Comments on the LANA Project Report of Kato et al.

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1 Introduction

Fumiharu Kato and the LANA Project Team have released their preliminary report (Kato, Commelin, Hoshi, Kedlaya and Topaz, July 17, 2026) documenting their unsuccessful attempt to formalize (in LEAN) some small fragment of (Mochizuki, 2021). The purpose of this document is to provide comments on (Kato et al., July 17, 2026) and the LANA Project and its claims regarding (Mochizuki, 2021) and is intended for a wider mathematical audience.

2 Flawed methodology of (Kato et al., July 17, 2026)

The methodology followed by (Kato et al., July 17, 2026) is described there in detail. They describe their reading of the key ideas, one of which is discussed in (Kato et al., July 17, 2026, 1.3, 1.4). However, this point, namely using the same object with different set of labels (e.g. ${}^{\dagger}A^{\rhd} \simeq {}^{\ddagger}B^{\rhd}$) to arrive at the conclusions of (Mochizuki, 2021), is already shown to be problematic and flawed by (Scholze and Stix, 2018) and (Joshi, 2025c). In May-June 2024, Scholze and I had robust discussions on this matter and both of us are in agreement on this point.

Attempting LEAN encoding of an assertion already know to be mathematically invalid is not going to change its status–as (Kato et al., July 17, 2026) have rediscovered (after two years of work).

To understand what Mochizuki actually wants to do, one must first understand Classical Teichmüller Theory (which is the model on which (Mochizuki, 2021) is based). For genus one, compact connected Riemann surfaces, the Teichmüller space is identified with the complex upper half-plane $\mathfrak{H}$. So let $\tau \in \mathfrak{H}$, then $-\frac{1}{\tau} \in \mathfrak{H}$. Let E_τ (resp. $E_{-\frac{1}{\tau}}$) be the elliptic curve with period lattice $[1,\tau]$ (resp. $[1,-\frac{1}{\tau}]$). Then the elliptic curves $E_\tau, E_{-\frac{1}{\tau}}$ are biholomorphic i.e. isomorphic complex elliptic curves.

Let $q_\tau = e^{2\pi i\tau}$ (resp. $q_{-\frac{1}{\tau}} = e^{-2\pi i\frac{1}{\tau}}$) be their Schottky parameters giving the Schottky parameterizations

$$E_\tau = \mathbb{C}^*/q_\tau^{\mathbb{Z}} \qquad \text{and} \qquad E_{-\frac{1}{\tau}} = \mathbb{C}^*/q_{-\frac{1}{\tau}}^{\mathbb{Z}}.$$

If $|\tau|$ is large then $\left|-\frac{1}{\tau}\right|$ is small i.e.

$$|q_\tau| \approx 0 \text{ and } \left|q_{-\frac{1}{\tau}}\right| \approx 1, \tag{2.1}$$

and both q_τ and $q_{-\frac{1}{\tau}}$ represent isomorphic elliptic curves, i.e. represent a single biholomorphism class of elliptic curves. Mochizuki asserts (recalled in (Kato et al., July 17, 2026, (1-1), Page 12)) that there is an analog of (2.1) in which the Schottky parameter q_τ is replaced by the Tate parameter q_E

$$^{\dagger}q_E^N \longmapsto {}^{\ddagger}q_E. \tag{2.2}$$

Notably, † and ‡ are supposed to be Teichmüller Labels just as τ and $-\frac{1}{\tau}$ are intrinsic Labels provided by Classical Teichmüller Theory. Mochizuki's "Links" must be understood in this Teichmüller sense. This brings us to the important question:

does Mochizuki provide such intrinsic labels in (Mochizuki, 2021)?

The answer is no. This was the main point on which (Scholze and Stix, 2018) is based–this topic too, was part of conversations between myself and Scholze in May-June 2024.

On the other hand my work demonstrates that there exists an intrinsic Arithmetic Teichmüller Theory which provides intrinsic Teichmüller Labels so that one can safely assert the existence of the phenomenon (2.2) which Mochizuki seeks to justify his claim.

