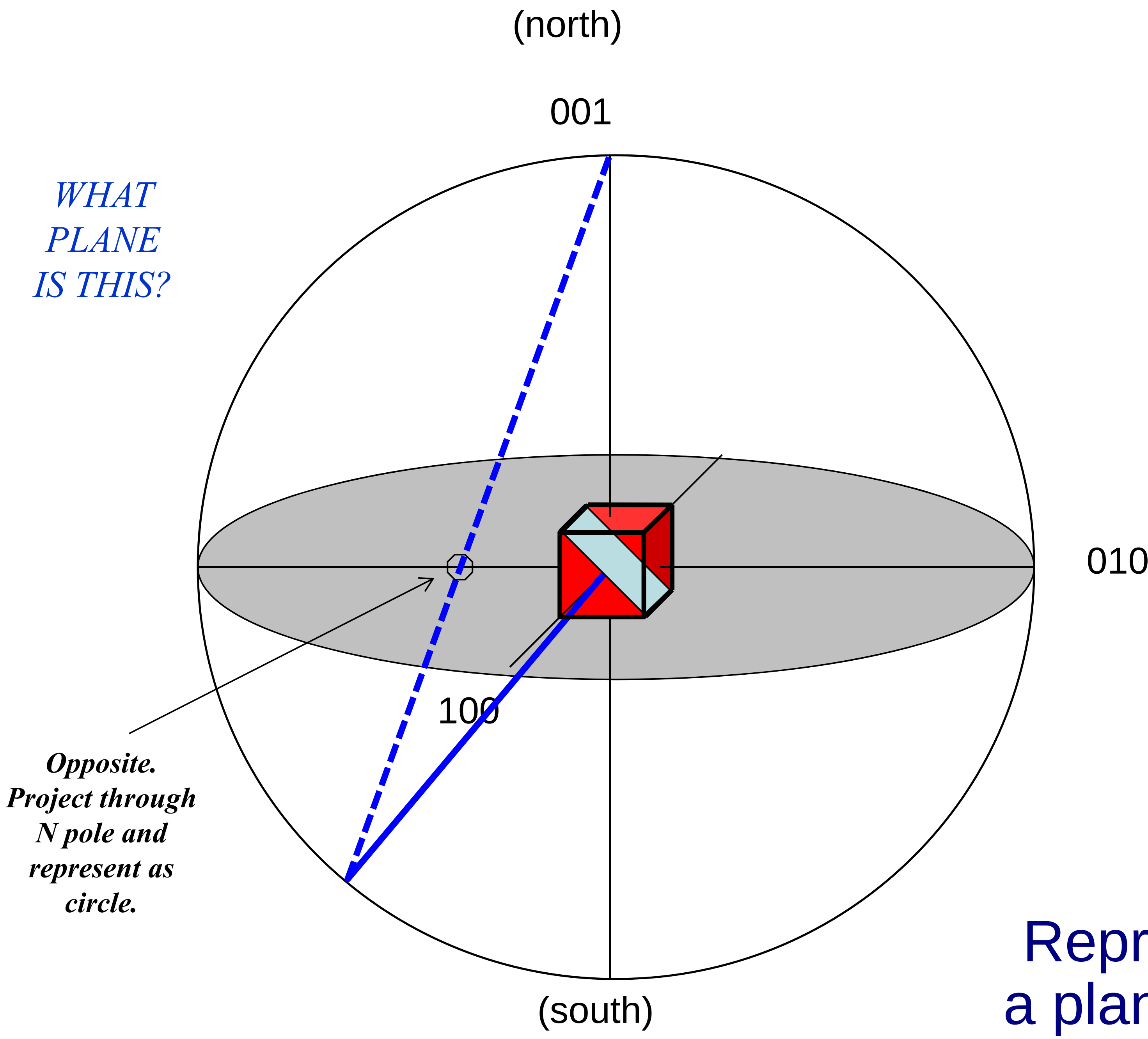


Representing a plane normal

WHAT PLANE IS THIS?

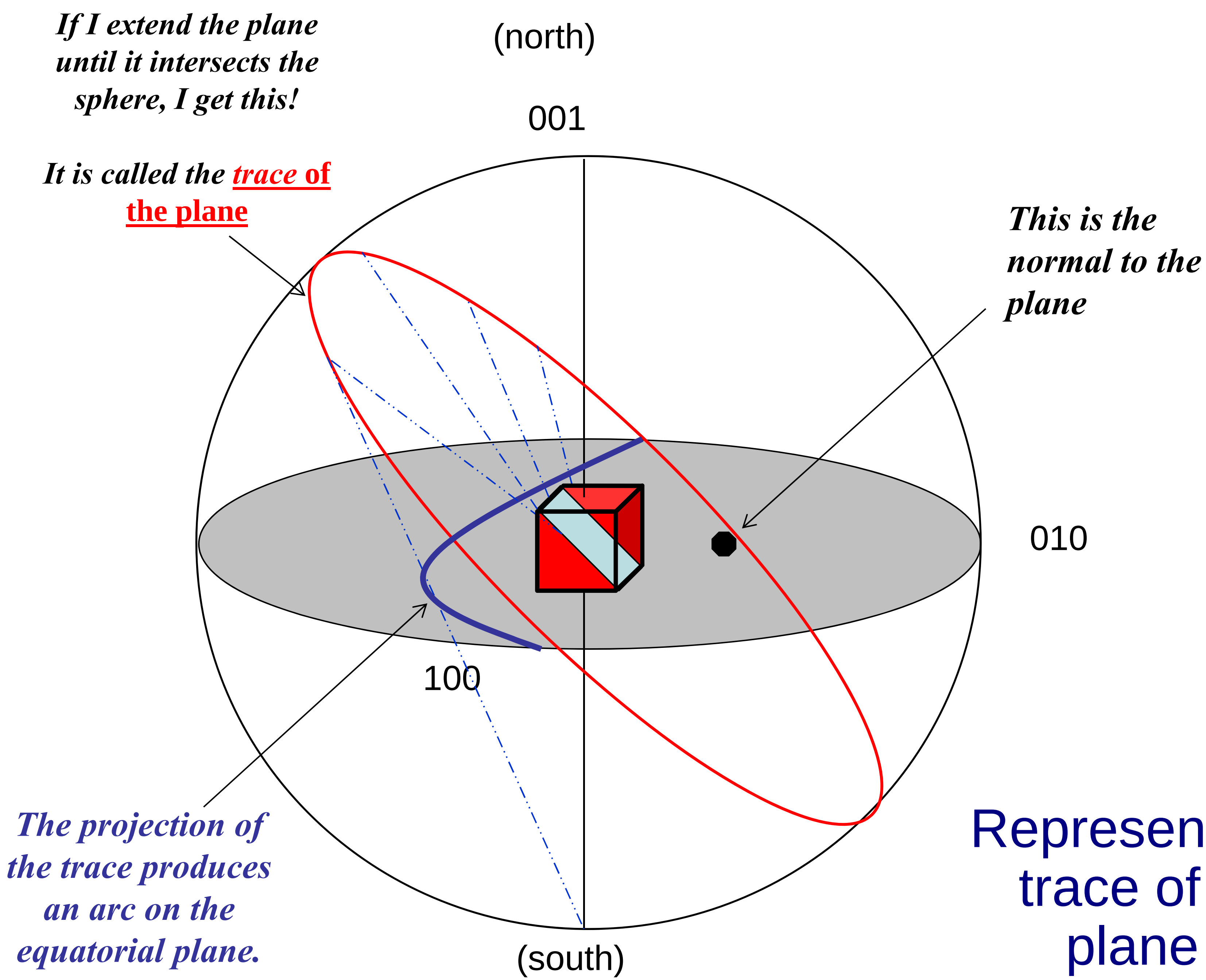


We can do this for any plane!



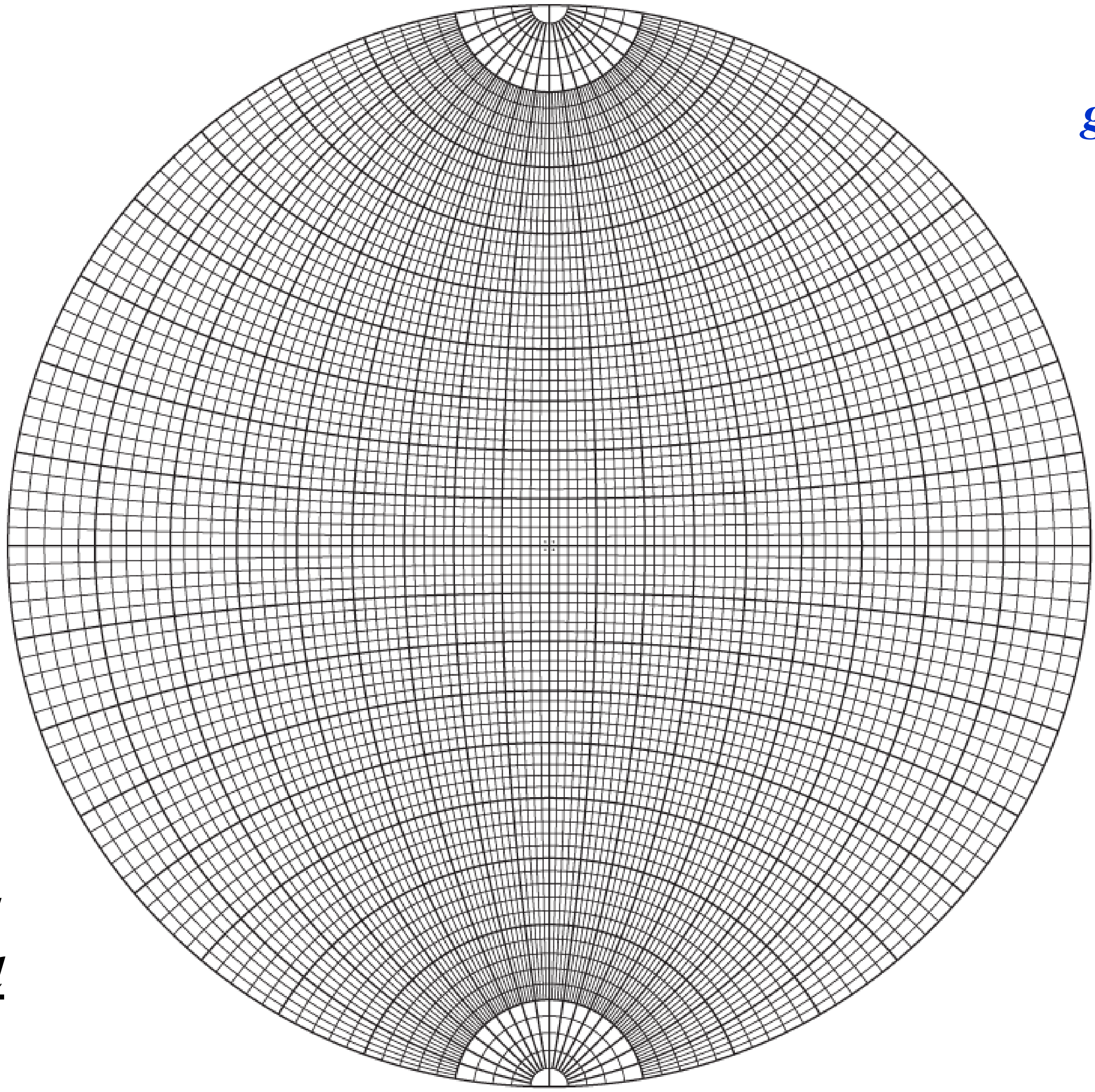
Representing a plane normal

Representing trace of a plane



Wulff Net

All vertical arcs and the equator are <u>great circles</u>.

