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Realism, functions, and the a priori: Ernst Cassirer's philosophy of science

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ABSTRACT

This paper presents the main ideas of Cassirer's general philosophy of science, focusing on the two aspects of his thought that—in addition to being the most central ideas in his philosophy of science—have received the most attention from contemporary philosophers of science: his theory of the a priori aspects of physical theory, and his relation to scientific realism.

Ernst Cassirer, the Neo-Kantian trained philosopher whose wide-ranging work spanned the first four decades of the twentieth century, was one of the most prominent and respected philosophers of science of his time. Easily the most subtle and mathematically well-informed of the Neo-Kantians, he was among the vanguard of early twentieth century philosophers seeking to understand the philosophical significance of the revolutionary advances made in logic, mathematics, and physics. Not only did Cassirer write some of the earliest philosophical works on general relativity and quantum mechanics,[1] but he was one of the first German academic philosophers to give serious attention to Russell's logicism and the new logic,[2] Dedekind's foundations of arithmetic, and to Hilbert's axiomatic foundation of geometry.[3] Cassirer also wrote extensively on some of the perennial issues in general philosophy of science: realism, confirmation, theory change, the nature of experimentation, the a priori elements in scientific theories, and the application of mathematics in physical science. Cassirer's commitment to a philosophy of science that engaged with cutting edge science ran deep and was widely known. For example, as a letter from Hans Reichenbach makes clear, Cassirer was the only philosopher to sign onto a petition, composed by Reichenbach in 1931 and co-signed by Hilbert and Einstein, petitioning the German government to create a chair in the philosophy of science.[4]

It is not surprising, then, that Cassirer's work has been studied extensively by historians of philosophy. But several prominent philosophers of science have also recently turned their attention to Cassirer, finding in his writings philosophical positions that only now many years later are receiving sustained philosophical interest. For example, many defenders of "structural realism" within the philosophy of science have explicitly pointed to Cassirer's writings as an historical anticipation of their own theories,[5] and Michael

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[1] Cassirer *Einstein's Theory of Relativity* [*ETR*] (1923 [1921]), *Determinism and Indeterminism in Modern Physics* (1954 [1936]). Throughout this paper, I will adopt the following practices. If there is a translation listed in the bibliography of a work not written in English, I will quote from the listed translation (except for occasional corrections that I make silently). Page citations are from the listed translations unless otherwise noted. Translations from works not in English for which there is no English translation in the bibliography are all my own.

[2] Cassirer (1907).

[3] Cassirer, *Substance and Function* [*SF*] (1923 [1910]), Chap. 2–3.

[4] Letter from Reichenbach to Cassirer, 5 June 1931, HR—025—11—04; 15 June 1931, HR—025—11— 03. These letters are reproduced in the CD-Rom accompanying Cassirer (2009).

[5] See French (2001), pp. 2–7, 14; French and Ladyman (2003), pp. 38–41; Gower (2000), pp. 87–95; Massimi (2010). Structural Realism is often split into "epistemic" and "ontic" versions. French and Ladyman see in Cassirer an anticipation of ontic structural realism; Gower and Massimi see in him an anticipation of epistemic structural realism.

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Friedman has identified Cassirer as an inspiration for the philosophy of science defended in his book *Dynamics of Reason* (2001, pp. 65–8; 2005).

Despite this attention, there is still disagreement among philosophers over the interpretation of many of the main ideas of Cassirer's philosophy of science. For example, one of the most striking and suggestive features of his philosophy is its conception of the a priori. Some interpreters (Richardson, 1998, chap. 5; Ryckman, 2005, chap. 2) have claimed that Cassirer's theory of the a priori is an anticipation (and, indeed, the historical source) of the theory of the relativized, but constitutive a priori later articulated by Reichenbach (who was, after all, Cassirer's student in Berlin). According to Michael Friedman, however, the a priori elements that Cassirer claims to find in science are absolute, but not relative, and are merely regulative, not constitutive (Friedman, 2000, 115ff). Alan Richardson, on the other hand, argues that Cassirer, in trying (unsuccessfully) to integrate a theory of the constitutive, relative a priori with a theory of the regulative, absolute a priori, succeeds only in presenting a "plurality of inconsistent accounts" (Richardson, 1998, p. 136). Last, Thomas Ryckman argues that for Cassirer (in his book *Einstein's Theory of Relativity*) the principle of general covariance is both constitutive and regulative (2005, p. 46)—thus agreeing with Richardson (against Friedman) that there are constitutive a priori principles in Cassirer's theory, while disagreeing with Richardson that this amounts to any kind of tension in Cassirer's thinking. Similar issues arise in discussions of Cassirer's relation to realism. As French, Ladyman, and Massimi all recognize,[6] there is a clear (though hard to articulate) tension between a reading of Cassirer's philosophy that brings him close to contemporary structural realism, and Cassirer's own rejection of realism in favor of idealism.

The goal of this paper is to present the main ideas of Cassirer's general philosophy of science. In particular, I will present the contours of the two aspects of his thought that—in addition to being among the most central ideas in his philosophy of science—have received the most attention from contemporary philosophers of science: his theory of the a priori, and his relation to scientific realism. I will argue (against Friedman) that Cassirer's theory assigns both constitutive and regulative, relative and absolute, roles for a priori representations. These a priori representations help explain the possibility of scientific objectivity and thus also objective reference. This theory of objectivity, and the need to secure it with a priori elements, flows out of the distinction between substance and function that is the main theme of Cassirer (1923 [1910]). In Section 1 of this paper I describe that distinction. In Section 2, I describe Cassirer's two part theory of the a priori and argue—contrary to Richardson—that Cassirer's second, absolute, part of the theory of the a priori is not inconsistent with the first, relativized, part, but *necessitated* by it.

The role of the theory of the a priori in Cassirer's philosophy of science is in explaining physical objectivity. In particular, objectivity is maintained in science even as physical theories change because the structure of science remains the same even as the fundamental ontologies of theories are replaced by successor theories. This naturally raises the philosophical question whether Cassirer was a "realist" at least about the structural features of physical theories, as are contemporary structural realists. In the last section of this paper, I critically evaluate this claim, distinguishing the senses in which Cassirer's philosophy of science is (and is not) a form of "realism."