Mochizuki's *Key Principle of Inter-Universality* (Mochizuki, 2021, Pages 24-27) holds the key to understanding Mochizuki's claim. Unfortunately, (Kato et al., July 17, 2026, 3.2) have missed the centerpiece of this Principle (namely the role of geometric base points). In Mochizuki's parlance he interprets $^{\dagger}q_E^N$ as living in a different mathematical universe (labeled by a geometric basepoint) different from $^{\ddagger}q_E$ to simulate distinct Teichmüller labels i.e. analogs of τ and $-\frac{1}{\tau}$. I have detailed Mochizuki's "different universes' point of view (with detailed proofs) in (Joshi, 2025b, Theorem 60). However, Mochizuki does not actually follow up with the details required to bring his strategy to fruition in his published papers. That is why I have asserted that (Mochizuki, 2021) is incomplete.

As I have explained in (Joshi, 2025b), I understood Mochizuki's strategy, and in my investigations (Mochizuki, 2021), I found a way to demonstrate the existence of an intrinsic Arithmetic Teichmüller Theory (naturally incorporating all geometric base-points) which allows us to correctly assert things like (2.2). This brings greater clarity to what Mochizuki was trying to achieve and complete his program in my work.

Mochizuki also has diagnosed why (Kato et al., July 17, 2026) arrives at its incorrect conclusion, and *I am in agreement with Mochizuki's diagnosis of the issue with (Kato et al., July 17, 2026).* I discuss this in Section 3**(6)**.

3 Summary of my findings regarding (Kato et al., July 17, 2026)

(1) (Kato et al., July 17, 2026) falsely assert that criticism of my work has appeared. So let me put the record straight: there is no manuscript on arxiv.org (or in print) providing any mathematical criticism of my work on the abc-conjecture (Joshi, 2025a). On the other hand, my criticism of (Mochizuki, 2021) appears on the arxiv (Joshi, 2025c). [Last update of Constr. IV (Joshi, 2025a) was in Feb 2025, both Mochizuki and Scholze received this version in October 2024. My 'Final Report' (Joshi, 2025c) was posted in June 2025.]

(2) As was already noted in my letter to Fumiharu Kato (Joshi, April 15, 2026), the LANA project is anachronistic, ignores findings which have appeared in my work 2018-2025, and rewinds the clock back to 2018. This is not how Science (or Mathematics) works.

(3) I note that (Kato et al., July 17, 2026) *finally* acknowledges that Mochizuki's discussion of Arithmetic Holomorphic Structures is woefully inadequate (a fact which was first observed by Scholze, and later by myself), and (Kato et al., July 17, 2026) show a lack of understanding of Mochizuki's work and proceed to dismiss it. But here is the long list of places where this notion (Arithmetic Holomorphic Structure) is invoked and critically used by Mochizuki:

(Mochizuki, 2021, Pages 7, 24, 25, 29, 33, 35, 105, 111, 209, 212, 214, 228, 229, 248, 257, 258, 259, 260, 261, 265, 269, 270 274, 329, 330, 340, 358, 368, 369, 370, (II, Cor. 4.11, Page 390), 391, 453, 454, 458, 460, 461, 462, 463, 464, 487, 488, 489, 490, 561, 462, 463, 464, 487, 488, 489, 490, 561, 562, 570, 571, (III, Theorem 3.11, Page 573), (III, Remark 3.11, 580), 581, 582, 583, 584, 585, 586, 587, 589, 590, (III, Cor. 3.12, Page 597), 598, 600, 604, 606, 607, 608, 609, 615, 619, 620.)

So this notion can neither be dismissed nor avoided as (Kato et al., July 17, 2026, Page 20) does. The lack of such objects in (Mochizuki, 2021) was a point which featured prominently in my conversations with Peter Scholze in May-June 2024 which led to our agreement and the public release of (Joshi, 2024c). My work proceeds by explicitly and carefully defining Arithmetic Holomorphic Structures in (Joshi, 2021), (Joshi, 2022) and establishing their properties.