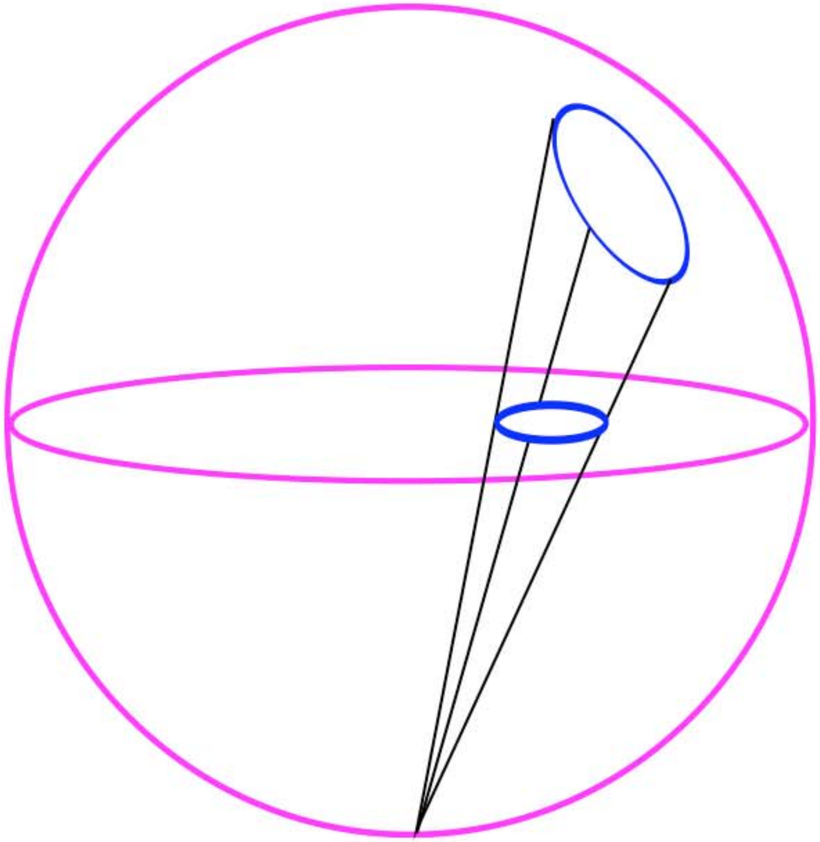


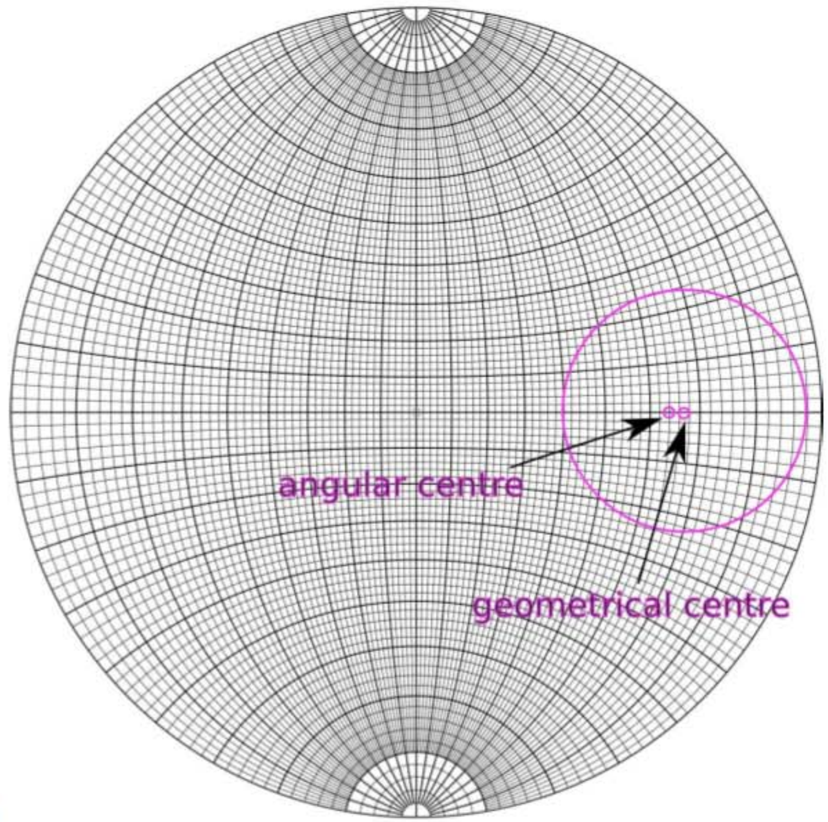
We can use great circles to measure angles

All horizontal arcs are <u>small circles</u>.

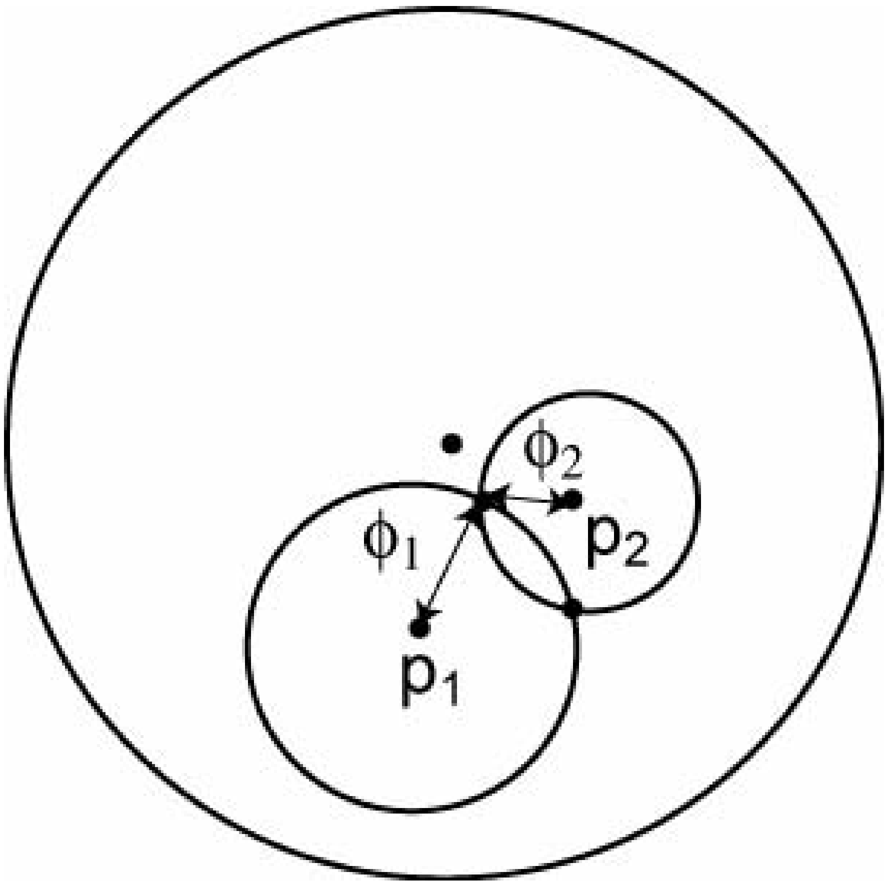
When small circles are projected onto the stereographic sphere, they project as circles.

However, the angular center is shifted due to angular distortion.





Using small circles



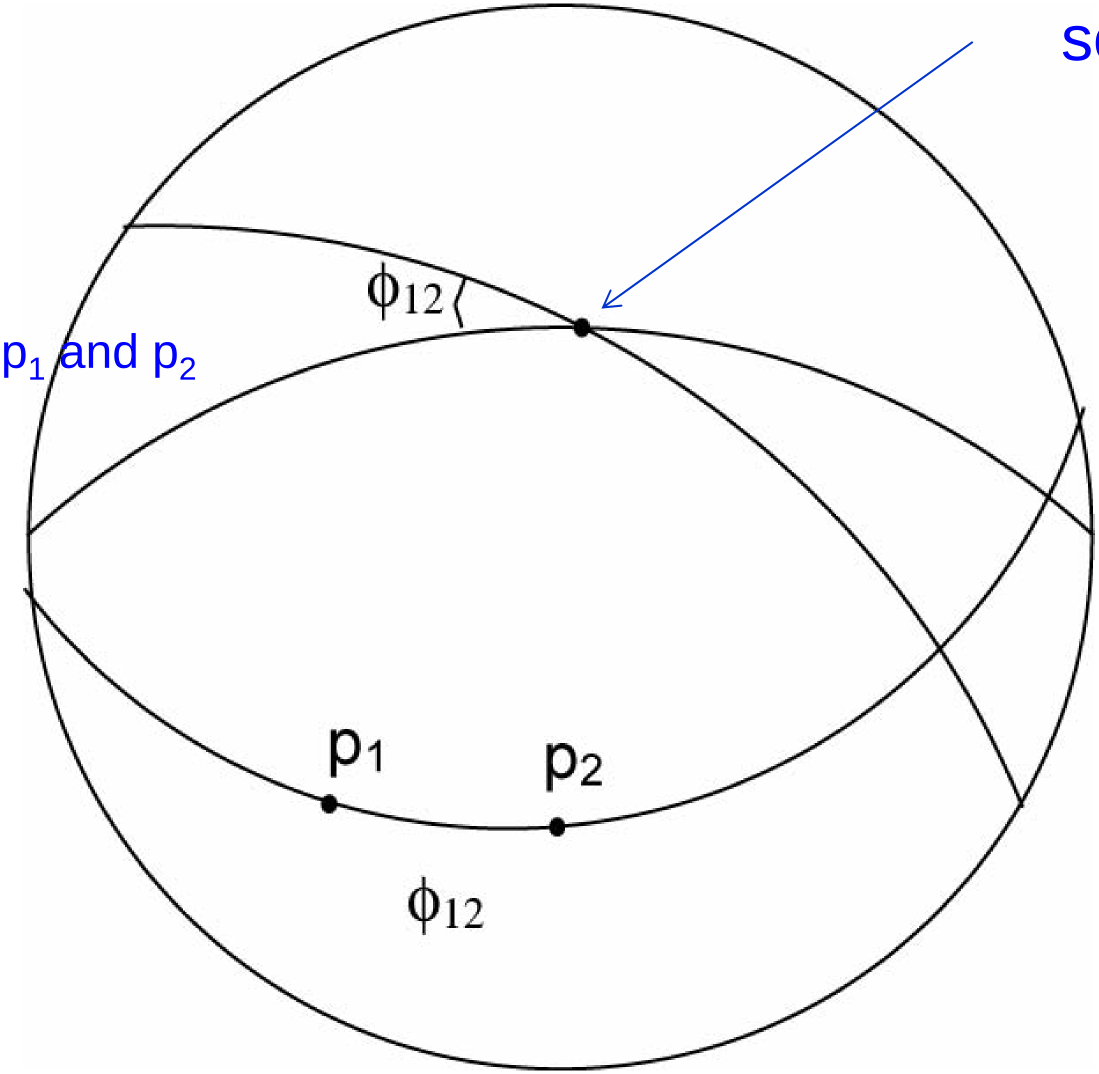
- To locate a pole at angles ϕ_1 from p_1 and ϕ_2 from p_2 draw the two small circles with angular centers on the two poles.

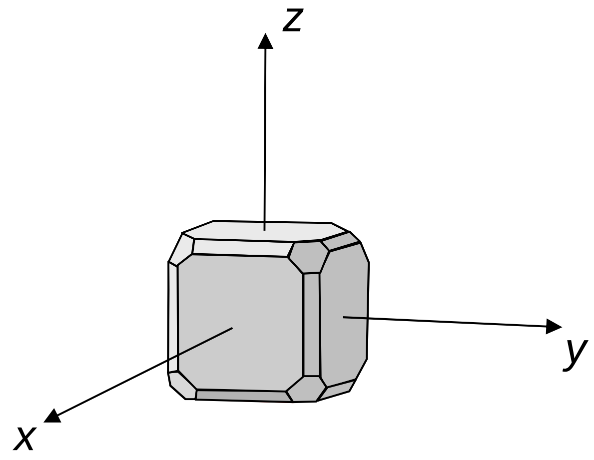
- 2 solutions at intersections of small circles.

Angle between two planes

Intersection is one of solutions from last slide.

Traces of p_1 and p_2




z
y
x

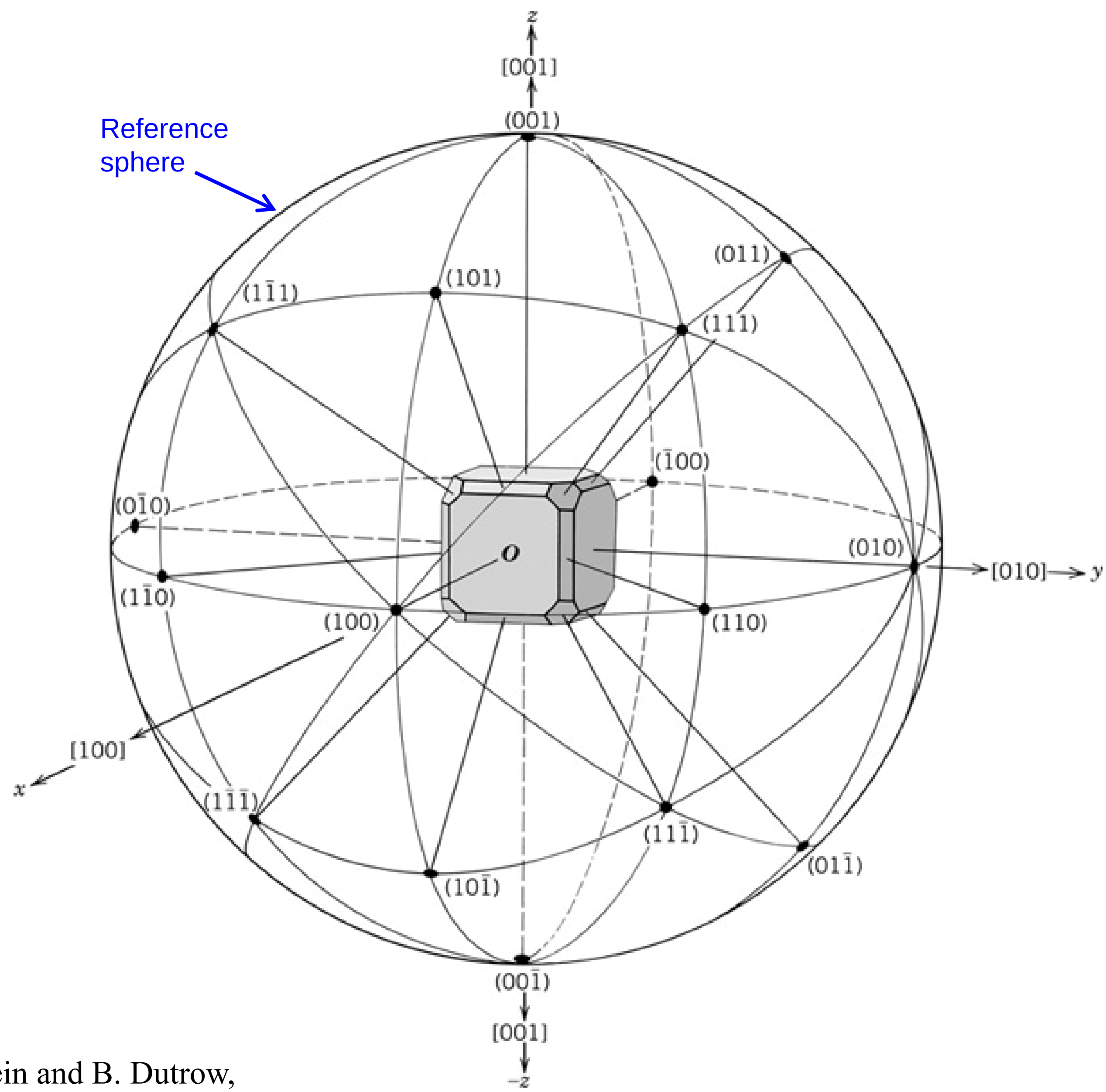


Fig. 8.1 from C. Klein and B. Dutrow,
Manual of Mineral Science, 23rd Edition,
John Wiley and Sons, New York (2002)

