Modern Physics Lab U1: The Ratio of Electron Charge and Mass

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Introduction

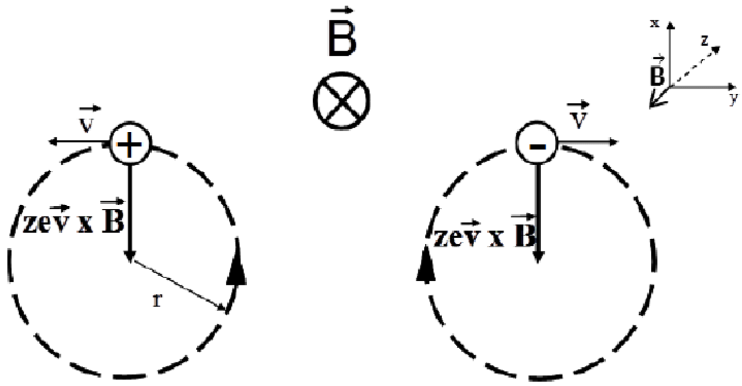
The electron was first theorized in the mid-19th century as scientists studied the nature of electricity and atomic structure. In 1897, the English physicist J.J. Thomson made the groundbreaking discovery of the electron through experiments with cathode ray tubes. He demonstrated that these rays were composed of negatively charged particles, which he called "corpuscles," later known as electrons. This discovery revealed that atoms were not indivisible as previously thought, but contained smaller subatomic particles. Thomson's work led to the development of the plum pudding model of the atom, which was later refined by further research, notably by Ernest Rutherford's nuclear model and Niels Bohr's atomic theory. The electron's discovery was pivotal for the evolution of quantum mechanics and modern understanding of chemical bonding and electricity.

Principle

A charged particle placed in a uniform magnetic field will move in a circle. This is due to the Lorentz force, which is exerted on the particle as it travels through the magnetic field and is always perpendicular to both the particle's velocity and the magnetic field itself. Since the force acts at right angles to the velocity, it continuously changes the particle's direction without affecting its speed, causing the particle to follow a circular trajectory. The magnetic force acts as a centripetal force, which is responsible for keeping the particle in circular motion. The radius of the circular path depends on the particle's mass, velocity, charge, and the strength of the magnetic field, as described by this equation:

$$r = \frac{mv}{qB} \quad => \quad \frac{e}{m} = \frac{v}{r \cdot B}$$

The particle's speed remains constant, but the continuous change in direction leads to circular motion. Here is a diagram detailing the forces:



If the quantities for velocity, orbital radius, and external magnetic field can be measured, then the fundamental constant $\frac{e}{m}$ can be calculated.