(4) (Kato et al., July 17, 2026) continues to project an incorrect view of (Scholze and Stix, 2018) objections. [I have often said that many people have a poor understanding of the objections of (Scholze and Stix, 2018). Scholze made his point of view clear to me during our conversations in May-June 2024 (and we are in agreement on those points).] The key issue in (Mochizuki, 2021) is Mochizuki's claim (and the claim of (Kato et al., July 17, 2026)) that working with less rigid objects such as Basic Prime Strips (BPS) and Anabelian Reconstruction Theory provides verifiably distinct Arithmetic Holomorphic Structures and leads to a proof of the abc-conjecture–this is exactly where the problems with (Mochizuki, 2021) are located.

(5) (Kato et al., July 17, 2026) uses terms from (Mochizuki, 2021) such as "algorithms" and "execution" of the said "algorithms" to justify the claims such as (Mochizuki, 2021, IUT3, Theorem 3.11). But "algorithm" as used in (Mochizuki, 2021) is not a finitistic procedure as commonly understood by mathematicians. Thus, the usage of the term "execution of algorithm" is also deeply problematic. Furthermore, these procedures are highly ineffective, notably they do not produce the claimed outputs, and do not certifiably produce outputs which can be distinguished from each other (this is necessary in the proof). This lead to issues raised in (Scholze and Stix, 2018), and my 'Final Report' (Joshi, 2025c).

(6) *On the other hand, I am in agreement with Mochizuki regarding why (Kato et al., July 17, 2026) arrives at its incorrect conclusion.* My analysis of (Kato et al., July 17, 2026) is that they have missed the fact that at a very intrinsic level the proof of (Mochizuki, 2021, IUT3, Corollary 3.12) requires, the Arithmetic Teichmüller Datum of a fixed number field $L = \mathfrak{arith}(L)_{\mathbf{y_0}}$, and also its Frobenius twisted version $L^{(1)} = \mathfrak{arith}(L)_{\varphi(\mathbf{y_0})}$ (this notation and the concept underlying it are detailed in (Joshi, 2023, Definition 5.9.1) and remarks following (Joshi, 2025a, Theorem 6.10.1) provide additional comments about how this plays out in proving the inequality one seeks). Here $L^{(1)}$ is not some new label for L, but a genuine topological deformation of L. (Kato et al., July 17, 2026) completely ignore Arithmetic

Holomorphic Structures and therefore encounters the same problem as (Scholze and Stix, 2018).

(Kato et al., July 17, 2026, 10.5) provides Mochzuki's analysis of why (Kato et al., July 17, 2026) arrives at its conclusion–Mochizuki asserts (and I quote) "[this situation] occurs whenever one works within a single holomorphic structure/ring theory."

I am in complete agreement with Mochizuki's reasoning: one does need to work with distinct Arithmetic Holomorphic Structures to complete the proof of (Mochizuki, 2021, IUT3, Theorem 3.11 and Corollary 3.12). As I have explained in (Joshi, 2025a), my (Joshi, 2024b) release of the proof of the abc-conjecture had omitted this central point and this deficiency was removed in October 2024 and the updated version was sent to Mochizuki and Scholze with my comments on the changes. In February 2025, (Joshi, 2025d) was publicly released on the arxiv. Notably, I employ the very strategy which Mochizuki employs and calls "log-shifting" (Mochizuki, 2021, IUT3) to prove the main inequality.

[As detailed in the 'Rosetta Stone' (Joshi, 2024a, § 8.9), Mochizuki's $\mathfrak{log}$-Link is the relationship $L \longmapsto L^{(1)}$ and this takes place on the same side of the Θ-Link, but with different Arithmetic topological data and ring structures.]

(7) If the LANA Project Team were hoping to imitate Scholze's Liquid Tensor Experiment as a model to coerce everyone into accepting (Mochizuki, 2021), I believe that this LANA Project is just not going to convince anyone because (a) we are in the year 14 of this controversy (and *still* the proof cannot be communicated in human terms, but only by attempting a LEAN certification), (b) the abc-conjecture has many deep consequences, so its proof must be canonical, intrinsically human-readable, (c) resorting to some naive oversimplifications envisaged by (Kato et al., July 17, 2026) is just not going to explain the innards of a complex and subtle proof, and (d) experts on the LANA Project TEAM still does not seem to understand what Mochizuki's Theory is really about.

