## 2025 SEC GEOMETRY-TOPOLOGY WORKSHOP- OPEN PROBLEMS

- 1. (Sabloff's tlak) Is there a theory of Legendrian cobordisms between curves in  $(M, \xi) \propto \mathbb{R}$  or Legendrians in  $\mathbb{R}^3/M$  (in Arnold sense) that is different/richer than lifts of exact Lagrangian cobordisms between Legendrians?
- 2. (Sabloff's talk) Is there a Floer-type/ LCH type theory? How does this relate to Lagrangian cobordism map? (Motivation from conormal construction)
- 3. (Breen's talk) Can you generalize Gay-Licata front diagrams in open books to Legendrian surfaces in *S*<sup>5</sup> (or other 5-manifolds)? Can you use this to draw pictures of special Legendrians? Put on page of an open book?
- 4. (Breen's talk) Can you simultaneously build  $S^5$  and Legendrian surfaces via handle attachment (w/ Legendrian objects in cores)? Relation to weaves? Relation to Lagrangian cones in  $\mathbb{C}^3$ ?
- 5. (Fernandez's talk)Does there exist a formally trivial but nontrivial loop in a tight  $(M, \xi)$ ?
- 6. (Fernandez's talk) Can you get anything about cabling/ satellite constructions? Higher dimensions?
- 8. If  $\varphi \in \text{Cont}(M, \xi)$  has property that  $\varphi(\Lambda) \sim \Lambda \forall$  Legendrians  $\Lambda$ , is  $\varphi \sim$  id? This is smoothly true for prime manifolds (cf. Etnyre–Van Horn-Morris). It is easier for links, spatial graphs.
- 9. Can we allow higher genus components in rational blow-down configurations?
- 10. When do small Seifert fibered spaces bound QHB<sup>4</sup>? Specifically, is there a non L-space example?
- 11. Is there a higher dimensional example of an exotic (formally isotopic to id, but not (contact) isotopic to id) contactomorphism of an overtwisted manifold? Or even examples beyond Vogel's in dimension 3?
- 12. Which smooth knots have Legendrian reprentatives w/ Lagrangian fillings? (Conjecture: *K* is quasi-positive and max to representative  $\Lambda$  has  $r(\Lambda) = 0$
- 13. Slice-Bennequin  $\leq$  in any tight  $(M, \xi)$ ? (Consider  $M \ge [0, 1]$ ). Look at minimal genus orientable  $\Sigma$  for  $K \subset M \ge \{1\}$ , there. Is it true that  $\overline{sl}(k) \leq -\chi(\Sigma)$ ? What about when  $\Sigma$  is non-orientable?
- 14. If *K* is fibered in  $(S^3, \xi_0)$  and supports  $\xi_0$ , is  $\overline{tb}(k) > 0$ ? ( $\geq 0$  yes, > 0?)
- 15. Fix a finitely presented group *G*. Let *W* minimize  $\chi(W)$  overall  $(W^4, \omega) \le \pi_1(W^4) = G$ . Is *W* unique up to symplectomorphism? This is known as "Generalized symplectic Poincare Conjecture"