Cassirer's philosophy of science was first presented in his 1910 *SF*. Starting with the publication of the first volume of the *Philosophy of Symbolic Forms* in 1923, these ideas were broadened in two ways. First, Cassirer came to appreciate that the various special sciences have their own distinct methodologies that need to be investigated individually, whereas the philosophy of science presented in *SF* is almost exclusively concerned with physics. In particular, in *The Logic of the Cultural Sciences* (2000 [1942]), Cassirer argues that cultural sciences such as history can attain objectivity despite not relying on the experimental and mathematical methods characteristic of physics. Second, Cassirer came to believe that the kind of human activities investigated by cultural sciences such as linguistics, anthropology, history, and comparative religion each make possible a distinct kind of objectivity. These various ways of constituting objectivity Cassirer calls "symbolic forms," and he argues that myth, art, and language are symbolic forms alongside the symbolic form of knowledge (which is the form exhibited most perfectly by science). The "functionalist" account of knowledge that I describe in Section 1 and the account of a priori elements that I describe in Section 2 are then carried over to these other symbolic forms, though of course in significantly modified ways. Unfortunately, given the confines of a paper, I cannot do any more than mention the wider vistas of Cassirer's philosophy of symbolic forms (though I believe that understanding Cassirer's philosophy of (mathematical and physical) science is an ideal way of working one's way into this larger system).

1. "Substanzbegriff" and "Funktionsbegriff"

The animating idea in Cassirer's philosophy of science—indeed, in his theoretical philosophy in general—is the contrast between substance-concept [Substanzbegriff] and function-concept [Funktionsbegriff], a contrast that provides the title for his first systematic book-length work in the philosophy of science.[7] Cassirer means this contrast to cover a number of different interrelated epistemic, logical, and metaphysical contrasts. Of these many uses, the most fundamental use that Cassirer makes of "Substanzbegriff" and "Funktionsbegriff" is *epistemological* and *Kantian*: it contrasts philosophical views that overlook the epistemic preconditions of various kinds of knowledge, with those that recognize the "functions" [or "preconditions"] that make certain kinds of knowledge possible.[8] To oppose the point of view of "Substanzbegriff," then, is to oppose various forms of epistemological atomism: the view that certain kinds of knowledge (be they scientific concepts, experiences, or measurements) could be acquired all by themselves, without any other epistemic conditions.

Cassirer's paradigm example of "Substanzbegriff," is an atomistic theory of concept acquisition (thus the phrase "substance-concept" to describe the approach he rejects). But the fundamental kind of atomism that he wants to oppose in the philosophy of physical science concerns measurement.[9] After analyzing a series of

[6] French and Ladyman (2003), p. 38. French (2001), p. 14. Massimi (2010).

[7] The interpretation in this section is defended and presented in greater detail in Heis (2014).

[8] Cassirer uses the term "function" ["*Funktion*"] in many ways, one of which is to refer to what mathematicians call "functions," especially when that function orders a series (*Reihe*), as the successor function orders the series of natural numbers. In §II of Heis (2014), I argue that the epistemic notion of function is primary, since it is only by recognizing the necessity of epistemic preconditions that the philosophical importance of mathematical functions becomes clear. As I will explain shortly, mathematical functions order contents into the kind of "unifed" whole that the functional theory of objectivity claims is a precondition of knowledge.

[9] See *SF*, 267: "If we take as given the whole of experience, as it is represented in any definite stage of knowledge, the whole is never a mere aggregate of perceptual data [Wahrnehmungsdaten], but is articulated and brought to unity according to definite theoretical points of view. It has already been shown from all sides that, without such points of view, no single assertion concerning facts, in particular no single concrete *measurement*, would be possible."

cases of measurement in physical science, Cassirer argues that these most basic results of scientific experimentation presuppose not only concepts and laws of pure mathematics, but also laws of nature and natural scientific concepts. Here are some of the cases Cassirer discusses: assigning a real number to a temperature by measuring a volume of mercury presupposes laws of geometry and the law relating temperature to the expansion of volume of mercury (*SF*, pp. 142–3); when Regnault measured the volume of a gas in his tests of Boyle's law, he used an instrument the design of which presupposes the "abstract principles of general mechanics and celestial mechanics" (p. 143; cf. Duhem, 1977 [1906], pp. 145–7); the measurement of time requires the identification of a unit, the choice of which presupposes the law of conservation of energy, or the principle of inertia (p. 145; cf. Poincaré, 2001 [1905], pp. 210–22); the determination of the position of a body requires constructing a Langean "inertial system" (p. 182); Ampère's measurement of the intensity of an electric current required a galvanometer, whose operations presuppose various physical laws (p. 280).

The failure of epistemological atomism about measurement, on Cassirer's view, generalizes to all of our scientific knowledge. Since even these most elementary experimental results presuppose a system of mathematical and theoretical concepts and even certain theoretical laws, Cassirer concludes that no scientific concept or principle could be acquired all on its own. Moreover, as Cassirer learned from Duhem, no single empirical statement of natural science can be confirmed atomistically.[10]

Cassirer believes that these non-atomistic conceptions of measurement and empirical confirmation require a distinctive, "functional" theory of objectivity. Since it is impossible to know any fact about a physical object outside of a system of concepts and principles, the objectivity of a particular scientific claim cannot be determined except by answering the prior question of the objectivity of a total theory. This motivates for Cassirer the view that objectivity is primarily a property of a total theory (*SF*, p. 284). Cassirer explicates the notion of objectivity of a total theory using two further notions: *unity* and *permanence*.[11] "Unity" for Cassirer includes not only logical coherence (both among the various sentences in a theory and with the measurements that provide its empirical support), but also a certain systematic form. This form is provided by a theory's laws, which logically structure the total theory.

