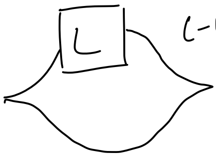
Legendrian surgeries, LOSS invariants and contact invariants

↳ Lisca - Ozsváth - Stipsicz - Szabó



Question: Given leg. $L \subset (Y, \xi)$, to what extent could we know the contact structure from doing legendrian surgery on L from L

More precisely, in (S^3, ξ)

1. Suppose we have L_1, L_2 in same (S^3, ξ) with same knot type, tb, rot. number but $L_1 \neq L_2$ when $(S^3, \xi_{L_1}) \neq (S^3, \xi_{L_2})$

2. Suppose we have non-loose L in (S^3, ξ^{ot}) when (S^3, ξ_L) tight
   ↳ complement is tight

Contact Invariant & LOSS invariant

Given $(Y, \xi) \Rightarrow c(\xi) \in \widehat{HF}(-Y)$ (Ozsváth-Szabó)

Given L in $(Y, \xi) \Rightarrow \mathcal{L}(L) \in HF\bar{K}(-Y, L)$ (LOSS)

↳ bigraded $\begin{cases} \text{alexander} \\ \text{maslov} \end{cases}$

1. $\mathcal{L}(L_1) \neq \mathcal{L}(L_2)$ when $c(\xi_1) \neq c(\xi_2)$?

2. $\Big(\underset{?}{\mathcal{L}(L) \neq 0} \Rightarrow L \text{ non-loose}\Big)$ when $\Big(c(\xi) \neq 0 \Rightarrow \xi \text{ tight}\Big)$

(LOSS)
<u>Lemma:</u> For a smooth knot $K \in Y$, $\exists$ map

$$g: CFK^-(-Y, K) \to \widehat{CF}(-Y)$$

and for L a legendrian rep. of K

$$G: HFK^-(-Y, K) \to \widehat{HF}(-Y)$$

$$\in (\mathcal{L}(L)) = c(\xi)$$

Theorem: Given L, S 2 disjoint leg. let S' be the corresponding leg. in (Y, ξ_L), we have

$$F: HFK^-(Y, S') \to HFK^-(Y, S)$$
$$\mathcal{L}(S') \to \mathcal{L}(S)$$

1. & 2.

when is G injective?

Theorem:
If $L \subset (S^3, S)$ is HF thin, $tb(L) - rot(L) = 2g_K(L) - 1$ then G is injective

A legendrian L is HF nice if it is thin, $tb(L) - rot(L) = 2g - 1$, $\mathcal{L}(L) \neq 0$

Corollary: $\mathcal{L}(L_1) \neq \mathcal{L}(L_2)$ and nice, then $c(\xi_{L_1}) \neq c(\xi_{L_2})$

Corollary: If L is nice in (S^3, ξ^{ot}), then $c(\xi_L) \neq 0$

example 1.: there are non-simple nice legendrian two-bridge knots found by Ozsvath - Stipicz, Foldvari, Wan

example 2: take $T_{(p,-q)}$, look at open book supported by $T_{(p,-q)}$. take legendrian push off the binding it is nice

Theorem: If K is thin and $\tau(K) < g(K)$, then K admits a nice legendrian rep. L in some (S^3, ξ^{ot}) with $tb(L) = 0$.

→ Observation: If L is nice then the negative Stabilization (L^-) of L is also nice.

Corollary If K is thin, $\tau(K) < g(K)$

$$S_p^3(K)$$

admits a tight contact structure for $p < 0$

Conjecture: tb could be any $\mathbb{Z}$

Theorem: If L is thin and $tb(L) - rot(L) = 2g-1$ with $tb < 0$, then

$$G : HFK^-(-S_L^3, S') \to \widehat{HF}(-S_L^3)$$

is injective at $A(\mathcal{L}(S'))$

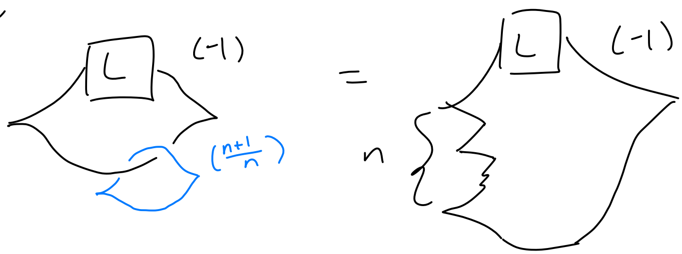
Proposition: (Smooth) If K is thin and K' be the dual knot in $S_n^3(K)$, $n \gg 0$, then

$$G : HFK^-(S_n^3(K), K') \to \widehat{HF}(S_n^3(K))$$

is injective at top grading

(going back to tb) 2

lemma



the naturality theorem of contact invariant under positive contact surgeries

$$F : \widehat{HF}(-S_L^3) \to \widehat{HF}(-S_{L^{-n}})$$

$$c(\xi_L) \longrightarrow c(\xi_{L^{-n}})$$

$$tb(L^{-n}) \ll 0$$