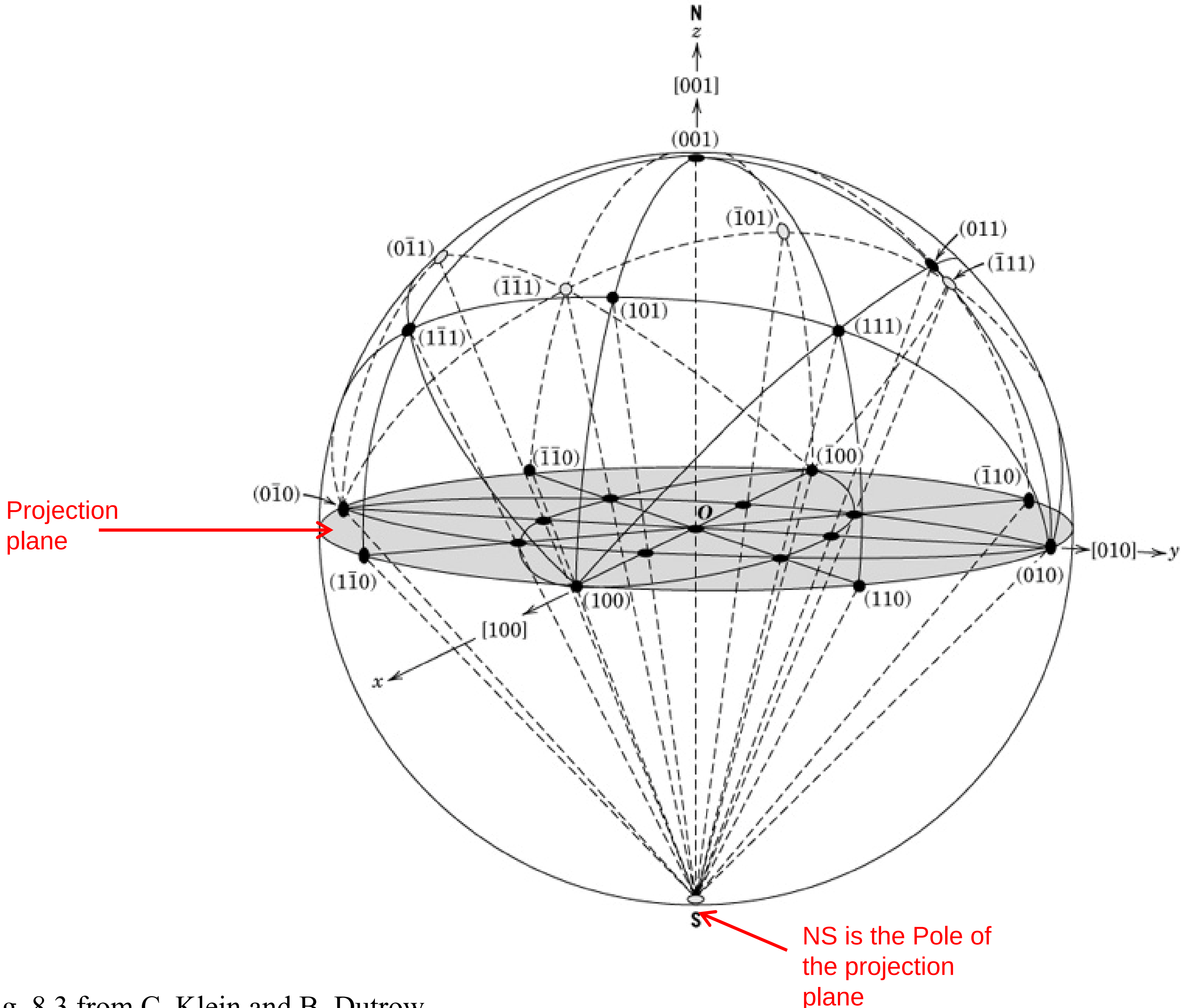


Fig. 8.3 from C. Klein and B. Dutrow, Manual of Mineral Science, 23rd Edition, John Wiley and Sons, New York (2002)

2-D map of 3-D crystallographic planes and directions

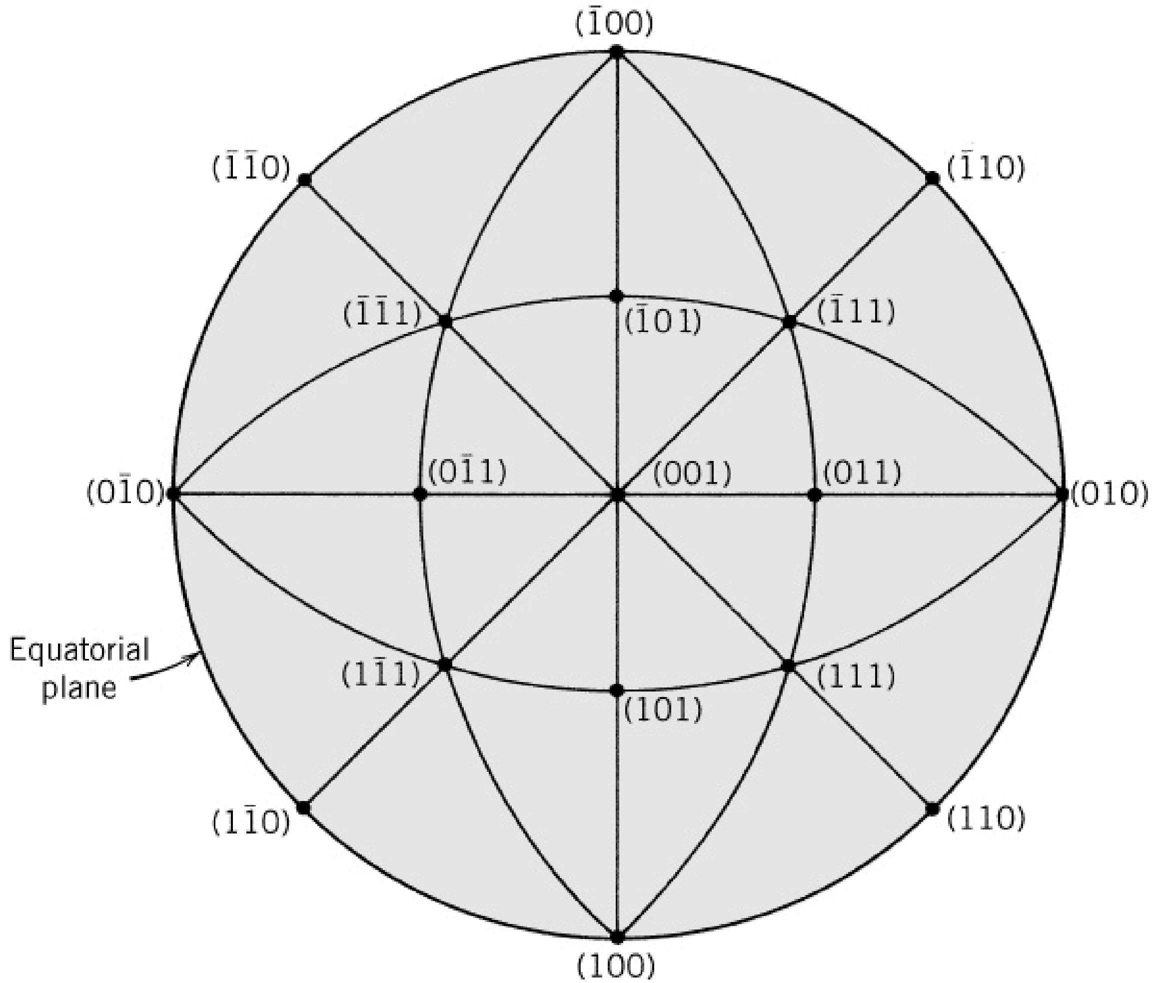
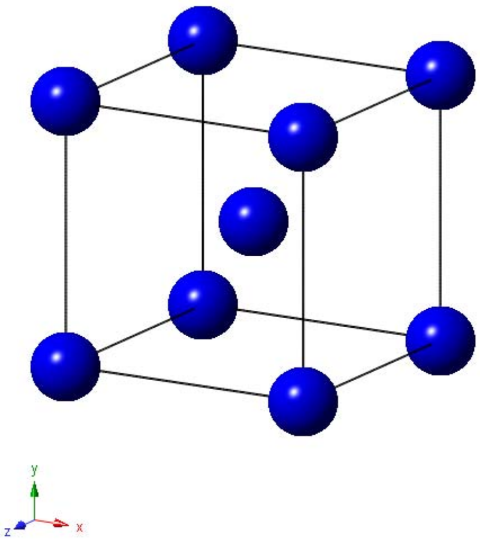
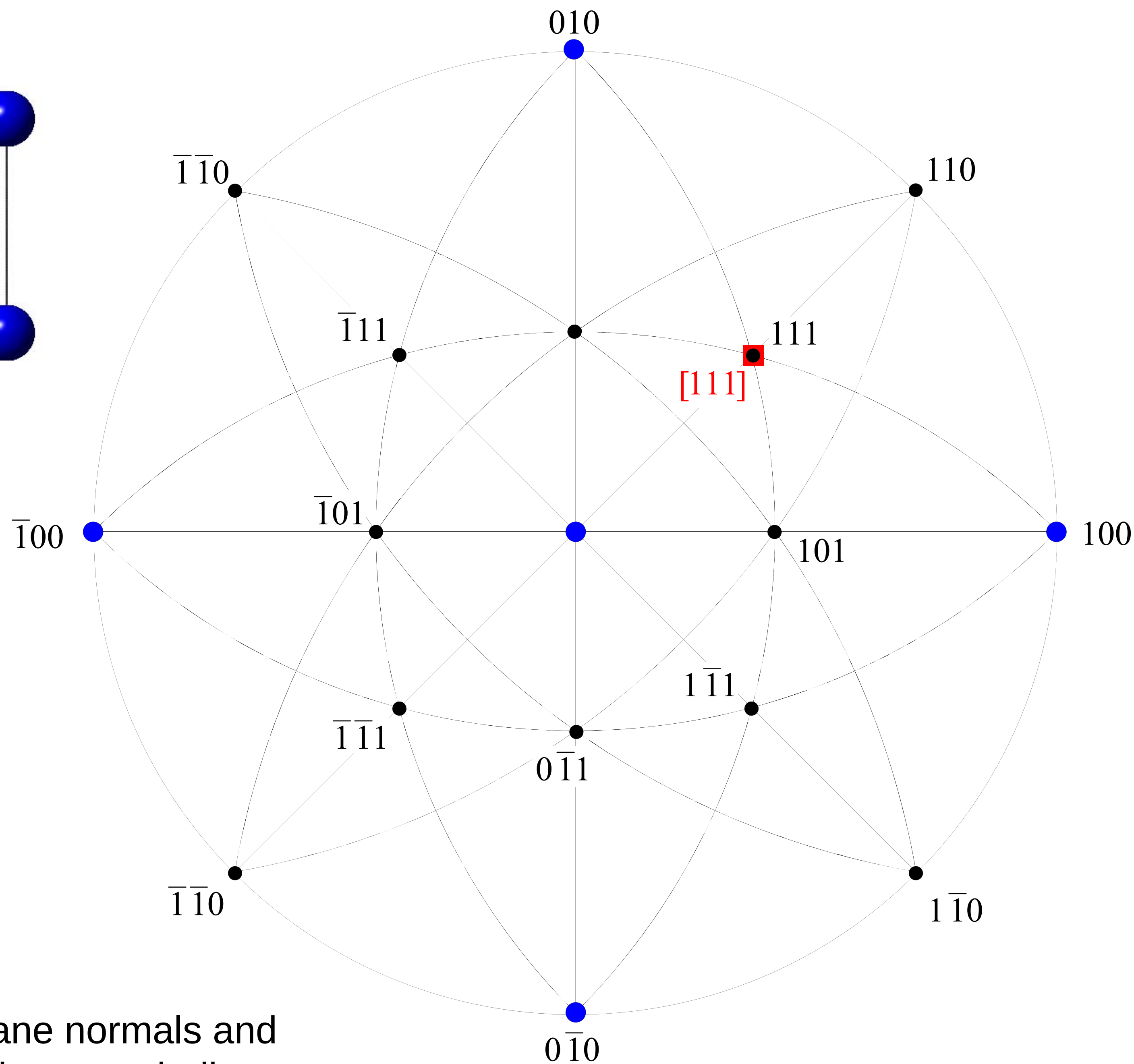