The significance of "permanence" or "constancy" for Cassirer's conception of objectivity is difficult to overstate. As Cassirer puts it, "'objective' ['Gegenständlich'], in the critical sense of the word, means that which is 'constant' ['beständig'] in our cognition" (1922 [1907], p. 639; cf. *SF*, p. 273). This permanence operates on multiple levels. In the simplest case, the properties of an individual object (say, its spatial location with respect to other objects in its environment) are objective, while the various appearances of that object to different observers are subjective, since the appearance of an unchanging object can alter as an observer moves about with respect to it. The permanence of the observed object makes itself manifest in the constant law that would predict its appearance as a function of the position of the observer. Here the "law" is objective, while the appearances are not (*SF*, p. 261). Particular facts about individual objects can then be united into a total theory by means of higher level physical laws that describe the behavior of physical objects over time. Though these higher laws are permanent in their own sphere (the laws giving, say, the forces acting on a moving body are permanent even as the body constantly changes its position and velocity), still no physical theory will be the final word. The objectivity of the revision or replacement of the highest laws in a physical theory is a particularly pressing issue, since (following Duhem) Cassirer allows that there could be two distinct theories equally compatible with the empirical evidence. For this reason, Cassirer argues that there are supreme "laws" or norms that guide the formation and selection of new physical theories. Science can remain objective, then, even as theories come and go, since there are permanent laws shaping this process of theory formation and selection.[12]

Cassirer builds on this "functional" theory of objectivity to give a functional theory of knowledge and truth: a content is knowledge if it plays the sort of role within a system of contents that would make objectivity possible (1957 [1929], pp. 4–5). The concept <object>[13] is understood in terms of <objectivity> and <knowledge>, as that which is known by fully objective knowledge (*SF*, p. 314). Cassirer emphasizes that the conceptual ordering among the core concepts <objectivity>, <knowledge>, <truth>, and <object> on the functional theory is opposed to the traditional ordering in the "substance" theory of knowledge. According to *the "substance" or "copy" theory of knowledge,* the primitive concept is the metaphysical concept of an <object>.[14] <Truth> is explained in terms of the concept <object>, where a representation[15] is true if it faithfully mirrors the properties of objects. Knowledge is then a certain species of *true* representation—a "copy" of objects, as it were. Last, objective knowledge is a kind of knowledge whose special status is explained in terms of the peculiarities of its object: objective knowledge is about "external" objects, not the inner states of a subject.

2. Theory change and the a priori

The fundamental concept of the functional theory of knowledge is objectivity, and objectivity is understood in terms of the "unity" and "permanence" of physical theory. On Cassirer's view, there is a system of a priori concepts and principles that are conditions of the possibility of this unity and permanence. In this section, I will outline Cassirer's theory of the a priori and show why Cassirer considered it to be a consequence of the polemics against

[10] Cassirer alludes to Duhem (1977 [1906]) frequently: see *SF* 143–7, 280; (1957 [1929]), p. 421; cf. 416, 461.

In arguing that no experimental result, scientific concept or scientific principle could be acquired atomistically, Cassirer is drawing on the opposition to the "given" first put forward by his teacher Hermann Cohen, under whom he studied at Marburg. Cassirer's fellow Marburg Neo-Kantian Paul Natorp well expressed this opposition to the given in Natorp (1912), 200ff, where he famously contrasts the mistaken view that the object is "given" [gegeben] to knowledge with the preferred Marburg view that the object is "given as a task" [aufgegeben]. That is, the ability of a thinker to have even the most elementary experiences depends on the subject's possessing a system of concepts and judgments—a system that is slowly acquired and refined as science develops (but not before).

[11] See *SF*, p. 322.

[12] *SF*, p. 287: "[Thought's] spontaneity is not unlimited and unrestrained; it is connected, although not with the individual perception, with the *system* of perceptions in their order and connection. […] [I]n so far as our intellectual construction is extended and takes up new elements into itself, it appears that it does not proceed according to caprice, but follows a certain law of progress. This law is the ultimate criterion of 'objectivity.'"

[13] I refer to concepts in angle brackets, words in double quotes.

[14] See (1955 [1923]), p. 77. On the "copy theory of knowledge" and the "functional" theory, see also *ETR*, pp. 391ff.

Cassirer's endorsement of a "functional" theory of objectivity draws heavily on the conception of objectivity and objecthood put forward by his Marburg School teachers, who argue repeatedly that the objectivity of our knowledge depends on our representing things as standing in lawlike relations. See (Natorp, 1981 [1887]).

[15] The word "representation" often is used by philosophers to denote a mental "copy" of a mind-independent object. Cassirer notes this standard usage and objects that this notion of representation makes sense only in the mistaken point of view of *Substanzbegriff* (*SF*, pp. 282–5). Cassirer therefore prefers to speak in *SF* of knowledge as a kind of "content"—and later after the development of the Philosophy of Symbolics Forms as a kind of "symbol." (Thanks to Sebastian Luft for prompting me to clarify Cassirer's view of "representation.")

epistemological atomism (and the "substance theory of knowledge") I've just described.

I noted above that the interpretation of Cassirer's conception of the a priori has been a matter of dispute. Some have argued that it anticipates liberalized or relativized versions of the theory of a priori, such as the theory defended by Reichenbach (1965 [1920]) and revived more recently in Friedman (2001). Others, most notably Friedman himself, have denied that Cassirer's theory includes relative, but constitutive a priori principles.

I believe that there is ample textual evidence that Cassirer does in fact maintain that there are relativized, constitutive a priori principles:

> That we in [science] find only a *relative* stopping point, that we therefore have to treat the *categories*, under which we consider the historical process itself, themselves as variable and capable of change, is obviously correct: but this kind of relativity does not indicate the limits, but rather the particular life of cognition. (1922 [1906], p. 16)

At *ETR* (p. 415), he gives examples of these a priori but nevertheless changeable principles.