Experimentation and Procedure

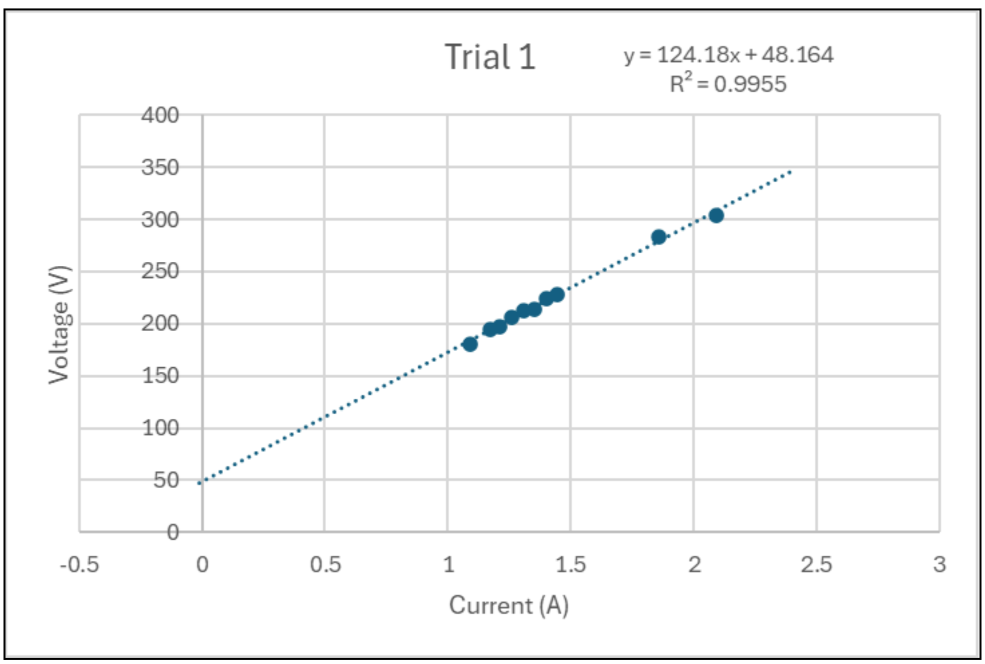
In order to utilize the behavior of an electron in a uniform magnetic field, an apparatus must be built allowing for the measurement of the above quantities. This experiment involves a diode with a heated metal cathode that emits electrons with negligible speed. The anode, which is a metal disk with a small hole in the center, is positively charged, and accelerates the electrons across the gap between cathode and anode. The speed of the electrons is calculated from the electron's energy. Some of the electrons pass through the hole in the anode at high speeds. The equation $\frac{m}{2}v^2 = eV$ uses the potential difference between the anode and cathode to find the kinetic energy of the electrons. Now, only the magnetic field is left to be measured, which can be done by measuring the diameter of the electron's orbital path, as well as the radii of two circular, parallel, coaxial coils, each with radius R and N turns, which produce magnetic field B. Once all of these quantities are measured, the electron's charge/mass constant can be calculated using the equation $\frac{e}{m} = 1.46 \times 10^8 \frac{VR^2}{I^2r^2}$. Once the data is plotted, uncertainties in the slope must be accounted for as well.

Data

The first measured quantity was the diameter of the coils, which was 30.5 cm ± 1mm. This will be used later in the analysis. Here are the data tables collected for the 5 trials:

Trial 1:	D = 10cm ± 3mm				
Current (A)	I^2	Volts (V)	Uncertainty (± V)		
1.044	1.089936	180	1	124.1802	48.16428
1.083	1.172889	195	1	2.966985	4.301623
1.099	1.207801	197	0.5	0.995454	2.817561
1.123	1.261129	206	1	1751.757	8
1.144	1.308736	212	1	13906.59	63.50921
1.162	1.350244	214	0.5		
1.183	1.399489	224	1		
1.202	1.444804	228	1		
1.362	1.855044	283	1		
1.447	2.093809	304	1		
		Avg	0.9		

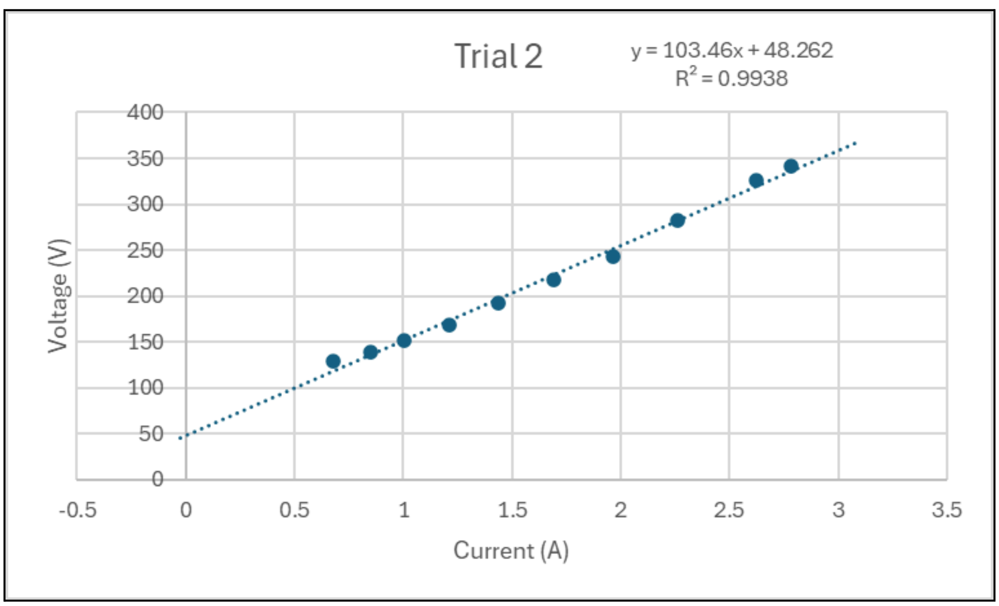
[**Table 1** is the data table for the first trial using an electron ring of radius ~ 5 cm, with the I, I^2, V, and uncertainty values shown, as well as the slope and intercepts for the linear estimations of the I^2 versus V graphs.]