Fig. 8.6 from C. Klein and B. Dutrow, Manual of Mineral Science, 23rd Edition, John Wiley and Sons, New York (2002)





Tungsten

Body Centered

Cubic

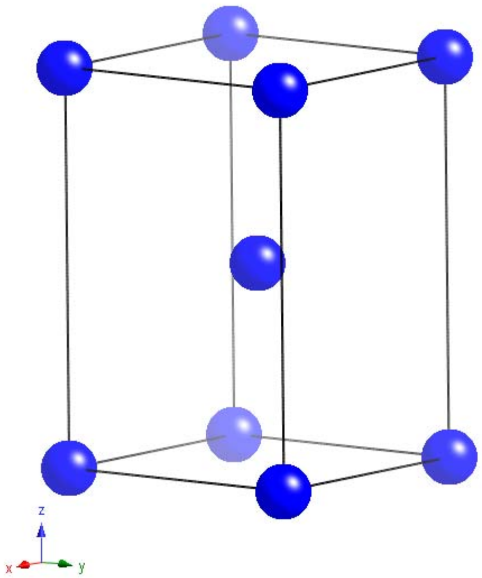
$a = 3.165$ Å

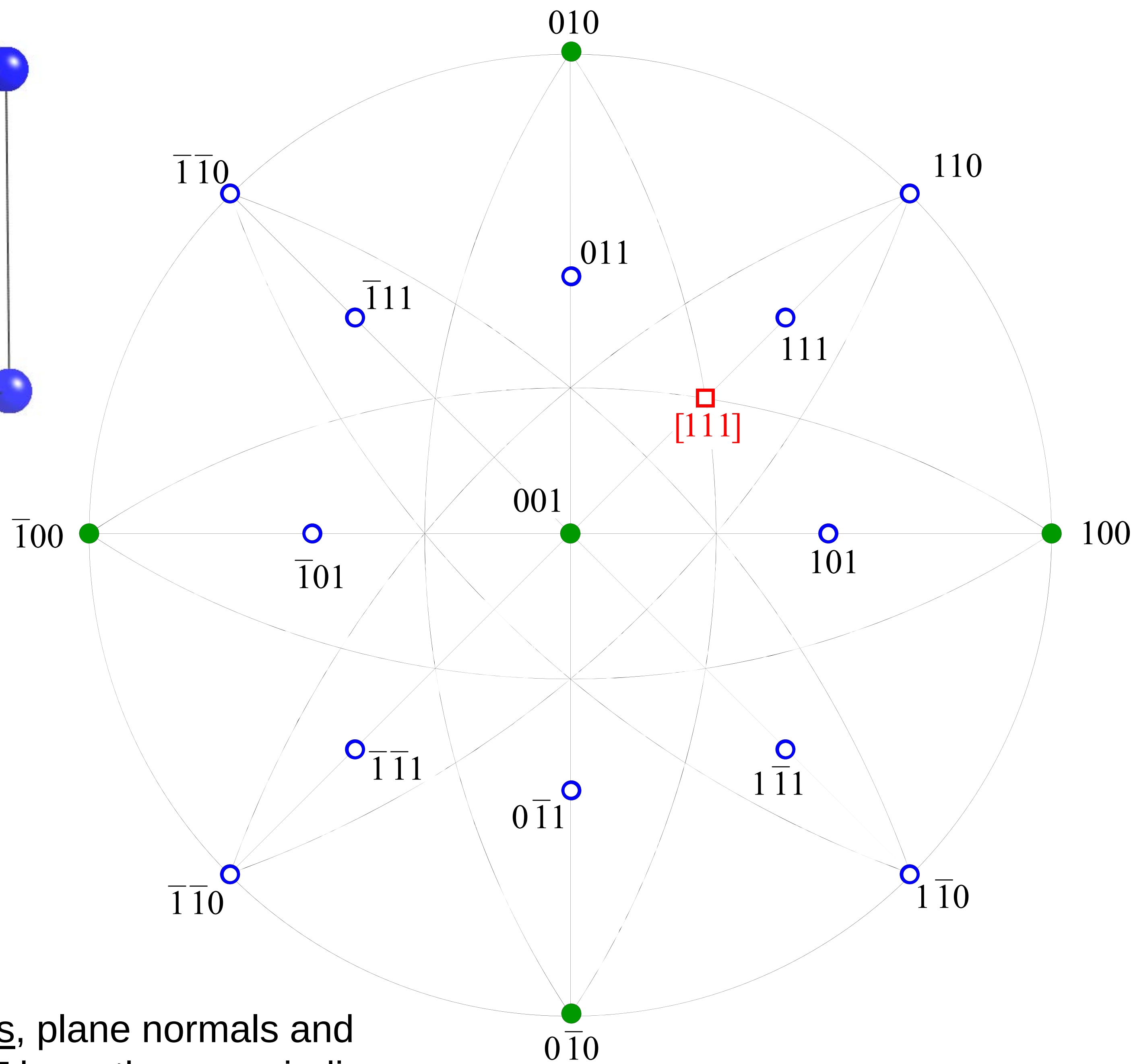
$b = 3.165$Å

$c = 3.165$ Å

$\alpha = \beta = \gamma = 90°$

In cubic systems, plane normals and directions will have the same indices.

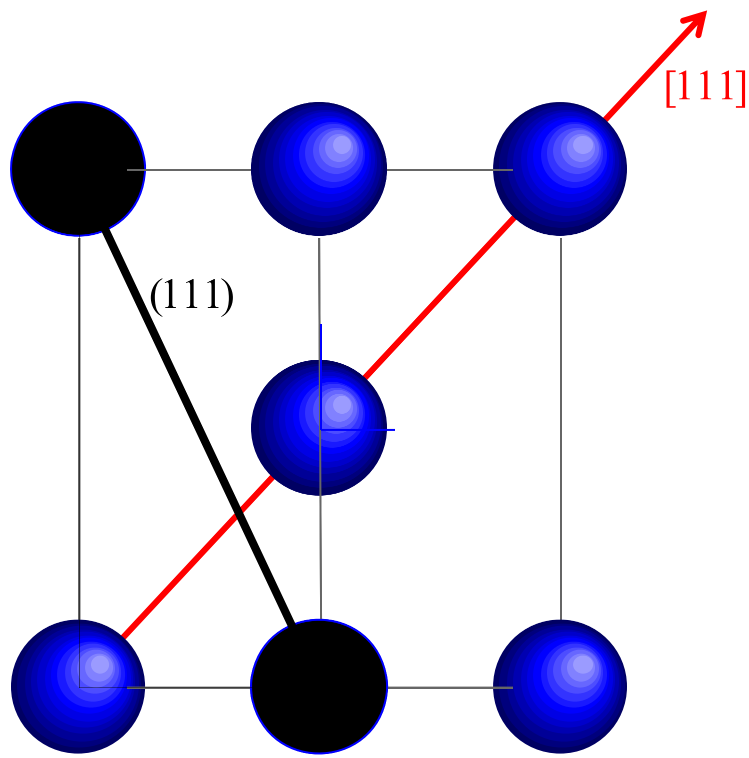




Indium

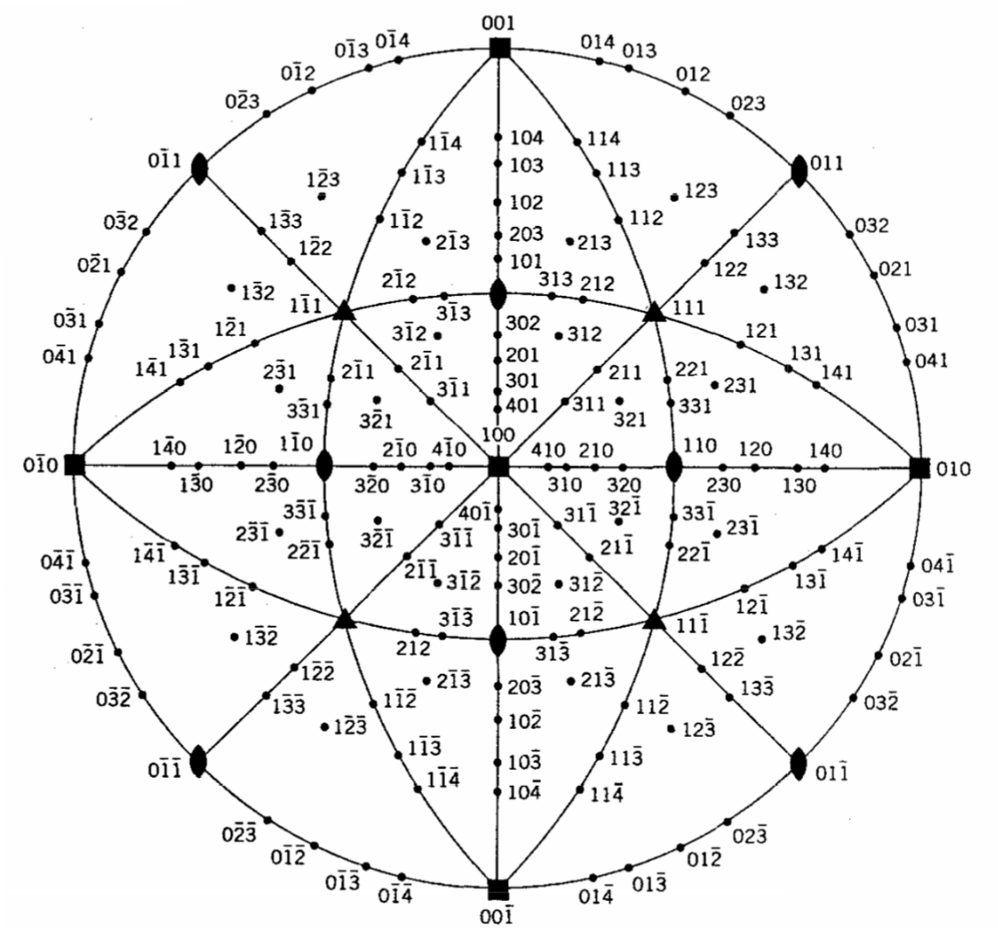
Body Centered
<u>Tetragonal</u>
a = 3.2512 Å
b = 3.2512 Å
c = 4.9467 Å
$\alpha = \beta = \gamma = 90°$

In <u>non-cubic</u> <u>systems</u>, plane normals and directions <u>WILL NOT</u> have the same indices.



They are not perpendicular to each other

Standard Projection for Cubic Crystal



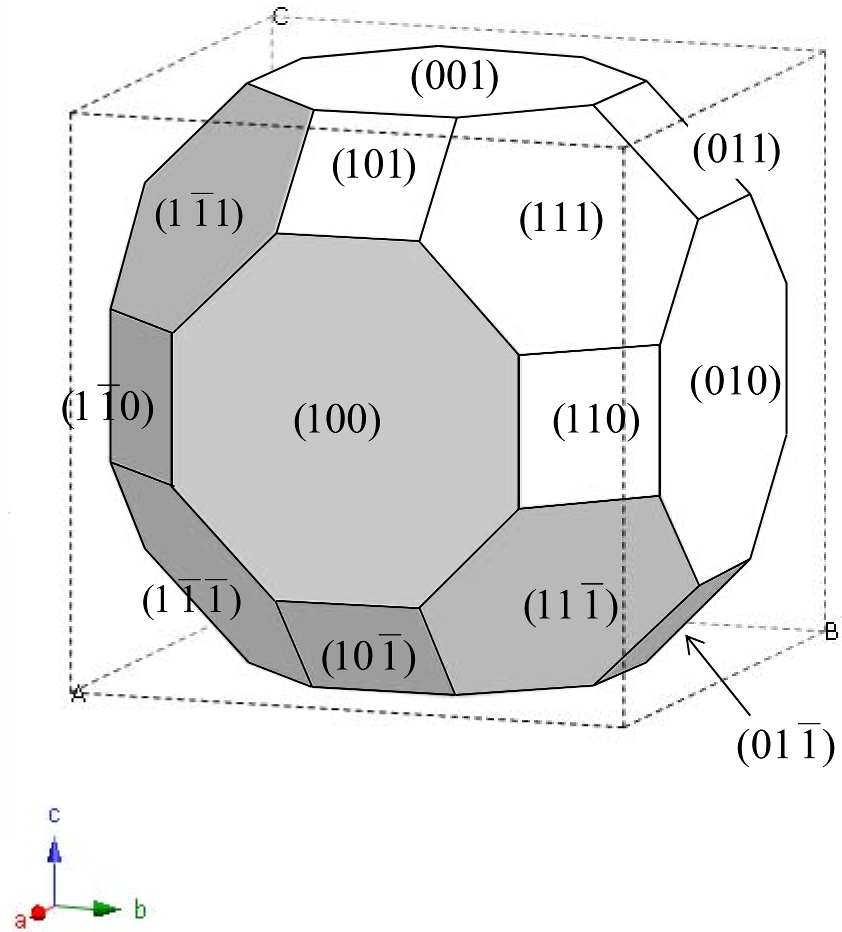


Fig. 3.53 from S.M. Allen and E.L. Thomas, *The Structure of Materials*, John Wiley & Sons, New York, 1999