> That a step is thereby taken beyond Kant is incontestable; for he shaped his "Analogies of Experience" essentially on the three fundamental Newtonian laws; the law of inertia, the law of the proportionality of force and acceleration, and the law of the equality of action and reaction. But in this very advance the doctrine that it is the "rule of the understanding", that forms the pattern of all our temporal and spatial determinations, is verified anew. In the special theory of relativity, the principle of the constancy of the velocity of light serves as such a rule; in the general theory of relativity this principle is replaced by the more inclusive doctrine that all Gaussian coordinate systems are of equal value for the formulation of the universal natural laws. It is obvious that we are not concerned here with the expression of an empirically observed fact, but with a principle which the understanding uses hypothetically as a norm of investigation in the interpretation of experience.

Though these principles are a priori (they are not "expressions of an empirically observed fact"), Cassirer nevertheless explicitly denies that they are apodictic, certain, or self-evident, and he denies that we have any conclusive reason to think that even the constitutive principles of the general theory of relativity[16] will be constitutive in all of our future physical theories (*SF*, p. 268). These principles (and the concepts they contain) are therefore relative but constitutive a priori.

However, this relativization of Kantian categories seems *prima facie* to be radically at odds with Cassirer's theory of objectivity. As I emphasized above, Cassirer, associating objectivity with permanence, thinks that the objectivity of experience is grounded in physical laws, since—though physical objects may change their properties over time—the laws describing these changes are permanent. Granted: these relativized constitutive a priori principles play an essential role in grounding the objectivity of experience, since lower level laws that describe the behavior of objects (like the laws of planetary motion) are made possible by constitutive principles (like Newton's principle of inertia).[17] However, if the very highest laws of our physical theories are not permanent but only relative, then the very possibility of physical objectivity seems threatened.

For this reason, it is of the utmost importance to Cassirer to identify elements in physical theories that *do* remain the same even as the fundamental concepts and principles (the relative, constitutive a priori elements) evolve. In a series of passages in *SF* (pp. 268–70; 321–2), Cassirer identifies seven such permanent elements:

1. Mathematical concepts and principles (though which mathematical concepts and principles get employed in a given theory may change).
2. Some questions posed by the older theory (which get answered in the new theory). For example, the question Why do planets travel in elliptical orbits? could be posed in Kepler's day and only later answered by Newton.
3. Some empirical facts (though they are interpreted in a new way in the new theory). For example, Tycho's observations of the position of Mars can be carried over from the Ptolemaic to the Newtonian system.
4. "Ultimate invariant" concepts, like <space>, <time>, <magnitude>, <cause> (though these concepts may be understood in new ways in the new theory).
5. The mathematical form of the old theory (which is maintained at least as a limiting case in the new theory).[18] For example, Newton's laws become approximately true in the small in the theory of relativity.
6. Some principles of theory selection.
7. The principle of the "unity of nature."

Numbers 2, 3, and 5 are empirical; 1, 4, 6, and 7 are then a priori, though in an absolute, not relative sense.

Cassirer describes these a priori concepts and principles as elements that remain "invariant" throughout the entire history of science.

> The goal of critical analysis would be reached, if we succeeded in isolating in this way the ultimate common element of all possible forms of scientific experience; *i.e.*, if we succeeded in conceptually defining these moments, which persist in the advance from theory to theory because they are the conditions of any theory. At no given stage of knowledge can this goal be perfectly achieved; nevertheless, it remains as a demand, and prescribes a fixed direction to the continuous unfolding and evolution of the systems of experience.
>
> From this point of view, the strictly limited meaning of the "a priori" is clearly evident. Only those ultimate logical invariants can be called a priori, which lie at the basis of any determination of a connection according to natural law. A cognition is called a priori not in any sense as if it were *prior* to experience, but

[16] In addition to the principle of general covariance (*ETR*, p. 415, 428), Cassirer also claims that Riemannian differential geometry (p. 433) and the equivalence of inertial and gravitational mass (p. 428) are a priori principles in GTR.

[17] Cassirer also suggests that without these relative but constitutive principles "no single assertion concerning facts, in particular no single concrete *measurement*, would be possible" (*SF*, p. 267).

[18] See *ETR*, p. 379. That this mathematical form is typically only retained as a limiting case: *SF*, pp. 259, 266; *ETR*, pp. 365, 366, 379.

Michael Friedman has strongly emphasized this fifth element in Cassirer's picture of theory change. Contrasting Cassirer's conception of theory change with Thomas Kuhn's, he writes:

> [W]hen Kuhn famously considers the relationship between relativistic and Newtonian mechanics, he rejects the notion of a fundamental continuity between the two theories on the grounds that the "physical referents" of their terms are essentially different, and he nowhere considers the contrasting idea, characteristic of Cassirer's work, that continuity of purely mathematical structures is sufficient. (2008, p. 241)

Friedman is correct to emphasize for Cassirer the role of constancy of mathematical form in explaining the objectivity of theory change. But given the place of items 2–4 and 6–7 in Cassirer's model, it overstates the case to say that continuity of mathematical form alone is *sufficient*.

> because and in so far as it is contained as a necessary premise in every valid judgment concerning facts. (*SF*, p. 269)

This second, absolute, sense of "priori" is then distinct from—and supplementary to—the relativized constitutive a priori elements we mentioned earlier.

As Cassirer makes clear throughout his writings, the a priori elements in this second sense are a varied lot, including: the concepts and propositions of mathematics[19]; <space> and <time>; Kantian categories like <magnitude>, <number>, <cause>, and "functional correlation"[20]; and principles of theory selection—like principles of simplicity and generality[21]—which Kant would have considered to be principles of reason. These a priori elements can be collected together in the following table:

Concepts	Regulative/ Constitutive?	Absolute/ Relative?	Examples
Mathematical Concepts	Constitutive	Absolute	<Riemannian manifold>, <3-d Euclidean space>
"Categories" of a given theory	Constitutive	Relative	<ether>, <gravitational mass>
"Ultimate Invariant" concepts	Regulative	Absolute	<space>, <time>, <magnitude>, <cause>
Principles	**Regulative/ Constitutive?**	**Absolute/ Relative?**	**Examples**
Mathematical Propositions	Constitutive	Absolute	Unique decomposability of prime numbers, propositions of Euclidean geometry
Principles of a given theory	Constitutive	Relative	Newton's three laws; Constancy of the speed of light (STR)
Principles of theory selection	Regulative	Absolute	Theories should be as simple, general, and fruitful as possible.
Principle of the "unity of nature."[22]	Regulative	Absolute	Any physical phenomenon can be explained by a simple, general, unitary, and empirically adequate physical theory (or set of coherent theories).