[**Figure 1** is the graph of Voltage versus Current for the first trial, with the accompanying slope value.]

Trial 2:	D = 9cm ± 3mm				
Current (A	I^2	Volts (V)	Uncertainty (± V)		
0.822	0.675684	129	1	103.46109	48.26168
0.923	0.851929	139	1	2.9005026	5.204617
1.001	1.002001	151	0.5	0.9937517	6.458423
1.1	1.21	168.3	0.3	1272.3528	8
1.199	1.437601	192	1	53071.406	333.6899
1.301	1.692601	218	1		
1.401	1.962801	243	1		
1.504	2.262016	283	1		
1.62	2.6244	326	1		
1.669	2.785561	340.9	0.1		
		Avg	0.79		

[**Table 2** is the data table for the second trial using an electron ring of radius ~ 4.5 cm, with the I, I^2, V, and uncertainty values shown, as well as the slope and intercepts for the linear estimations of the I^2 versus V graphs.]



[**Figure 2** is the graph of Voltage versus Current for the second trial, with the accompanying slope value.]

Trial 3:	D = 8cm ± 3mm				
Current (A)	I^2	Volts (V)	Uncertainty (± V)		
0.978	0.956484	132	1	83.77422	43.88229
1.041	1.083681	137	1	2.092558	4.266568
1.135	1.288225	152	1	0.995033	5.00782
1.218	1.483524	168	1	1602.75	8
1.279	1.635841	177	1	40194.18	200.6261
1.322	1.747684	187	1		
1.468	2.155024	219	1		
1.51	2.2801	230	1		
1.686	2.842596	282	1		
1.86	3.4596	340.9	0.1		
		Avg	0.91		