(8) There are other oddities and incorrect statements in (Kato et al., July 17, 2026). Here is a small list:

- **(i)** The toy model (Kato et al., July 17, 2026, 1.3) is quite wrong in the context of (Mochizuki, 2021) because that sort of comparison *never* occurs in (Mochizuki, 2021); on the other hand (Joshi, 2021, Theorem 2.5.1) offers a far better way of understanding the sort of changes in ring structures which occur, and it is in parallel with the discussion of Section 2.
- **(ii)** For understanding Mochizuki's decoupling of additive and multiplicative structures, (Joshi, 2019) will be more useful.
- **(iii)** Mochizuki's Indeterminacy Ind_1 (see (Kato et al., July 17, 2026, Section 6)) is *Anabelomorphy* of (Joshi, 2020)–Mochizuki himself explains this in (Joshi, 2020, Appendix). There is no mention of this in (Kato et al., July 17, 2026). The existence of Mochizuki's Ind_2, Ind_3 is substantially deeper and exceedingly difficult to claim via (Mochizuki, 2021). My proofs of their existence is documented in (Joshi, 2024a, § 8.11).
- **(iv)** The discussion of (Kato et al., July 17, 2026, 3.5) is misleading. The key point in (Mochizuki, 2021) is that the Tate parameter changes in non-trivial ways with changes of Arithmetic Holomorphic Structures and in particular with Ind_1, Ind_2 and Ind_3 (to be clear Mochizuki does not prove this–that is why the theory is incomplete). In the parlance of (Joshi, 2020), the Tate parameter is not amphoric. A proof of how Ind_1 impacts the Tate parameter can be read from (Joshi, 2020, Theorem 6.5.1) which

shows that the L-invariant ((Colmez, 2010)) of a two dimensional semi-stable p-adic representation is not amphoric, and the non-amphoricity of the L-invariant implies that the Tate parameter of a semi-stable elliptic curve over a p-adic field is not fixed by Ind_1.

4 Conclusion

The methodology and interpretations of (Mochizuki, 2021) by the LANA Project TEAM, as described in their report (Kato et al., July 17, 2026) were already known to be problematic (Scholze and Stix, 2018), (Joshi, 2025c). I have said on several occasions that people around Mochizuki do not have a sound understanding of his theory. *Finally* in 2026, there is admission that they do not understand (Mochizuki, 2021).

As explained in Section 3**(6)** (and despite our differences), I am in agreement with Mochizuki as to why (Kato et al., July 17, 2026) have gone wrong. Scholze and I (in resp. 2018 and 2020) came to the conclusion that Mochizuki's proof, from the very beginning, and even critically needs a precise definition of Arithmetic Holomorphic Structures. Based on the lack of the evidence for their existence in (Mochizuki, 2021), Scholze had concluded that such structures cannot exist. But starting with (Joshi, October 2020), I present a direct construction of such structures (this renders Anabelian Reconstruction Theory irrelevant for (Mochizuki, 2021). This is an important conceptual development because Anabelian Reconstruction is inadequate for (Mochizuki, 2021)). As Mochizuki also informed Kato in the context of (Kato et al., July 17, 2026), that at the key point in (Mochizuki, 2021, IUT3, Corollary 3.12), requires working with two different Arithmetic Holomorphic Structures (that is exactly what I do in (Joshi, 2025a)). It is unfortunate that people around Mochizuki and some others (including the LANA Project Team) have failed to recognize this key point for so many years.

This episode surrounding (Mochizuki, 2021) is a remarkable tale of bruised egos and Ivy-League Exceptionalism and (Kato et al., July 17, 2026) is yet another (unsuccessful) attempt at settling priority for Mochizuki. So this sorry tale continues.

I invite Mochizuki for conversations with myself and Scholze, so that this issue can be put to rest, once and for all. Considering that between Mochizuki and myself, we do have a proof of the abc-conjecture, I had suggested to Mochizuki in January 2024, that it would be best to settle the matter by issuing a document coauthored by both of us. I believe that such a document is still needed.

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