The function of these a priori elements is to make possible the objectivity of science in the face of constant revisions in theories. As Cassirer succinctly puts it: "The system of these unchanging elements," that is, the absolutely a priori concepts and principles, "constitutes the pattern [Muster] of objectivity in general" (*SF*, p. 277; cf. *SF*, p. 309). A priori principles function like Kantian categories: they provide the conditions of the possibility (and thereby also the *limits*) of empirical meaning. If there were only relative a priori concepts and principles, then there could then be no common stock of meanings—no shared concepts—between two scientists at different times (or worse yet: between two scientists who disagree on a new theory). But this would undermine the *objectivity* of theory change, turning the history of science into a sequence of logically and semantically isolated belief systems. According to the functional theory of knowledge, this failure of the objectivity of theory change would undermine the "unity" of knowledge and thereby even the *objective* reference of our scientific knowledge: the fact that our science is about objects at all (*SF*, p. 308). But we know that the history of science is a progressive history, moving asymptotically toward the truth about the natural world. What's more, a philosophy of science with only relative a priori principles would threaten the fundamental truth that various scientists at various stages in the history of science are all trying to understand one and the same natural world.[23] Given the undeniable changeability of the relative, constitutive categories, then, a historically progressive science is possible only if there are some a priori cognitions that remain invariant through all stages of the history of science.[24]

Cassirer thus introduced the theory of the invariant a priori to address concerns about the objectivity of theory changes. These concerns would not arise if the epistemological atomism that forms the basis of the substance theory of knowledge were true. For on such a view, there could be elementary facts of experience (for example, primitive measurements) that are immediately given and independent of interpretation. These elementary facts would be theory-neutral and could thus form a common, shared basis from which our physical theories could be derived in an objective way. The invariance of these elementary experiences would take the place of Cassirer's invariant a priori. However, if we follow Cassirer in rejecting the possibility of this common stock of experiences, the only alternative is to posit a privileged set of constant cognitions—the a priori—to secure the objectivity of science. In this sense, Cassirer's theory of the a priori ultimately emerges from his core opposition between *Substanzbegriff* and *Funktionsbegriff*.

Cassirer's theory of the a priori is sophisticated enough to have the resources to answer some of the well known objections to theories like it. In the opening of the paper, I noted Alan Richardson's claim that the relative and absolute elements of Cassirer's theory of the a priori are inconsistent with each other. I have claimed that—instead of being inconsistent—the second, absolute sense of the a priori is in fact necessitated by the first. Moreover, the a priori/empirical distinction was famously attacked by Quine, employing as an essential part of his argument Duhemian confirmation holism. From a Quinean perspective, it is then historically surprising that Cassirer's own theory of the a priori was in fact partially motivated by Duhemian arguments. But Cassirer would not grant to Quine an unconstrained holism: at least some elements of our theories must remain invariant, lest our theories cease to be objective and capable of objective reference. In fact, Cassirer would simply deny that *every* element of our current best science could be revised. In particular, Cassirer would deny that the propositions of pure mathematics are confirmed or disconfirmed, even indirectly, by their role in physical theory[25]; he would insist that, though our *beliefs* about space, time and the highest causal laws may shift, the *concepts* <space>, <time>, and <cause> themselves remain invariant; and he would lastly insist that the highest principles employed in forming and evaluating theories must themselves remain invariant.[26]

Cassirer's theory can also withstand the most famous and influential line of criticism leveled against it. In his famous review

[19] SF, pp. 257, 322–3.

[20] *SF*, pp. 269, 309, 321; *ETR*, pp. 394, 420, 445.

[21] *SF*, p. 260; *ETR*, pp. 365, 374.

[22] *SF*, pp. 248, 304, 315, 321-1; *ETR*, p. 381; Letter to Schlick 23 Oct 1920 (2009, pp. 50–1).

[23] *SF*, pp. 321–2: 'Going back to such supreme guiding principles [i.e., the '*form* of experience' that persists in all stages of the asymptotic progression toward the fully empirically adequate theory] insures an inner homogeneity of empirical knowledge, by virtue of which all its various phases are combined in the expression of *one* object. The 'object' is thus exactly as true and as necessary as the logical unity of empirical knowledge;—but also no truer or more necessary…We need, not the objectivity of absolute things, but rather the objective determinateness of the *method of experience*.'

[24] See Cassirer (1922 [1906]), p. 16: "The concept of the *history of science* itself already contains in itself the thought of the *maintenance of a general logical structure* in the entire sequence of special conceptual systems."

[25] On this point, see Heis (2011), §5.

[26] Indeed, Cassirer points out that, given Duhemian confirmation holism, there needs to be a principle that determines which parts of a theory should be revised in the face of empirical disconfirmation. Cassirer articulates what it is essentially Quine's maxim of minimum mutilation (*SF*, p. 267), calling it a "principle of methodological advance"—that is, a regulative, absolute a priori principle of theory formation and selection.

of *ETR*, Schlick argued that Cassirer's a priori principles—lacking the certainty and self-evidence characteristic of Kant's own notion of the a priori—are better understood simply as conventions, and thus as *analytic* a priori principles (1979 [1921], pp. 323–4). Indeed, both Schlick and Reichenbach worried that the Kantianism in Cassirer's philosophy of physics had been so weakened as only to differ verbally from their own empiricism.[27]