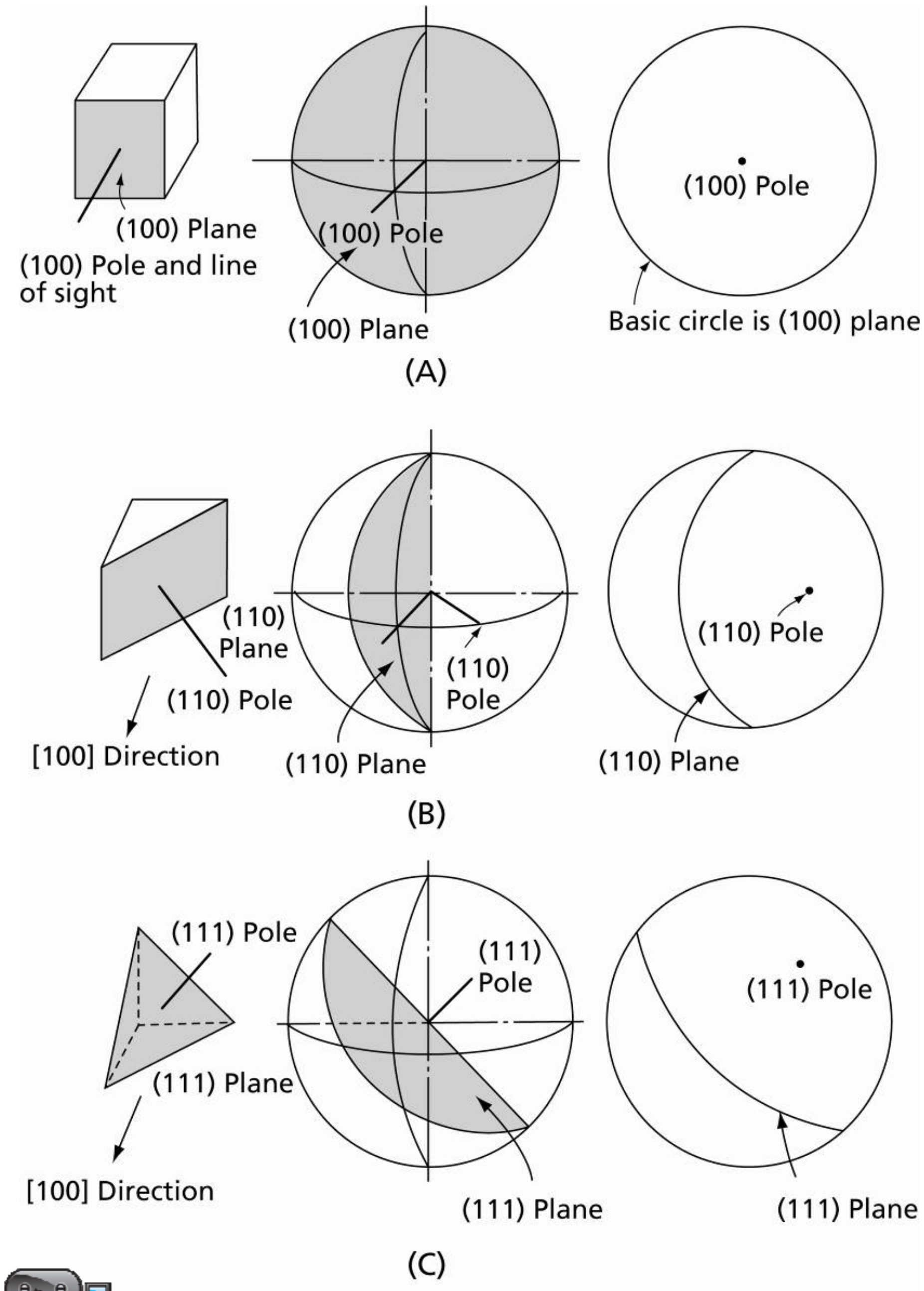


Fig. 1.20 Stereographic projections of several important planes of a cubic material.

(A) the (100) plane and line of sight along the [100] direction.

(B) The (110) plane and line of sight along the [100] direction.

(C) The (111) plane and line of sight along the [100] direction.

R. Abbaschian, L. Abbaschian, and R.E. Reed-Hill, Physical Metallurgy Principles, 4th Edition (Cengage learning, Stamford, CT, 2009), pp. 16.

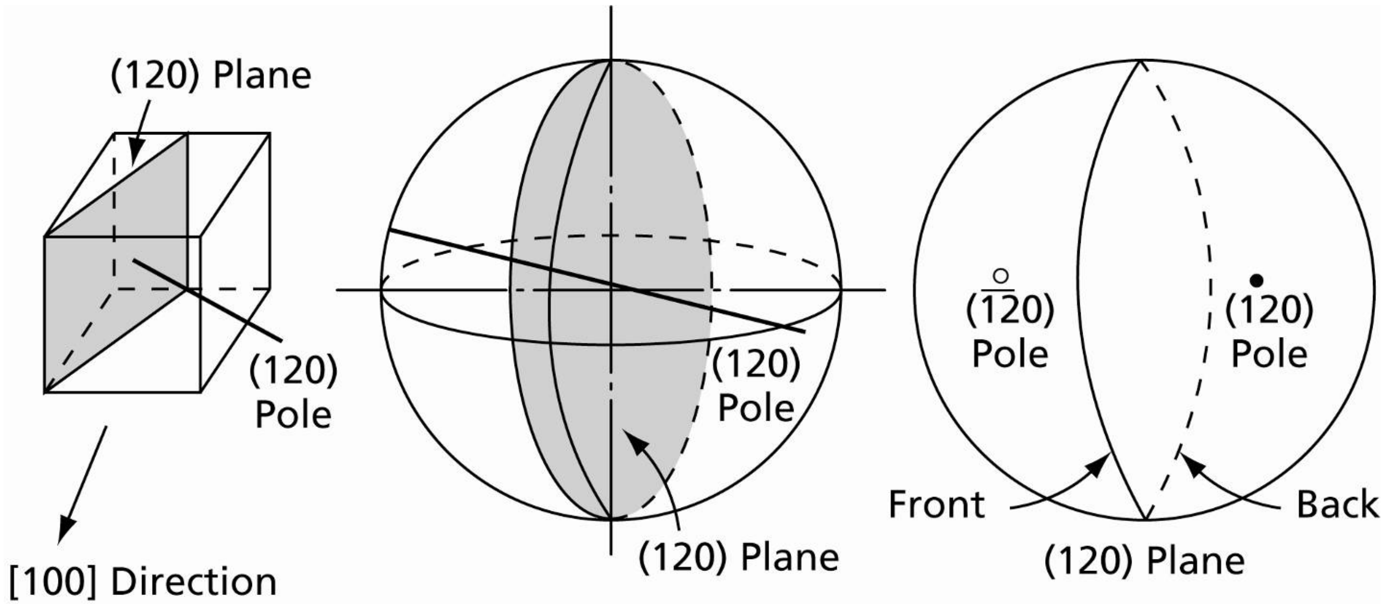


Fig. 1.21 Cubic system, the (120) plane showing the stereograpic projections from both hemispheres and line of sight along the [100] direction.

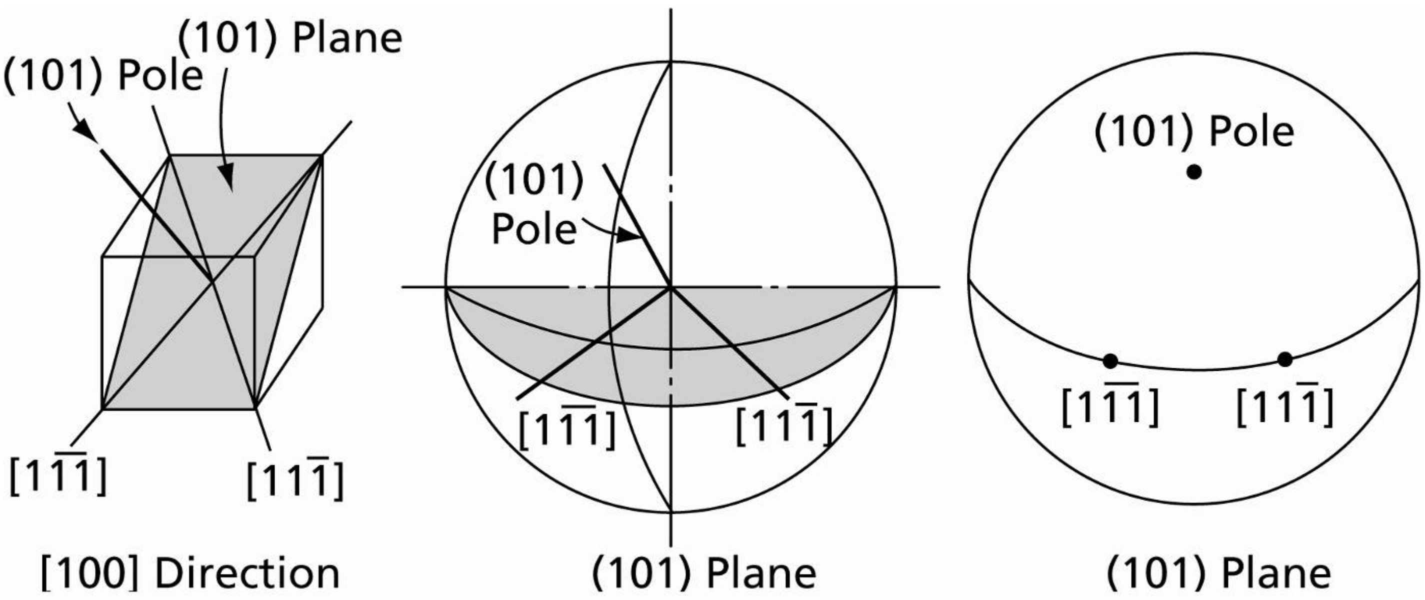


Fig. 1.22 Cubic system, the (101) plane and two <111> directions lying on that plane. Line of sight along the [100] direction.

R. Abbaschian, L. Abbaschian, and R.E. Reed-Hill, Physical Metallurgy Principles, 4th Edition (Cengage learning, Stamford, CT, 2009), pp. 17.

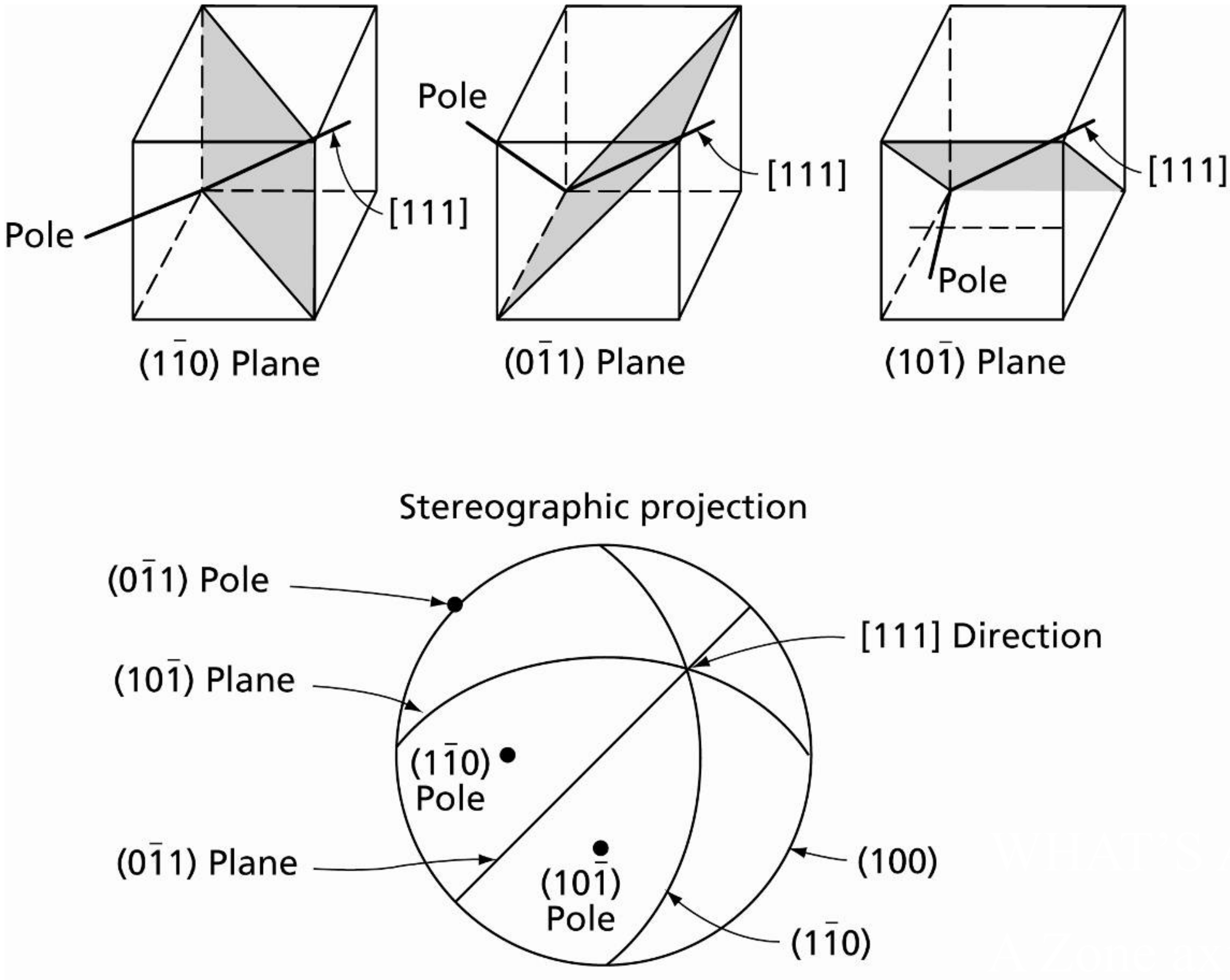


Fig. 1.23 Cubic system, zone of planes, the zone axis of which is the [111] direction. The three {110} planes that belong to this zone are illustrated in the figures.

