Half Atwood's Machine Introducing a Changing Mass

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Designed by Cambridge mathematician George Atwood and published by Cambridge University Press in 1784, the Atwood's machine demonstrates the effects of gravity on falling objects. This two object system of unequal masses connected by a string and positioned over a pulley explores Newton's second law of motion, $F = ma$, by examining the measured acceleration due to gravity at the earth's surface. The forces in this experiment include the force of gravity on the two objects and the force of tension in the string while air resistance and friction between the string and the pulley are negligible. However, in a half atwood's machine, a static friction force and normal force are introduced, along with the previous forces, due to the addition of a surface under one of the objects.

In this experiment, a changing mass is introduced. By using a hollow box filled with water, the gravitational constant can be calculated by finding the change in the volume of water in the hollow box over time. The static friction force in this experiment can be neglected because the friction on the wheels neutralizes the force caused by the exiting water, so there is no net thrust.

The general materials needed in the experiment include some mass of .073 kg, a hollow box on wheels that will hold water and an attachment for the rope on the box, a stopwatch, a measuring cup to hold 750ml of water, one rope of negligible mass, one pulley wheel, a plug for the hollow box to keep the water inside, a mass on the water to keep a constant pressure on the water flow, a meter stick, and a surface to conduct the experiment on. For my experiment specifically, I used a fishing line for the rope, a tape measure instead of a meter stick, a wheel with duct tape on it as the pulley, a constant pressure piece using aluminum foil, duct tape, and a felt pad, and a rock to set on top of it, a 3-D printed hollow box, metal inserts for the axles, 3-D printed wheels, and a hydro brand water bottle lid for my hanging mass. The dimensions for the box are .10m x .10m x .10m with a .01m diameter hole on the side of the box opposite from the rope attachment .01m from the base of the box, two 6.5mm metal rods slid into four 10mm inserts, and a folding table to conduct the experiment on. Additionally, I used the clock app on my phone for the stop watch.

For this experiment to be conducted, all materials must be set up as shown in Figure 1 below. I put the plug into the hollow box and filled it with 750ml of water. Then, I placed it at the desired position on the table which was 0.676m from the edge of the table, measured from the front edge of the box, so that the hanging mass was 0.568m from the ground. Once the hollow box was positioned, I added the felt covered in aluminum foil and duct tape piece on top of the water, then added the rock on top of that piece. This piece is a crucial part of the experiment because of the need for constant fluid flow from the hollow box. By using Bernoulli's equation, $\frac{v^2}{2} + gz + \frac{P}{\rho} = C$, it is known that more pressure means gz, which is the gravitational influence of the liquid, matters less. Because of this, the amount of water that came out of the box over time is what will be used as the mass. When I had my stopwatch ready on my phone, I pulled the plug with one hand and simultaneously started the stopwatch with the other hand. I moved my hand to catch the hollow box before it fell off the edge of the table, and once it hit the edge, I pressed stop on the stopwatch. I recorded the time on a piece of paper and repeated this process for a total of nine runs.

I recorded all of this data into a spreadsheet on google sheets, then I graphed the data to get the slope of the line to be g.

The math used in this experiment is shown below.

Given:

$$\Delta x = \frac{1}{2}at^2$$

$$t = \sqrt{2a\Delta x}$$

$$a = \frac{dv}{dt}$$

Known:

$$\Sigma F_c = F^T$$

$$\Sigma F_h = F^T - F^G$$

$$\Sigma F = ma$$

$$F^T = ma$$

$$F^G = mg$$

Derived:

$$a_1 =- a_2$$

$$m_c(t)a_1 - m_h g = m_h a_1$$

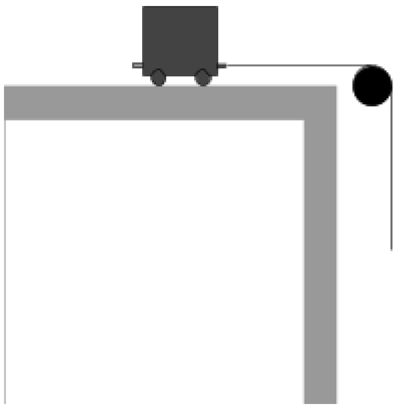
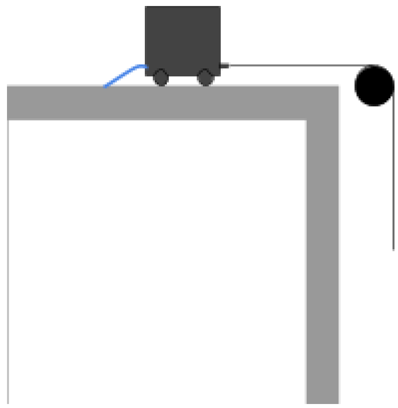
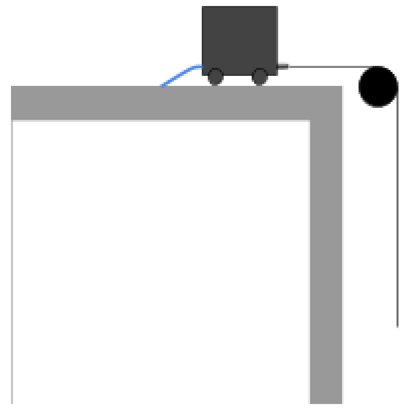
$$g = \frac{(m_h - m_c(t))a_1}{-m_h}$$