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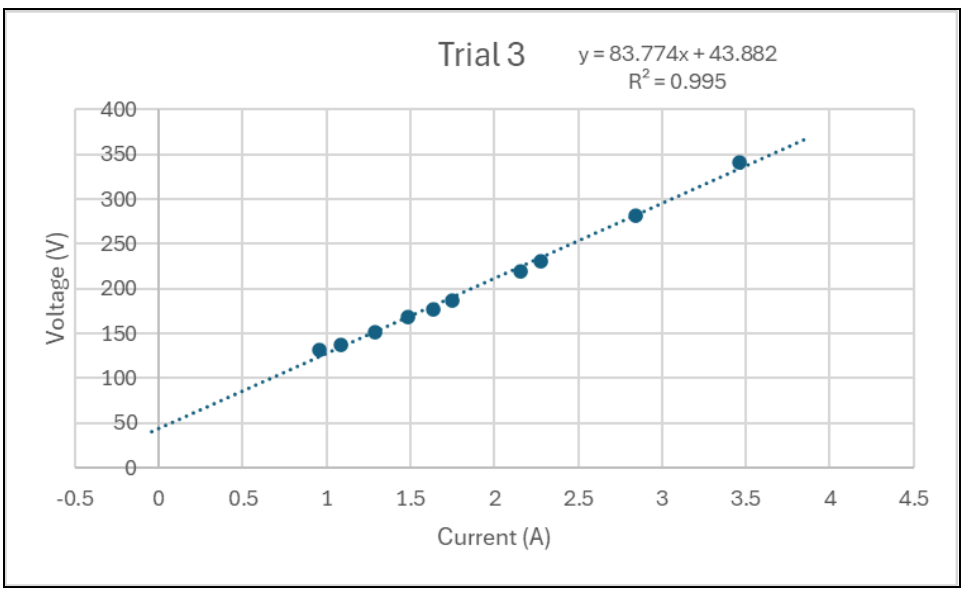
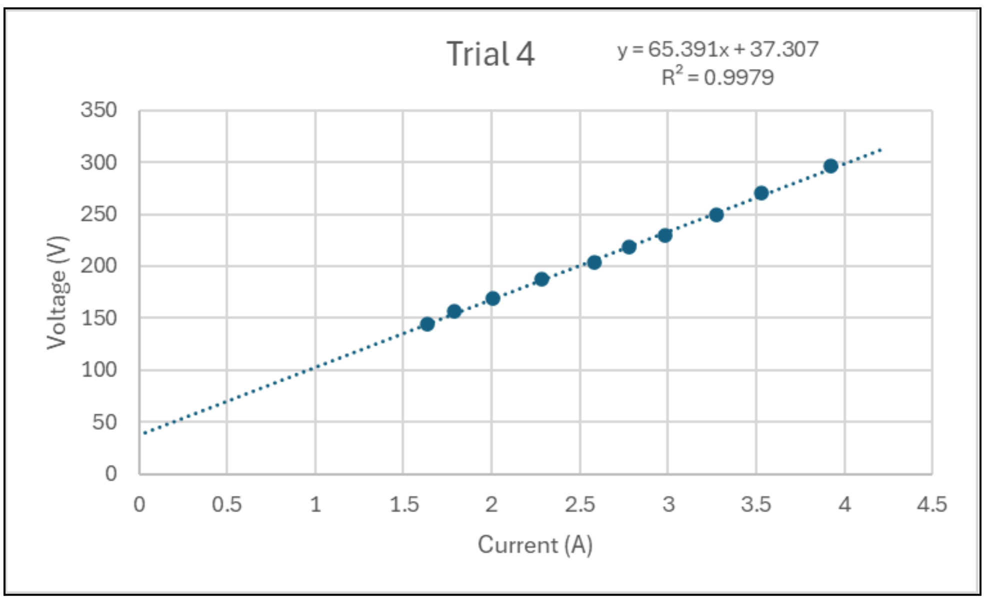
[**Table 3** is the data table for the third trial using an electron ring of radius ~ 4 cm, with the I, I^2, V, and uncertainty values shown, as well as the slope and intercepts for the linear estimations of the I^2 versus V graphs.]



[**Figure 3** is the graph of Voltage versus Current for the third trial, with the accompanying slope value.]

Trial 4:	D = 7cm ± 3mm				
Current (A	I^2	Volts (V)	Uncertainty (± V)		
1.278	1.633284	144	1	65.39107	37.30659
1.336	1.784896	157	0.5	1.062761	2.948664
1.417	2.007889	168.5	0.5	0.997891	2.433241
1.511	2.283121	188	0.5	3785.869	8
1.607	2.582449	204	1	22414.86	47.36531
1.667	2.778889	218	0.5		
1.728	2.985984	229	1		
1.809	3.272481	249	1		
1.879	3.530641	270	1		
1.981	3.924361	297	1		
		Avg	0.8		