Since Cassirer's theory of the a priori includes both relative and absolute elements, a defense of Cassirer against Schlick has to show that both his relative a priori, and his absolute a priori cognitions cannot be relabeled as conventions. Let's take the latter first. (The argument for why the relative a priori principles cannot be conventions will turn out to depend on the non-conventionality of the absolute a priori, and so I'll postpone the relativized a priori until the next section of the paper.) To start with, the fact that these elements are invariant throughout the history of science already makes trouble for a conventionalist reading of them. It makes no sense to talk of the very same conventions being laid down throughout the history of science. Furthermore, the conventionality of these a priori elements would fit very poorly with their being "regulative," and not "constitutive." As Cassirer put it in *SF* p. 269 (quoted above), "at no given stage of knowledge can this goal [of identifying the ultimate invariants] be perfectly achieved," but instead these absolute a priori elements remain "as a demand" that guide the "continuous unfolding and evolution" of our physical theories. There are three "regulative" features of Cassirer's theory that are relevant here. First, the principles of theory selection require (among other things) that a physical theory cover a wide range of phenomena using the simplest resources possible—an ideal that our current theories obviously do not fulfill completely and that will not be fulfilled until the postulated final theory. Second, though concepts like <space> and <time> are a priori in this absolute sense, it is still the case that we can be radically mistaken about their nature. Although these concepts remained invariant during the switch from Newtonian to relativistic physics, space went from being thought of as a 3-dimensional Euclidean container independent of time and the distribution of matter, to a 4-dimensional Riemannian space–time whose structure evolves dynamically. Just as an area of mathematics will, only after a long period of active research and discovery, settle on a mature set of axioms and definitions for the theory, so too the content of these a priori concepts will become clear only in the postulated final physical theory. The content of these concepts thus far outstrips what we now in our state of incomplete scientific knowledge can articulate.[28]

The task of identifying the a priori elements in science is always incomplete in still a third sense. For Cassirer, these a priori principles can be identified—not by reflecting on the nature of the mind, nor by some non-scientific insight into the nature of reality or knowledge—only through an analysis of our current best scientific theorizing. However, no amount of careful analysis of our current theories will give us anything more than an "educated guess" (Friedman, 2001, p. 66) about what these principles are, for the simple reason that we cannot foresee how science will develop in the future. A Newtonian might have thought that <Newtonian mass> is a priori, and someone else might have thought that <the ether> is a priori, only to be refuted as the constitutive principles of one theory are replaced with different principles in a successor theory (*ETR*, p. 358). All of these "regulative" features prevent these absolute a priori principles from being relabeled as conventions. It makes little sense to say that there are conventional rules that we fail to live up to, whose content we cannot know, and whose identity we may be mistaken about.

3. Realism

Consideration of Cassirer's story of which elements remain invariant in theory change naturally leads us to consider Cassirer's stance on another fundamental issue in philosophy of science: realism. By "realism" I mean the thesis that our best physical theories are (probably or approximately, etc) true, when interpreted literally and at face value, about a mind-independent physical world. Realism in this way contains three aspects: epistemic, semantic, and metaphysical, respectively (Chakravartty, 2007, pp. 8–13; 2014). I claimed above that the philosophical role of Cassirer's theory of the absolute a priori (and, in general, his story about the elements of a physical theory that remain constant during theory change) is to explain the objectivity of scientific theorizing, and so also the fact that the history of science is a progressive history, moving asymptotically toward the truth about the natural world. This sounds at first blush like an endorsement of realism, and indeed (as I noted earlier) some contemporary defenders of "structural realism" have invoked Cassirer as an ally.

The basic idea behind structural realism—here I have in mind its "epistemic" form—is to get beyond the impasse in contemporary debates over realism by emphasizing the continuity of mathematical structure between successor theories. On this view, realism is well motivated by the well-known "no miracles" argument: that the success of science can only be explained by its (approximate, etc.) truth. However, this compelling realist argument is opposed by an equally compelling anti-realist argument, that the historical fact that scientific theories that seem compelling at one time inevitably prove inadequate over time should lead us, by induction, to the pessimistic thought that our current best theories are no more likely to prove true than the discarded theories of the past. Structural Realism counters that, though the ontological claims of scientific theories are generally discarded by successor theories, the mathematical form of the theory is typically retained (at least as a limiting case). So we should be realists at least in believing that our current best theories make true claims about the mathematical structure of nature.

In claiming that the constitutive principles of a given theory (which, after all, determine the fundamental ontology of a theory) can evolve, Cassirer does strongly emphasize that the ontology of a theory is very likely to be discarded by its successor; and in claiming that the mathematical form or structure of a theory does typically remain constant during theory change, Cassirer does seem to "trace many of the steps towards structural scientific realism" as Barry Gower claims (2000, p. 91). However, the very fact that Cassirer calls his view "critical idealism" (*SF*, p. 298) or "logical idealism" (*SF*, p. 308), and contrasts it with "metaphysical realism" (*SF*, p. 301), should give us pause before we attribute the core ideas of "structural realism" to him. Of course, the words "realism" and "idealism" mean many different things to different philosophers (especially when we are comparing philosophers across time periods), and it is hardly self-evident what it means to deny that we can know things as they are "in themselves" (*SF*, pp. 277–8, 305; *ETR*, p. 387). My task in the remainder of this paper is to make clear what Cassirer means by this denial.

A first (but ultimately, I'll argue, mistaken) interpretation is that Cassirer is endorsing the argument that is most commonly put forward by contemporary anti-realists: the pessimistic induction. Cassirer clearly knew of this argument, and vividly describes its purported conclusion:

[27] Schlick (1979 [1921]), p. 326; Reichenbach (1957 [1928]), pp. 36ff.

[28] It is clear, then, that Cassirer owes us an account of the content of concepts such that the content of a concept can outstrip what any competent user of a concept at a given time can know, and such that the same concept can be grasped over time despite wide divergence in beliefs about it. For a discussion of this issue with respect to mathematical concepts, see Heis (2011), pp. 767–9, 776–7.

> From this standpoint, what we call science appears not as an approximation to any "abiding and permanent" reality, but only as a continually renewed illusion, as a phantasmagoria, in which each new picture displaces all the earlier ones, only itself to disappear and be annihilated by another. (*SF*, p. 266)

But Cassirer rejects this "radical skepticism": he denies that the repeated failure of physical theories undermines the idea that our current best theories do approximate to the truth about "abiding and permanent" reality.