R. Abbaschian, L. Abbaschian, and R.E. Reed-Hill, Physical Metallurgy Principles, 4th Edition (Cengage learning, Stamford, CT, 2009), pp. 18.

HW: Mathematically, prove that [111] is the zone axis for the 12 planes drawn on the figure below.

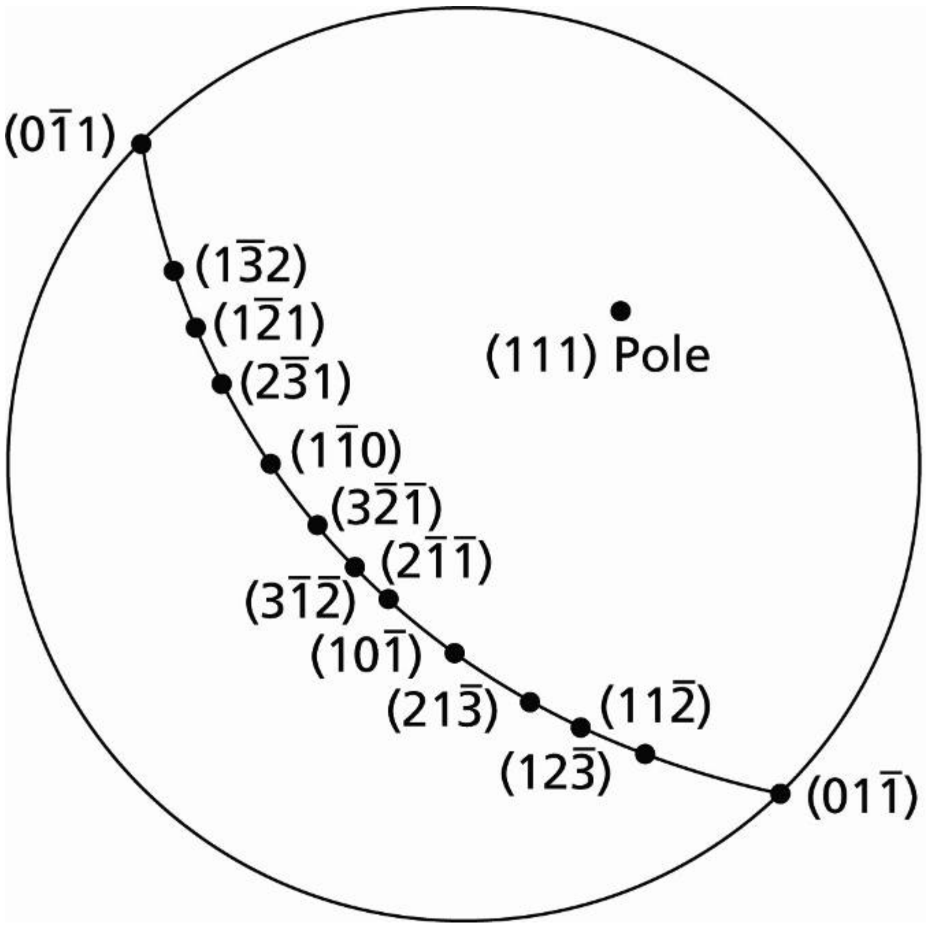


Fig. 1.25 Stereographic projection showing the zone axis for the 12 indicated planes. The zone axis is the [111] direction. Notice that all of the planar poles lie on the (111) plane.

Have you figured out how to determine whether the poles line on a plane yet?

Adapted from R. Abbaschian, L. Abbaschian, and R.E. Reed-Hill, Physical Metallurgy Principles, 4th Edition (Cengage learning, Stamford, CT, 2009), pp. 19.

The Wulff Net

W. Borchardt-Ott, *Crystallography, 2nd Edition*, Springer, New York, 1995

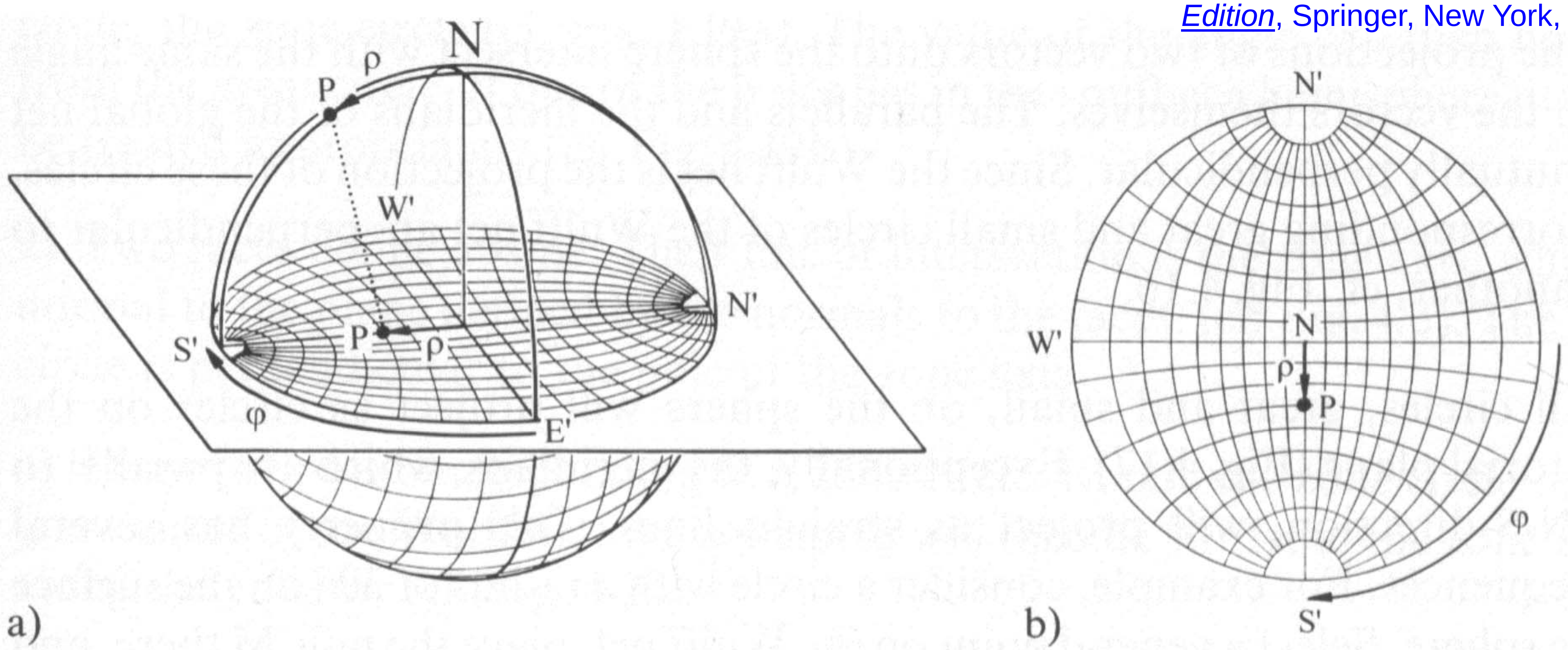
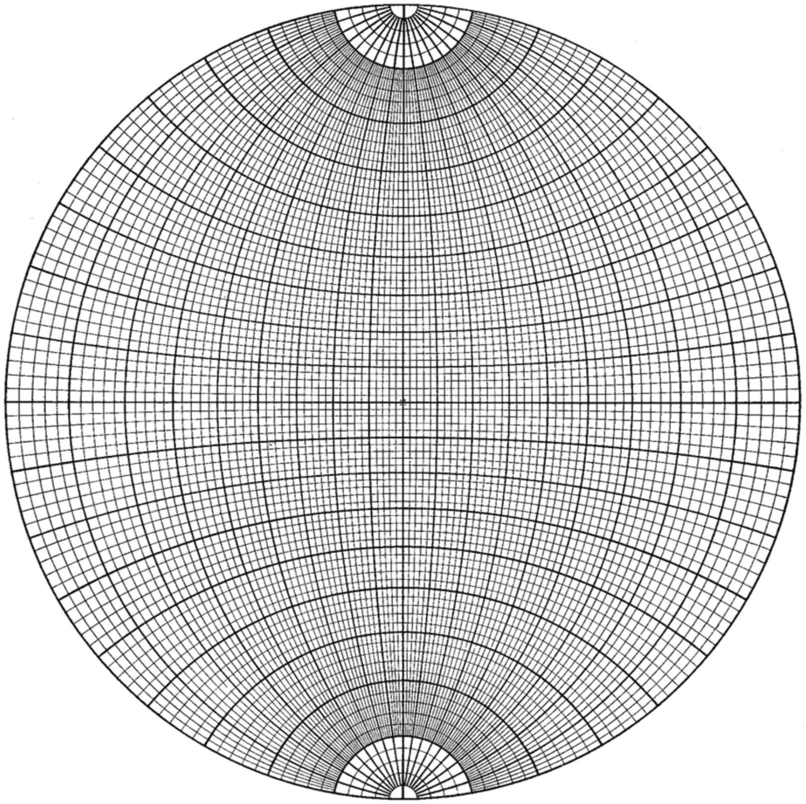
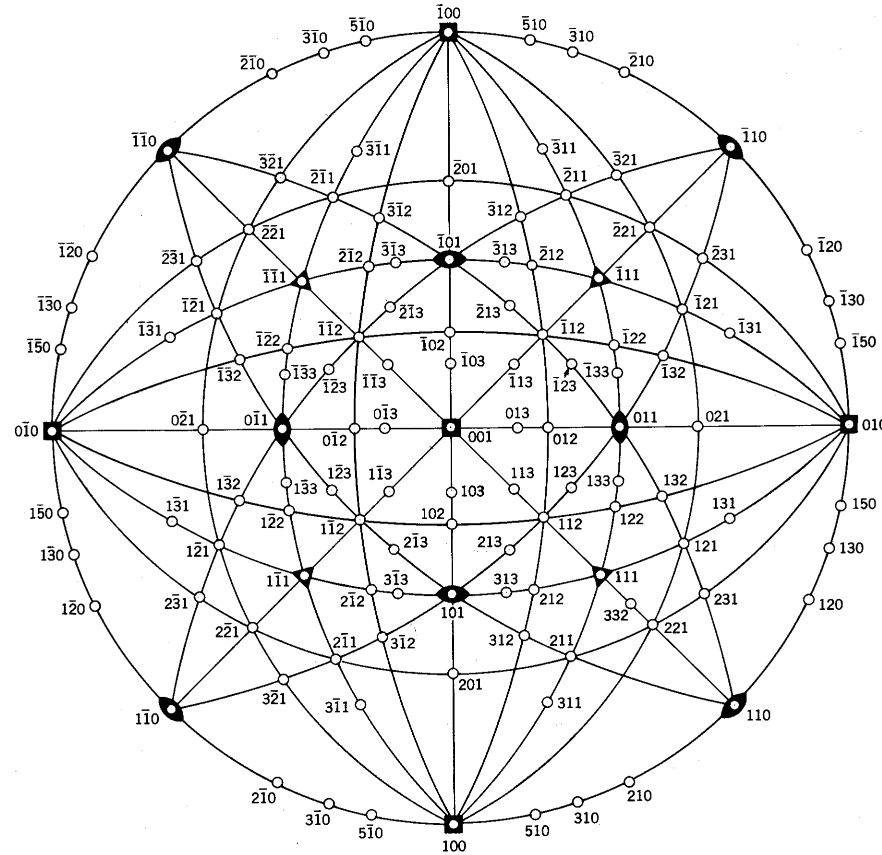


Fig. 4.16 a, b. The stereographic projection of the grid net of a globe (N′-S′ ⊥ N-S) produces the Wulff net; the positions of the angular coordinates ϕ (the azimuthal angle) and ϱ (the pole distance) are indicated. The pole P has coordinates $\phi = 90°$ and $\varrho = 30°$

- Enables measured crystal angles to be plotted directly on a stereographic projection.
- It is the stereographic projection of the grid of a conventional globe oriented so that the N´-S´ direction lies in the plane of projection.