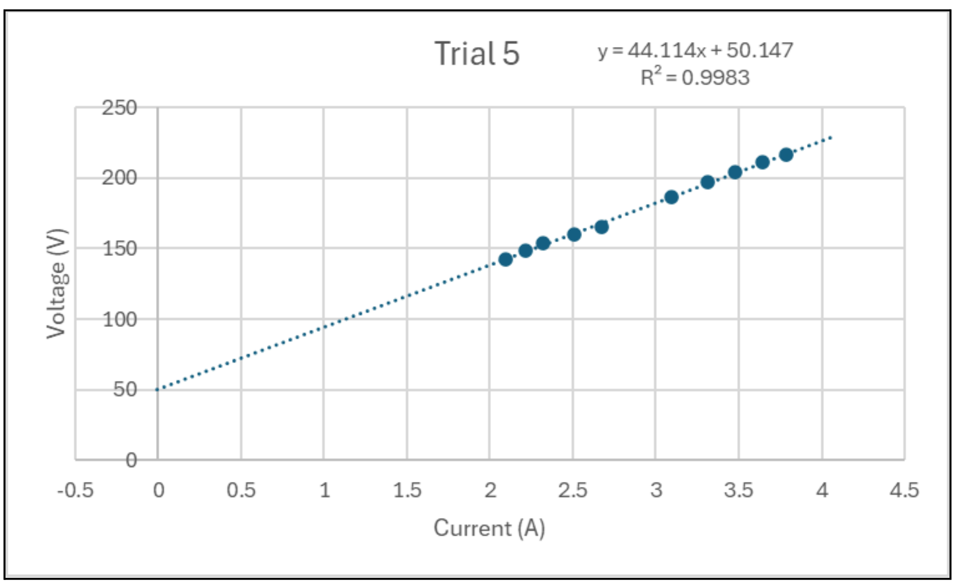
[**Table 4** is the data table for the fourth trial using an electron ring of radius ~ 3.5 cm, with the I, I^2, V, and uncertainty values shown, as well as the slope and intercepts for the linear estimations of the I^2 versus V graphs.]



[**Figure 4** is the graph of Voltage versus Current for the fourth trial, with the accompanying slope value.]

Trial 5:	D = 6cm ± 3mm				
Current (A	I^2	Volts (V)	Uncertainty (± V)		
1.448	2.096704	142.5	0.5	44.1136	50.14729
1.487	2.211169	149	0.5	0.651562	1.93644
1.522	2.316484	153.5	0.5	0.998258	1.225596
1.583	2.505889	160	1	4583.883	8
1.636	2.676496	165.5	0.5	6885.383	12.01668
1.76	3.0976	186.5	0.5		
1.82	3.3124	197	0.5		
1.864	3.474496	204	1		
1.908	3.640464	211	0.5		
1.946	3.786916	217	1		
		Avg	0.65		

[**Table 5** is the data table for the fifth trial using an electron ring of radius ~ 3 cm, with the I, I^2, V, and uncertainty values shown, as well as the slope and intercepts for the linear estimations of the I^2 versus V graphs.]



[**Figure 5** is the graph of Voltage versus Current for the fifth trial, with the accompanying slope value.]

Analysis

The equation, $\frac{e}{m} = 1.46 \times 10^8 \frac{VR^2}{I^2r^2}$, has a slope component of $M = \frac{V}{I^2}$. Rewriting the previous equation, you can equate the slope created by plotting the values of current and voltage measured in-lab with the known values of R and r: $\frac{e}{m} = 1.46 \times 10^8 \frac{R^2}{r^2}M$. For each of the five trials (R = 5, 4.5, 4, 3.5, 3 cm), slopes were calculated to be 124.18, 103.46, 83.77, 65.39, and 44.11. Plugging these values into the new equation gives the following estimations for the ratio of electron charge and mass:

	Found e/m	% Error
Trial 1	1.68657E+11	4.11
Trial 2	1.73478E+11	1.37
Trial 3	1.7778E+11	1.08
Trial 4	1.81249E+11	3.05
Trial 5	1.66427E+11	5.38
Average	1.73518E+11	1.35
Real	1.75888E+11	

The accepted values for the mass and charge of an electron are 9.1093×10^{-31} kilograms and 1.6022×10^{-19} coulombs, with a ratio of 1.7588×10^{11} coulombs per kilogram. Typically, when working with a value of this scale, procuring a measured value that's merely of the same order of magnitude is enough to prove the relative accuracy of an experiment, or of the measurements that were taken. In this case, the found values were delightfully accurate. The error percentages and found values are in the above table, and, notably, the average $\frac{e}{m}$ value had the corresponding error percentage of 1.35%. Considering the nature of the experiment, and the inaccuracy of the human eyes and hands, this low of an error percentage is particularly surprising, proving this experiment to be highly successful.