Mathematical Model:

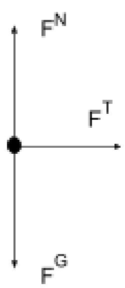
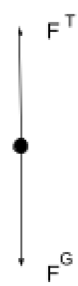
$$a = g\frac{-m_h}{m_c(t) - m_h}$$

From this mathematical model, the independent and dependent variable can be chosen. The independent variable is $\frac{-m_h}{m_c(t)-m_h}$ and the dependent variable is acceleration. This mean that in the line equation $y = mx + b$, $y = a,\ m = g,\ x = \frac{-m_h}{m_c(t)-m_h},\ and\ b = 0.$

Diagram of Model

At Rest	Water beginning to drain	Hanging Mass Lowering
 Figure 1: model of half atwood's machine	 Figure 2: model of half atwood's machine's water draining	 Figure 3: model of half atwood's machine in motion

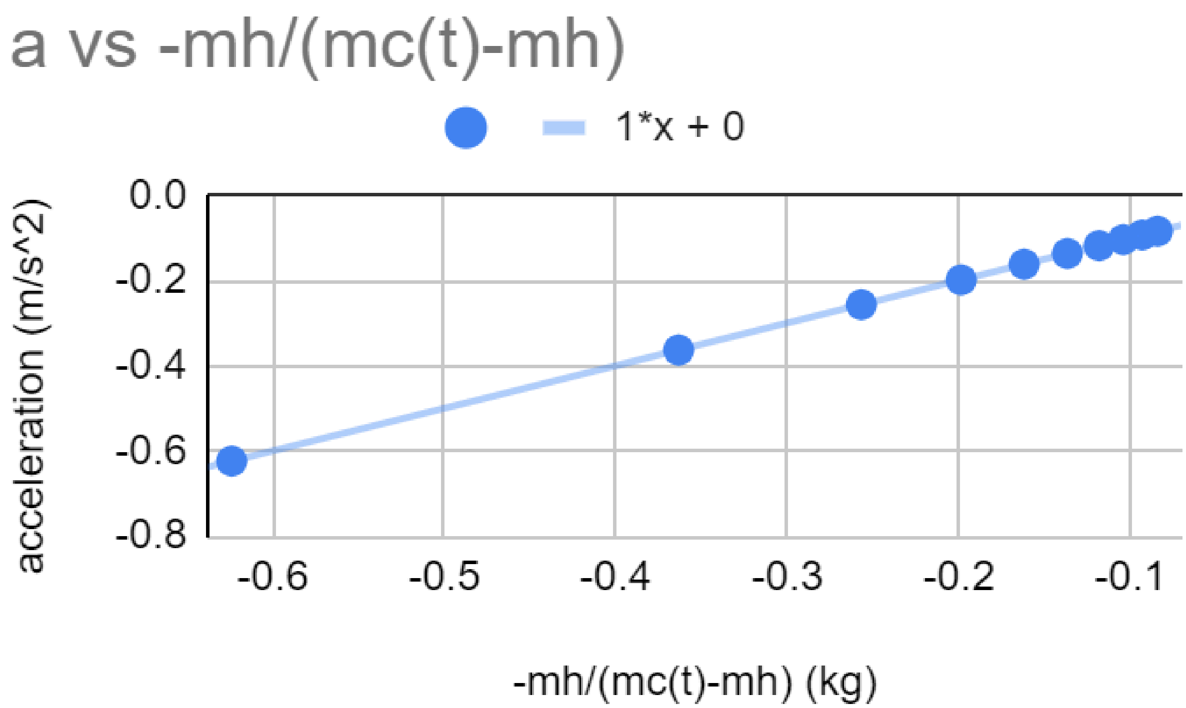
Free Body Diagrams

 Figure 4: Free Body Diagram of hollow box	 Figure 5: Free Body Diagram of hanging mass