> [T]he preceding member is not absolutely destroyed by the entry of the succeeding member; but certain fundamental determinations persist, on which rest the homogeneity and uniformity of the series. In the successive stages of *science*, this demand is most purely and perfectly realized. Each change in the system of scientific concepts places in a clear light the permanent structural elements to be ascribed to the system, since it is only under the assumption of these elements that [the change] can be fixed and described. (*SF*, p. 266; cf. 321)

And as Cassirer makes clear later, the "permanent structural elements" include not just the mathematical structure of a theory (as structural realists insist) but all seven elements listed in §2, including the a priori concepts and principles.

Cassirer would also reject some of the other "anti-realist" contentions that are common among contemporary philosophers of science. A common argument is that the available empirical data even in the best cases underdetermine which scientific theory to endorse, and that ultimately the best theory is chosen on the basis of factors that do not track the truth. Cassirer was very sensitive to cases like this (and indeed he saw them as crucial arguments for his position, since they demonstrate the necessity of non-empirical principles to ensure the objectivity of theory construction and selection). Here are a few cases he discusses. Kepler's claim that the motion of Mars traces out an elliptical orbit was empirically undeterdetermined, "for no matter how many points of the path of the planet might be given, it is nevertheless always possible to connect them by any number of lines of different and more complicated form." But the question is decided in Kepler's favor by "the methodic requirement [which] remains permanent": namely, the absolute a priori principle that "the processes of nature be reduced to definite simple rules" (*SF*, p. 260; cf. *ETR*, p. 354). Furthermore, Cassirer repeatedly argues in *ETR* that the choice between Einstein's special theory of relativity and Lorentz's alternative could not be based on "an experimental decision" (*ETR*, p. 375), since the results of the Michelson-Morley experiment could be accounted for "by a number of intellectual approaches between which a choice is possible only with reference to logical 'simplicity,' more exactly, to systematic unity and completeness, of scientific exposition" (p. 365).[29]

In discussing Poincaré's arguments for the conventionality of the measurement of time, Cassirer agrees that time cannot be measured without assuming some specific laws of nature, like Newton's law of inertia, and that therefore the ordering of events in time "always leaves room for different possibilities of exposition" (*SF*, p. 187). Poincaré inferred from this case that, since the choice of unit of time is chosen so as to make the natural laws the simplest, "we therefore choose these rules, not because they are true, but because they are most convenient" (2001 [1905], §XIII). Cassirer, however, rejects this conclusion, and objects to calling the principle of inertia in Newtonian mechanics and the choice of a unit of time "conventions." This case shows only that confirmation of empirical laws is holistic and that the choice among rival systems is determined, not by "caprice" [nach Willkür], but by "a determinate law of progress," which constitutes "the ultimate criterion of 'objectivity'" (*SF*, p. 187). And as we saw above, this "law" includes all of the invariant a priori concepts and principles Cassirer discusses. (This explains why Cassirer rejects conventionalism even about the relative a priori principles: the choice among candidate relativized a priori principles is constrained by higher level a priori principles, which are absolute, regulative, and non-conventional.) Again, all of these laws together provide the criterion of objectivity, and so by the functional theory of knowledge, give the criteria for scientific truth and knowledge.

A common form of contemporary anti-realism is instrumentalism, either as a semantic thesis (that the "theoretical" propositions of a science either do not make truth claims at all, or are ought to be interpreted as elliptical claims about observables instead of literal claims about unobservables) or as an epistemic thesis (that we are only warranted in asserting the truth of the claims that our theories make about observables). Cassirer would completely reject an instrumentalism that divides up the claims within our current best science into those that do make genuine, (approximately) true claims about objects and those that do not. "Critical idealism," as he understands it, is emphatically not distinguished from what he calls "realism" in *restricting* the set of knowledge claims to a privileged circle, whether this circle includes only sense data or observable objects (*SF*, pp. 294–5). Indeed instrumentalism for Cassirer is a paradigm instance of the point of view of "*Substanzbegriff*": it posits a set of objects (the observables) and a set of claims (sentences about the observable features of observables) that are "given" independently of the "functions" constitutive of natural science—the a priori principles and concepts Cassirer identifies. (Cassirer actually refers to the epistemology underlying instrumentalist anti-realism as a kind of realism: "naïve" realism (*ETR*, p. 357) assumes that we can have direct perceptual knowledge of things and their properties independently of a system of knowledge and the a priori principles that constitute the form of that system.)

Cassirer's idealism, then, is not a form of epistemic anti-realism: he does not claim that some or all of our best current theories fail to achieve (approximate, probable, etc.) *knowledge*. "Critical idealism" is not a form of semantic anti-realism, either—in the sense that it does not claim that "theoretical" scientific statements should not be read at face value as making truth claims about unobservable objects.[30] This

[29] I am not here giving a complete account of the a priori principles that Cassirer thinks count in favor of STR over Lorentz's theory. In particular, Cassirer believes that the special relativistic principle (that the laws of nature can be formulated in any rectilinear and non-accelerating reference frame) is preferable because it further "de-anthropomorphizes" the laws of nature, removing from physical laws the privileged perspective of an observer at rest. By "de-privileging" the perspective of one individual subject, Cassirer believes that STR better approximates the constitutive goal of a completely objective physical theory. See *ETR*, p. 381, and the illuminating discussion of Cassirer's point in Ryckman (2005), p. 45. This principle of "de-anthropomorphism" would then be a regulative, absolute a priori principle of theory selection (like the principles of simplicity and generality). In the text, I have focused on the principle of simplicity simply to stream line the exposition of the issues: I could make parallel points about the objectivity and truth-tracking character of any of Cassirer's regulative, a priori principles of theory selection.