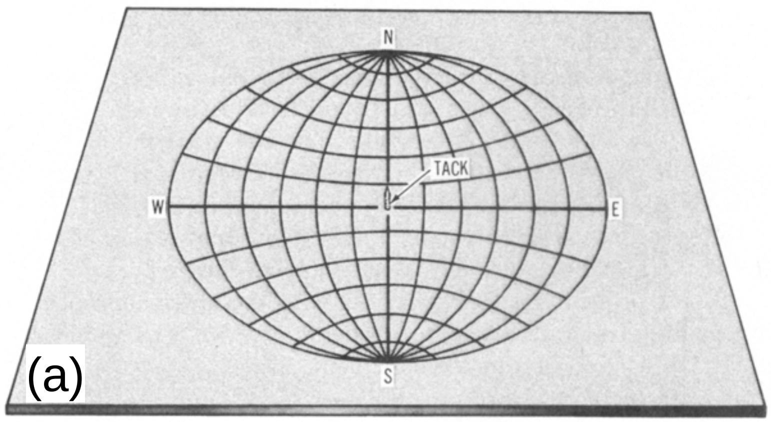
Wulff Net and Standard Projection

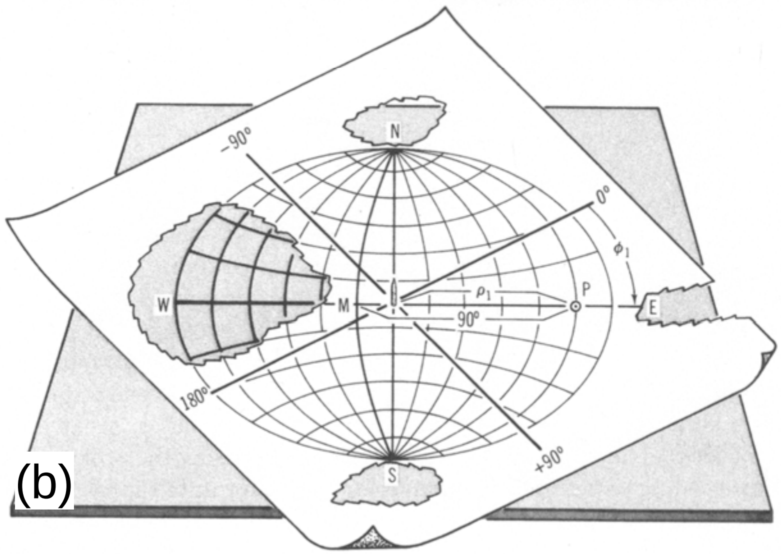


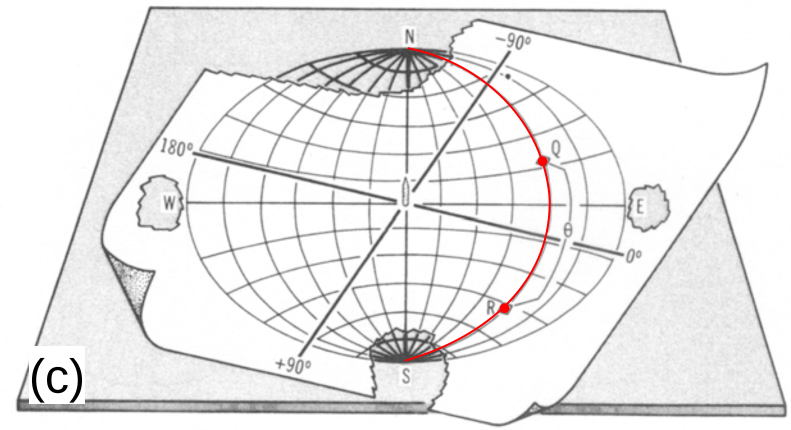


When you overlay a stereographic projection on it,
you can use it like a protractor.

Makes it easy to measure angles between directions and planes.







(a) Place a tack through center of Wulff net.

(b) Draw two perpendicular lines on a sheet of tracing paper or a transparency. Place tack through intersecting lines. The ends of the lines should be labeled with their ϕ values.

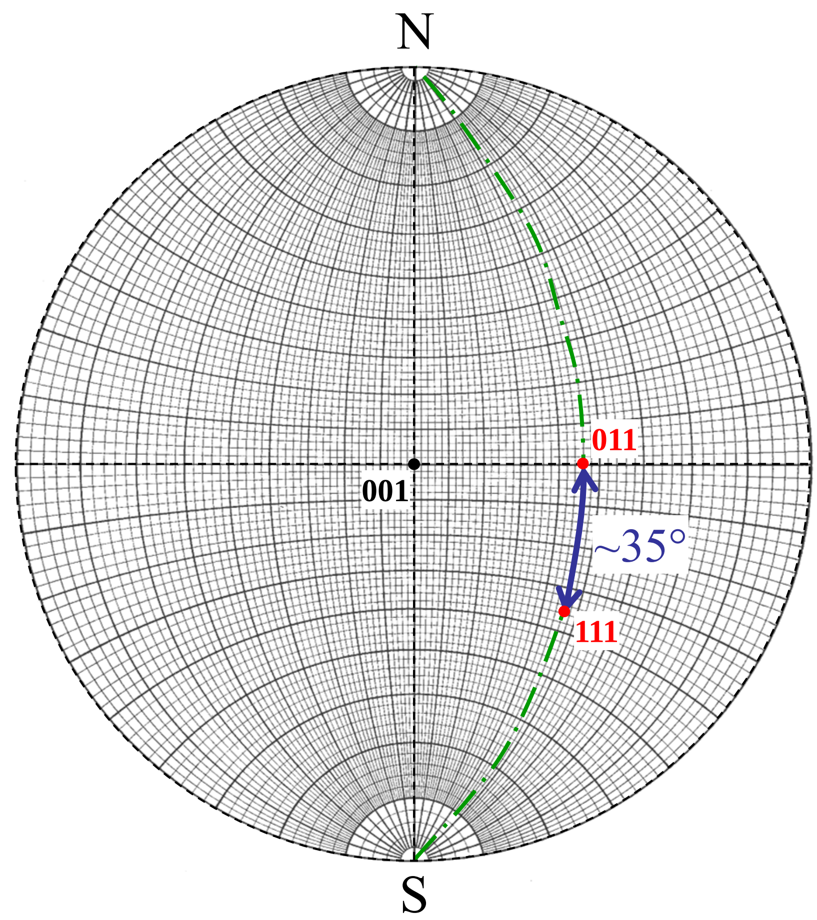
(c) The angle between two points can be measured by rotating until each point lies no the same great circle.

Adapted from F.D. Bloss, ***Crystallography and Crystal Chemistry, An Introduction***, Holt, Rinehart and Winston, Inc., New York, 1971, pp. 79, 81.

ANIMATED!

Rotate two poles until both are on the same great circle (*i.e.*, the same NS arc).

Read angle btw. on great circle.

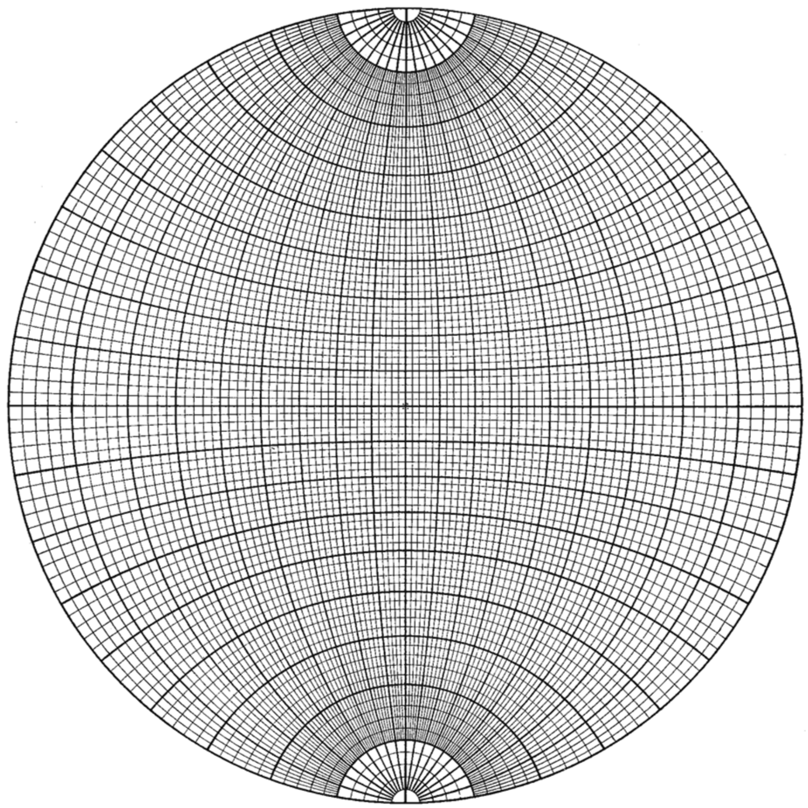


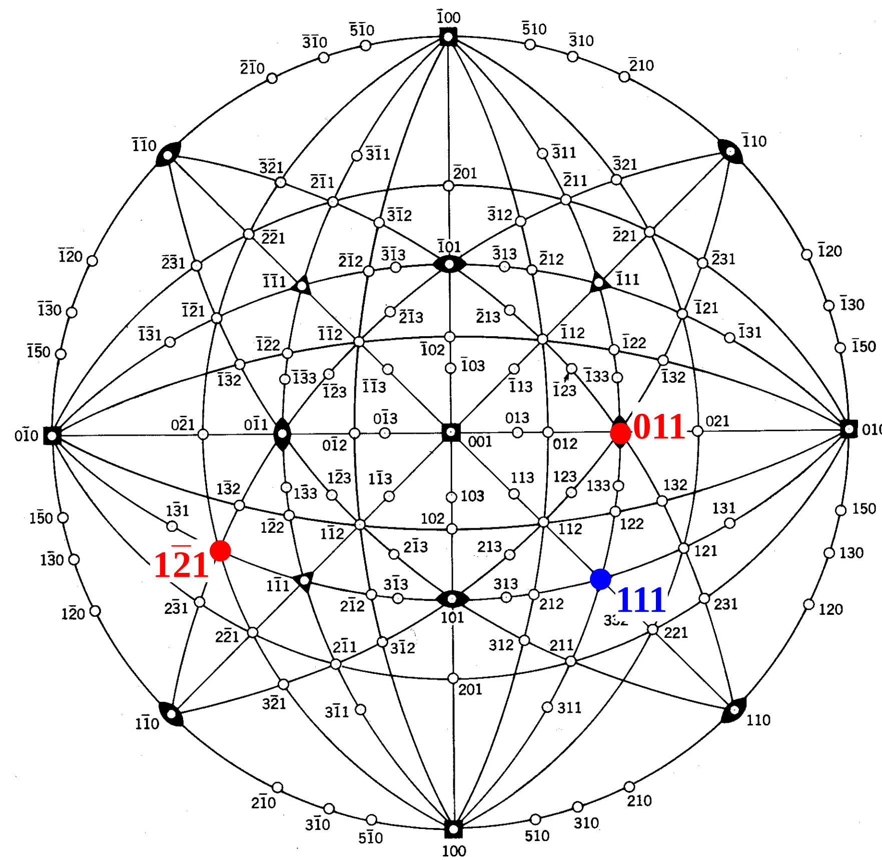
These two are already on the same great circle

What is the angle between [011] and [111]?

Or do it mathematically w/ the cosine law

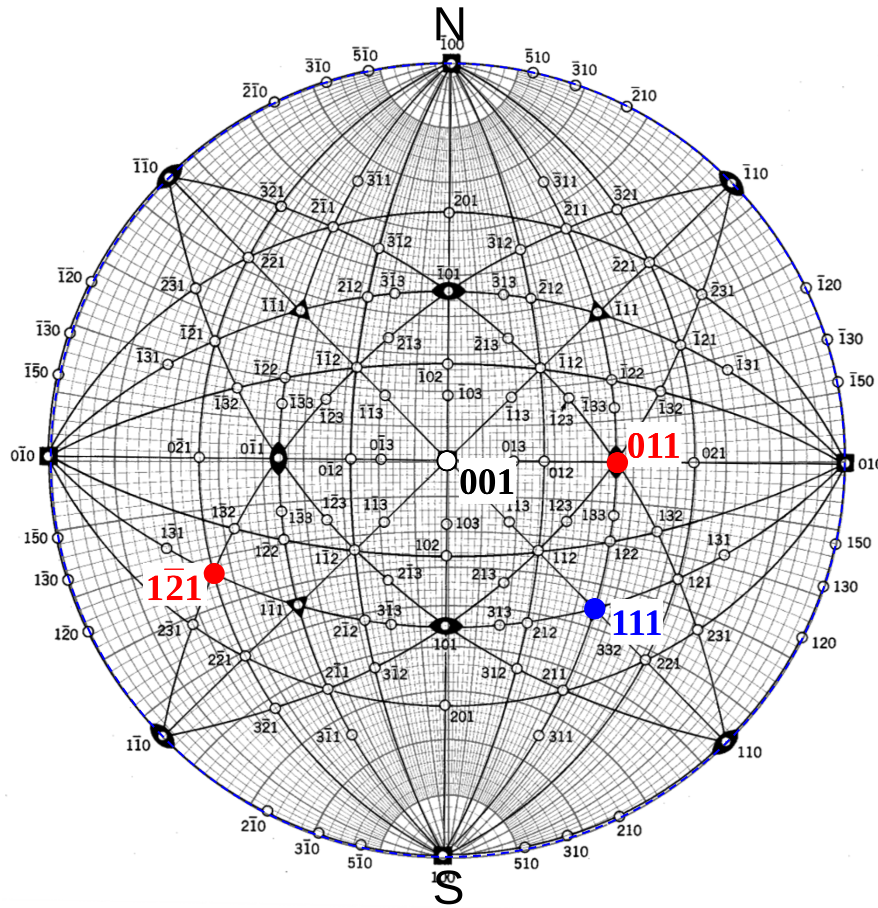
$$\cos\alpha =$$





What is the angle between [011] and [1$\bar{2}$1]?

Let's center this one and solve the problem.