Data Table 1

t (seconds)	m (meters)	t^2 (seconds squared)	x^2 (meters squared)	a (m/s^s)	-mh/(mc(t)-mh)
0	0.94	0	-1.35255	0	-0.08419838524
3.02	0.8567	9.1204	-1.35255	-0.1482994167	-0.09314788822
6.04	0.7735	36.4816	-1.35255	-0.03707485417	-0.1042112777
9.06	0.69022	82.0836	-1.35255	-0.01647771297	-0.118272253
12.08	0.60695	145.9264	-1.35255	-0.009268713543	-0.1367169211
15.1	0.523693	228.01	-1.35255	-0.005931976668	-0.1619727841
18.12	0.440432	328.3344	-1.35255	-0.004119428241	-0.1986762176
21.14	0.35717	446.8996	-1.35255	-0.003026518708	-0.2568884822
24.16	0.27391	583.7056	-1.35255	-0.002317178386	-0.3633467722
27.2	0.19	739.84	-1.35255	-0.001828165549	-0.6239316239

Graph



Because the data shows a 89.8% deviation, $\frac{1-9.81}{9.81} * 100 = 89.8$, this conveys that the data is not correct. However, the experiment itself did work, but I do not believe I took the data correctly.

For the calculations to be correct, the following math was required to be used.

$$\Delta x = \frac{1}{2}at^2$$

$$t = \sqrt{2a\Delta x}$$

$$a = \frac{dv}{dt}$$

$$\Sigma F_c = F^T$$

$$\Sigma F_h = F^T - F^G$$

$$\Sigma F = \frac{d\rho}{dt} = \frac{dm}{dt}v(t) + m(t)\frac{dv}{dt}$$

$$\frac{dm}{dt}v(t) + m_c a(t) - m_h g = m_h a(t)$$

$$m_c(t) = m_c(t_0) - \frac{dm_c}{dt}(t)$$

$$m_h g + \frac{dm_c}{dt}(2\Delta x t^3) + [m_c(t) - m_h](2\Delta x t^2) = 0$$

$$Ax^3 + Bx^2 + Cx + D = 0$$

x in the equation above is $2\Delta xt$, so $Cx = 0$.

From these equations, the quadratic formula is supposed to be used to get t alone. However, I could not figure out how to derive the quadratic formula from a cubic polynomial. This is why my experiment calculations failed.

The additional data I took can be seen below.

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9
Time (s)	15.26	14.49	14.49	13.15	15.3	12.28	12.59	11.69	12.87

The total time it took for all of the water to drain out was 27.2 seconds. A total of 0.75kg of water was lost, therefore the initial mass of the hollow box was 0.94kg and the final mass was 0.19kg.

There are multiple sources of error in this experiment that caused a skew and high percent deviation in the data. The first source, a systematic error, is neglected friction and air resistance in a non-controlled environment. The table was slightly textured and the contact with the wheels created friction between the cart and the table. Additionally, the friction between the wheel and the fishing line as well as the friction between the wheel and its axle was neglected, which causes a negative skew in the data points and overall result. However, the addition of water on the table from the previous trials decreased the overall friction between the table and the wheels. Air resistance was also not accounted for. The day this experiment was conducted was a slightly windy day and while the house blocked a majority of the wind, it still played a role in affecting the trials. This adds to the negative skew in the result. These errors can be prevented in a future run by conducting said experiment in a controlled environment with a truly frictionless surface and set of wheels as well as zero air resistance.

An additional source of error, a random error, was the measurement of water. While filling up the plastic measuring cup, I had to assume, from the perspective I was looking at the water. that the amount of water was exactly at the 750ml mark. This means the amount of water used in each trial varied slightly by a milliliter or two. As the water was poured into the hollow box, the plug holding in the water was a close fit, but it was not leak proof, so some of the water leaked out before I pulled the plug and started the timer. This error caused a negative skew in the result because the time for the water to drain out of the box would have been less since water leaked out. This error can be prevented by using an airtight seal on the plug for the box that has an easy release mechanism. A gram scale could also be used to measure exactly the mass of the water in the measuring cup.

The last source of error, a random error, in this experiment was my reaction time in starting and stopping the timer. The delay in pulling the plug on the box to start the timer caused a negative skew and my reaction time in stopping the timer also caused a positive skew because I stopped the time when the box hit my hand, so the signal from one hand to another caused the addition of time. So, the reaction time from starting compared to stopping the timer most likely balanced out the positive and negative skews. This error can be fixed by using a computer software and sensor to record the time and position of the box as it moves in the experiment. This will allow the reaction time to have no effect on the data.

The half atwood's machine is a way to derive the acceleration due to gravity for falling objects. With my experiment in particular, I added water to a hollow cube to introduce a changing mass over time to the original machine. While the machine itself did work and the hanging mass lowered as the water drained from the hollow box, my data was invalid. My slope came out to be $m = 1$ instead of 9.81 which resulted in a 89.8% deviation. This was caused not only by the sources of error of neglected friction, air resistance, eyeballing the water measurement, and my reaction time with the stopwatch, but the data I took was not able to be used in my calculations to derive a correct acceleration due to gravity constant.

Sources

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