To determine the uncertainty of the final value, this equation is utilized:

$$\Delta(\frac{e}{m}) = \frac{e}{m} * \sqrt{(4(\frac{\Delta R}{R})^2 + (\frac{\Delta M}{M})^2 + 4(\frac{\Delta r}{r})^2)}$$

Where ΔR is the uncertainty in the radius of the coils (±1mm), ΔM is the uncertainty of the slope (±.81), and Δr is the uncertainty in the radius of the electron ring (±3mm), two of which were all measured during the experiment. The uncertainty in the slope was not measured directly, rather, the uncertainty in the voltage was measured, and was accounted for in the calculation of the slope, producing uncertainty in the slope. These values in quadrature with one another, multiplied by the found value for $\frac{e}{m}$, produce an uncertainty in the value of $\frac{e}{m}$. This value was found to be $\pm 2.097*10^9$ C/Kg, which seems like a significant amount, but when comparing to the found value of $\frac{e}{m}$, the uncertainty is two orders of magnitude smaller than the found value. A better demonstration of $\Delta(\frac{e}{m})$ can be seen when finding the uncertainty in error percentage by using the difference of ± 2.097*10^9 C/Kg on the found value for $\frac{e}{m}$, and finding the difference in error percentages when accounting for ± 2.097*10^9 C/Kg. This changes the average error percentage from 1.35% to 1.35% ± 1.21%, which seems like a much more reasonable value for the uncertainty of electron mass-charge ratio.

This ratio should also be completely independent from the radius of the electron path. As the radius decreases, the value for the slope ($M = \frac{V}{I^2}$) decreases, as different amounts of current and voltage are required for different electron path radii. Fortunately, the collected data does appear to have random error, suggesting this independence to be true. The found values for $\frac{e}{m}$ neither increase nor decrease as the path radius decreases through the trials.

Post-Lab Questions

1.

$$\frac{e}{m} = \frac{2V}{B^2 r^2} \quad , \quad B = \frac{\mu_0 N I}{R}$$

=>

$$\frac{e}{m} = \frac{2V}{\left(\frac{\mu_0 N I}{R}\right)^2 r^2}$$

=>

$$\left(\frac{\mu_0 N I}{R}\right)^2 = \frac{\mu_0^2 N^2 I^2}{R^2}.$$

=>

$$\frac{e}{m} = \frac{2V}{\frac{\mu_0^2 N^2 I^2}{R^2} \cdot r^2}.$$

=>

$$\frac{e}{m} = \frac{2V R^2}{\mu_0^2 N^2 I^2 r^2}.$$

=> $$\frac{2}{\mu_0^2 N^2} = 1.46 \times 10^8$$, where N = 130 turns

=> $$\frac{e}{m} = 1.46 \times 10^8 \cdot \frac{VR^2}{I^2 r^2}$$

2. The lifetime of the excited state of helium is typically on the order of nanoseconds (10^{-9} s). Using the root-mean-square velocity formula for helium at room temperature gives an average thermal velocity:

=> $$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \approx 1.25 \times 10^2 \text{ m/s.}$$

For a time duration on the order of 10^{-9} s, a helium atom would move approximately 0.125 micrometers before emitting a photon. The visible path created by an electron in a magnetic field is primarily due to the diffusion of emitted photons from excited helium atoms. The displacement of helium atoms (0.125 µm) during the excitation-emission process is much smaller than the diffusion length of photons in the gas, as well as the spatial resolution of typical detection methods (e.g., the human eye or imaging equipment). Therefore, the motion of helium atoms does not significantly affect the width of the visible path. The width is dominated by the distribution of emitted light and any spreading of the electron beam itself.

3. After an electron excites a helium atom, it loses some of its kinetic energy, which slows it down and can slightly alter its direction due to momentum conservation. In a magnetic field, the electron continues along a helical trajectory, but with a smaller radius because the reduced velocity decreases the centrifugal force balancing the magnetic force. Over time, as the electron undergoes more collisions with helium atoms, it gradually loses more energy, its path becomes more randomized, and it eventually slows to thermal velocities if it remains in the system.