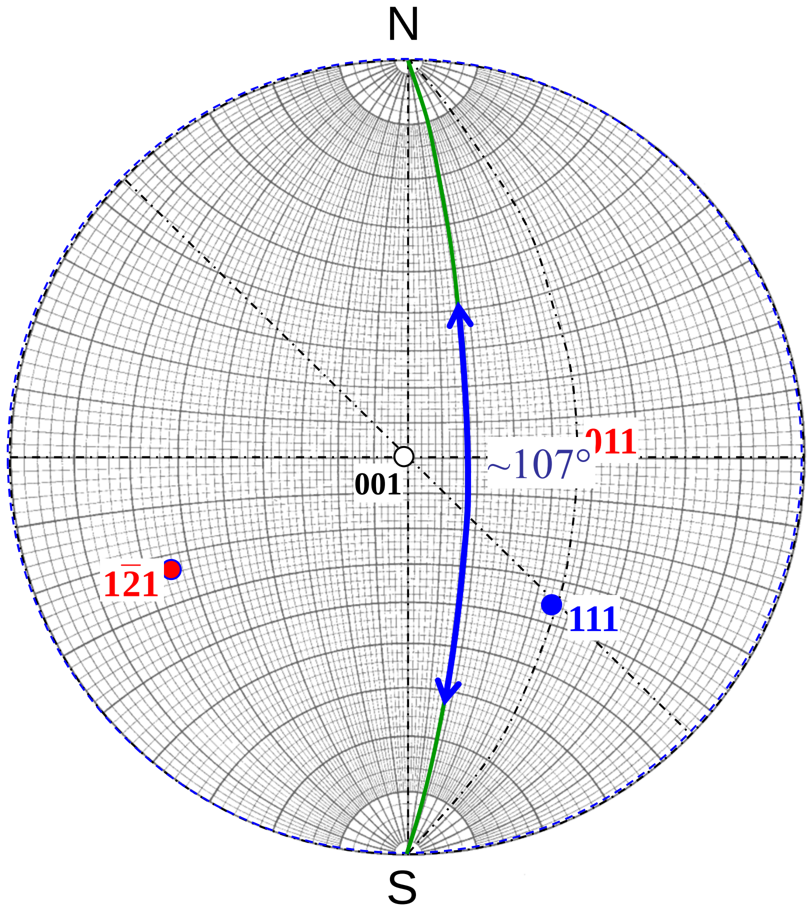


What is the angle between [011] and [1$\bar{2}$1]?

ANIMATED!

Rotate two poles about the center until both are on the same great circle (*i.e.*, on the same N-S arc).

Measure angle between on great circle.

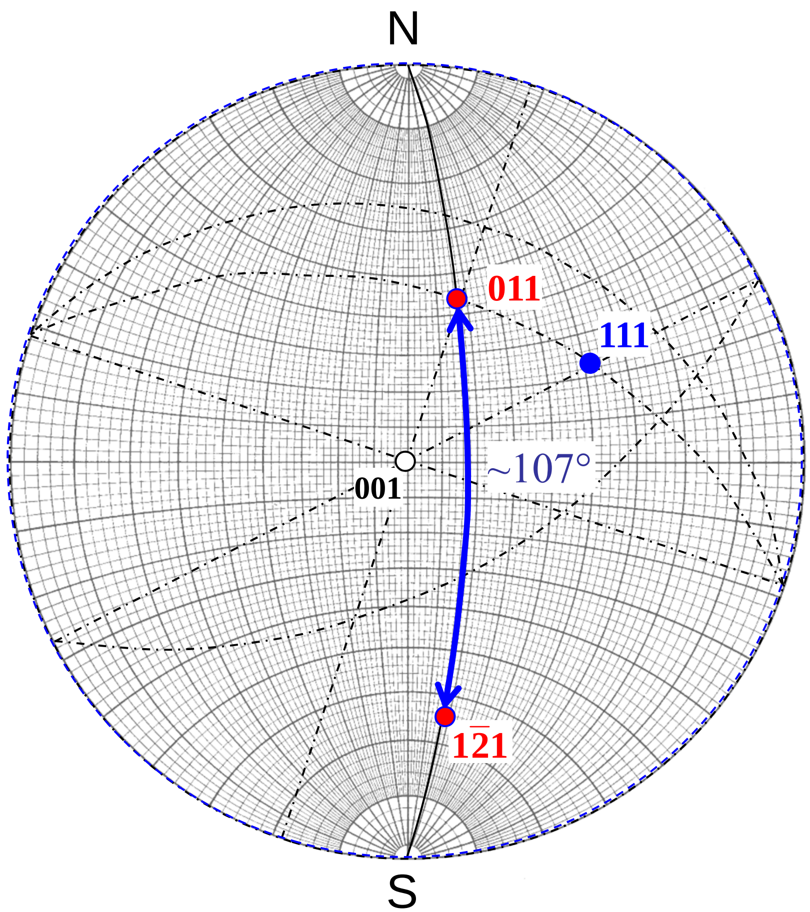


What is the angle between [011] and [1$\bar{2}$1]?

$\cos\alpha =$

Rotate two poles about the center until both are on the same great circle (*i.e.*, on the same N-S arc).

Measure angle between on great circle.



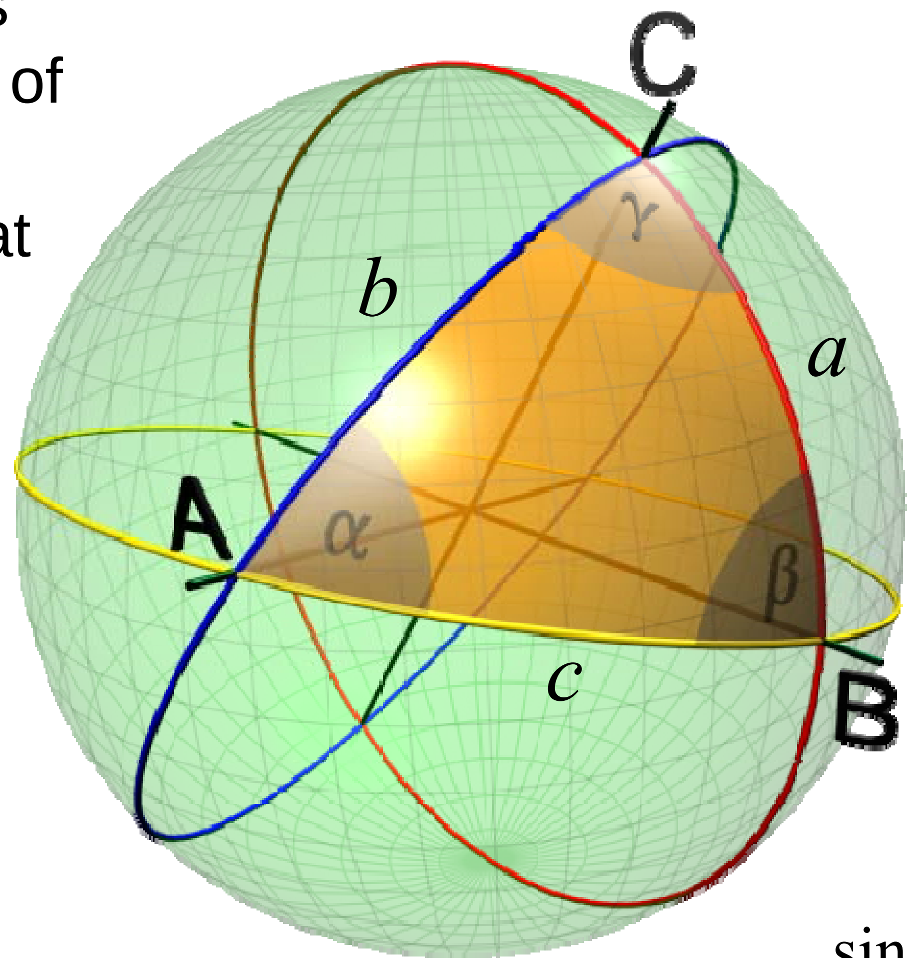
What is the angle between [011] and [$1\bar{2}1$]?

$$\cos\alpha = \frac{0(1)+1(-2)+1(1)}{\sqrt{(0^2+1^2+1^2)(1^2+(-2)^2+1^2)}} = \frac{-1}{\sqrt{12}}$$

$$\underline{\underline{\alpha = 106.8^\circ}}$$

Spherical Triangles

When triangles are on surface of sphere, the edges are great circles



$$\alpha + \beta + \gamma > 180°$$

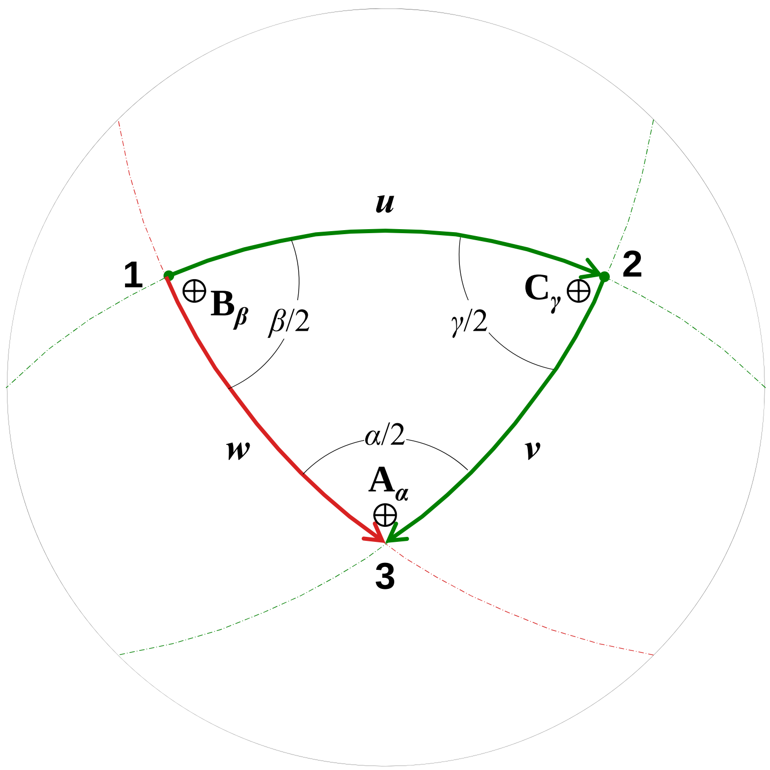
$$\frac{\sin\alpha}{\sin a} = \frac{\sin\beta}{\sin b} = \frac{\sin\gamma}{\sin c}$$

http://commons.wikimedia.org/wiki/File:Spherical_triangle_3d.png

Combined of Rotational Symmetries

- When several symmetries are present at the same time, they must be mutually consistent.

- A combination of two rotations about intersecting axes implies that a third rotation axis exists that is equivalent to the combination (i.e., $u + v = w$)



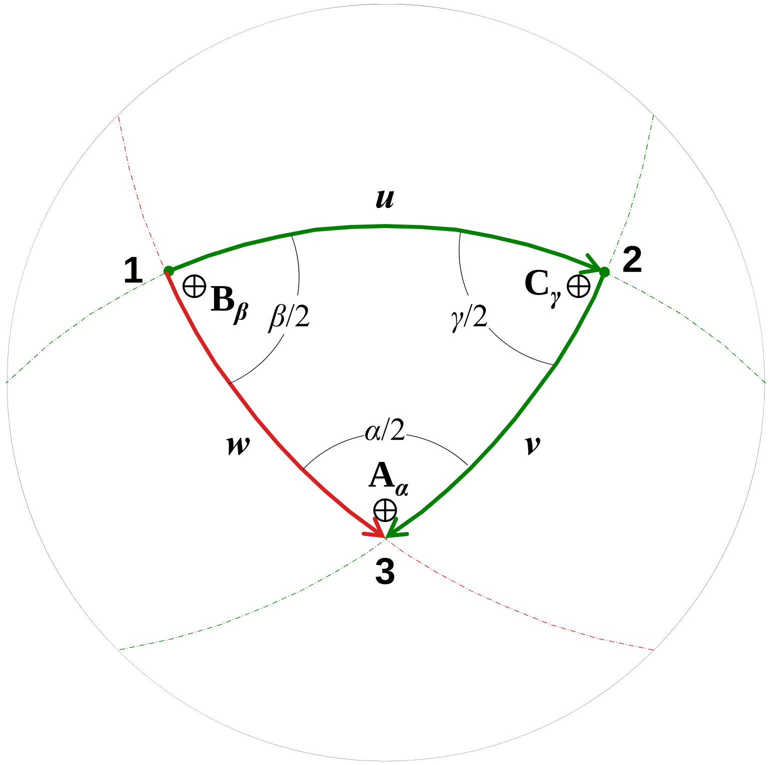
Euler construction

- Spatial relationships between axes (A_α, B_β, and C_γ) must conform. Spatial arrangement of axes at points of intersection is given by u, v, and w.

$$\cos u = \frac{\cos(\alpha/2) + \cos(\beta/2)\cos(\gamma/2)}{\sin(\beta/2)\times\sin(\gamma/2)}$$

$$\cos v = \frac{\cos(\beta/2) + \cos(\alpha/2)\cos(\gamma/2)}{\sin(\gamma/2)\times\sin(\alpha/2)}$$

$$\cos w = \frac{\cos(\gamma/2) + \cos(\alpha/2)\cos(\beta/2)}{\sin(\alpha/2)\times\sin(\beta/2)}$$



- Allowable combinations yield values in the range -1 to 1.
- Values outside this range are not possible.
- The matrix of allowed combinations is provided on the next page.

Summary

- Stereographic projections are useful quantitative tools for presenting 3-D orientation relationships between crystallographic planes and directions on a 2-D figure.

- They also make it relatively easy to visualize crystal symmetry.

- They are very valuable in TEM analysis.