4. The centripetal force is provided by the Lorentz force in a magnetic field:

$$F_c = F_L, \quad \frac{mv^2}{r} = qvB,$$

Rearranged for *v*:

$$\Rightarrow \qquad v = \frac{qBr}{m}$$

Since the magnetic field in this experiment changes depending on the inputted current and voltage, the speed of the electron varies as well, so for the time being, the B will remain a variable.

$$\Rightarrow \qquad v = 8.79 \times 10^9 \cdot B\,\mathrm{m/s}$$

The time T to go around the circle is the circumference of the path divided by the speed:

$$T = \frac{2\pi r}{v}$$

Substituting *v*:

$$\Rightarrow \qquad T = \frac{2\pi r}{\frac{qBr}{m}} = \frac{2\pi m}{qB}$$

$$\Rightarrow \qquad T = \frac{3.57 \times 10^{-11}}{B}\,\mathrm{s}$$

For larger magnetic fields, the time around the circumference decreases proportionally. With an average current of 1.5 A running through the coils, the magnetic field produced is roughly 5.75×10^{-4} T, or 5.75 G. For the sake of these questions, this value for B will be used in calculations. Thus, electron speed v is ~5×10^6 m/s, and the period T is 6.2×10^{-8} seconds.

5. Substituting v into the classical equation for angular momentum gives:

$$L = m \cdot \frac{qBr}{m} \cdot r = qBr^2$$

=> $$L = 4.00 \times 10^{-21} \cdot B \, \mathrm{kg \ m^2/s}$$

According to quantum mechanics, the angular momentum of an electron in a bound state is quantized, and its magnitude is given by:

$$L = n\hbar$$

Compare L to $\hbar$:

$$\frac{L}{\hbar} = \frac{4.00 \times 10^{-21} \cdot B}{1.055 \times 10^{-34}} = 3.79 \times 10^{13} \cdot B$$

The angular momentum of this free electron is not quantized because the electron is not in a bound quantum state, like in an atom. Instead, it follows classical mechanics. The magnitude of L is enormously larger than $\hbar$, with $L/\hbar$ proportional to B. For any realistic B field, L remains much greater than $\hbar$.

6. De Broglie wavelength for a particle is given by:

$$\lambda = \frac{h}{p}$$

Substituting for classical momentum and the previous velocity value gives:

$$\lambda = \frac{h}{qBr}$$

=> $$\lambda = 1.44028 \times 10^{-10} \, \mathrm{m}$$

With a path length of $2\pi(.05) = .314$ m, the de Broglie wavelength is roughly 9 orders of magnitude smaller than the path length, highlighting the classical nature of the electron's motion in this experiment.

7. The formula for the magnetic field at the center of a Helmholtz coil is given by:

$$B = \frac{\mu_0 N I}{R}\left(\frac{8}{\sqrt{125}}\right)$$

Where N = 130 turns, R = 30.5cm, and the average lowest and highest currents run through during the experiment were 1.114 A and 1.781 A. Thus, the magnetic field in this experiment ranges from roughly 4.14 G to 5.89 G. The Earth's magnetic field is typically between 0.25 and 0.65 Gauss, so the magnetic fields produced by the coils are roughly 6.4 to 9 times stronger than the Earth's magnetic field.

Conclusion

This experiment was remarkably successful. The Helmholtz coil proved efficient in measuring the ratio of the electron charge to mass yielded the result 1.73518×10^{11} C/Kg with an error rate of 1.35% ± 1.21% and an uncertainty of $\pm 2.097 * 10^{9}$ C/Kg. This level of precision suggests that the experimental setup and methodology were effective in obtaining an accurate measurement. The relatively small error margin reflects the reliability of the equipment and the accuracy of the measurements, while the associated uncertainty accounts for potential minor inaccuracies in the system, such as variations in the magnetic field strength or electron trajectory. Overall, the results were consistent with theoretical predictions, and the small error rate demonstrates the successful application of the Helmholtz coil for this measurement.