[30] As some philosophers understand the term, "realism" is committed to a correspondence theory of truth. Cassirer does not deny that true scientific claims correspond to physical objects and their properties. But he certainly *does* think that this account inverts the proper explanatory order among the concepts <objectivity>, <true>, and <object>. On the functional theory of knowledge, a true proposition is one that could be incorporated into a completely objective and systematic total physical theory. (It would be a sentence in the final complete theory: "the completed system of experience" (*SF*, p. 248). That there could be such a completed system is postulated by the principle of the "unity of nature.") For philosophers who think that any denial of a correspondence theory of truth amounts to *semantic anti-realism*, Cassirer would be an anti-realist in this sense. It should be repeated, though, that Cassirer would be a semantic anti-realist in this sense about *every domain of discourse*, since he emphatically does not believe that there is something in principle unique about the theoretical claims of natural science.

leaves the "metaphysical" thesis that our current best science makes true claims about a *mind-independent reality*. Though it is certainly not surprising that Cassirer's idealism would run afoul of realism here, it is again worthwhile to get as clear as we can about the precise senses in which Cassirer denies that our scientific knowledge is "mind independent." Before we begin, it is essential to recognize that Cassirer uses "mind" or "reason" in a peculiar way: according to the heterodox reading of Kant advocated by Cassirer and his Marburg teachers, philosophical talk of the "mind" or "reason" is just a roundabout way of talking about science.[31] "Idealism" in this peculiar sense then asserts the "science dependence" of our scientific knowledge. I see Cassirer's philosophy of science as "idealistic" in three ways: philosophy of science is *methodologically* dependent on science; scientific claims are *logically* dependent on a priori principles; and the concepts <object>, <knowledge>, and <truth> are *conceptually* dependent on the concepts <objectivity> and <unity of science>.

First, Cassirer—following his teacher Hermann Cohen and the other members of the Marburg School—thinks that the proper method of the philosophy of science is the "transcendental method": we take as the starting point for our philosophical reflection the *fact* of modern science and mathematics and then seek to investigate the conditions of their possibility. Since our best current science—crucially including the highest scientific concepts, principles, and norms—is our starting point, there is no extra-scientific standard that we could invoke to call into question the established results of science. "The critical philosophy … does not seek to dominate [meistern] the sciences but to understand them."[32] Now, this non-skeptical and non-revisionary attitude toward science is a kind of "idealism," as Cassirer and the other Marburg Neo-Kantians use the term. Idealism, we might say, asserts that "reality must conform to the nature of our minds"; unless this is to be understood psychologically, our "mind" must really mean the norms and methods of our best science,[33] and so we get the "idealistic" conclusion that the physical world is as our best science says it is.

We've already seen Cassirer's "idealism" at work in his rejection of instrumentalism and anti-realist arguments from underdetermination. He drew no skeptical conclusions from the fact that scientists choose between two empirically adequate theories on the basis of theoretical simplicity, because he denied that we could make an in-principle distinction between the truth-tracking scientific norms ("choose empirically adequate theories") and the purportedly conventional norms ("choose the simplest theory compatible with the empirical evidence"). This anti-realist attitude is opposed to the transcendental method, since it does not take the scientific method as a given, but instead calls some aspects of scientific practice into question based on some non-scientific criterion of truth. This leads us to a second, related sense of "idealism": Cassirer denies that we can have knowledge of things "as they are in themselves" (that is, independently of the a priori concepts and principles of science[34]), and in fact he thinks the very question Do objects really conform to the a priori form of science? is non-sensical. The transcendental method led Cassirer to investigate the conditions of the possibility of the objectivity of science. Locating objectivity in constancy, Cassirer identified the invariant concepts and principles of science as conditions of the possibility of scientific objectivity (and so also, by the functional theory of knowledge, scientific knowledge and truth). "Idealism," in this second sense, then maintains that there are substantive a priori principles that limit our knowledge to those objects that conform to these principles (*SF*, p. 297).

Once we recognize these a priori elements in science, it would seem that it would be a sensible question to ask whether reality really does conform to these principles. Do physical objects really have a mathematical structure? Are they really spatio-temporal, and do they interact according to physical laws? Is reality really simple, or is it complex in a way that necessarily eludes our grasp? Are there really general laws that describe the behavior of all physical phenomena, or is there an irreducible multiplicity in nature? On some philosophical views, these are perfectly sensible questions. For instance, on one reading of Kant, Kant finds the question whether things in themselves are really spatial to be a legitimate question, and answers it in the negative. Similarly, many philosophers of science (following Poincaré) think that it is legitimate to question whether we have any reason to believe that a simpler explanation is more likely to be true, and see no reason to think that it is.[35] Cassirer, however, thinks these questions are nonsensical:

> The content of experience becomes 'objective' for us when we understand how each element is woven into the whole. If we attempt to characterize this whole itself as an illusion, it is a mere play of words; for the difference between reality and appearance presupposed here is itself only possible within the system of experience and under its conditions. (*SF*, p. 284; cf. 278, *ETR*, pp. 381, 387)

When Cassirer rejects the external question whether the concepts, principles, and norms of science reflect or distort reality, he is in this very specific sense denying that reality is "mind independent." But we should be careful to distinguish this kind of mind independence from other "idealistic" positions. The objects of science are not on this view thoughts or representations; they are not produced by the mind as effects of a cause, and the existence of

[31] See (1981 [1918]), p. 193: "The 'subjectivity' that was the starting point of transcendental reflection has now been presented in a precisely defined, terminologically restricted sense… The 'subject' spoken of here is none other than reason itself, in its universal and its particular functions. In this sense alone could we style Kant's system 'idealism'; the ideality to which it is related and on which it rests is that of the highest rational principles." Or as he puts it a few pages earlier, the "understanding" whose authority idealism asserts "stands directly for that entity which we designate by the name 'science' and for its axiomatic presuppositions" (pp. 154–5).

[32] Cassirer (1907), p. 31. Richardson (2006). Massimi (2009). On the transcendental method in the Marburg School, see Natorp (1912), pp. 196ff.

[33] "The energy with which Kantian idealism is constantly pointed back to science as the proper and indispensable correlate of the transcendental method finds its explanation in the necessity of preserving its "objective" meaning. […] The organization of the "mind" [Geist] that idealism seeks can be read off nowhere else than in the structural connection of natural science" (Cassirer, 1912, p. 257).

Cassirer is here following Cohen. Cohen (1902), pp. 507ff. argues that adherence to the transcendental method must be the true meaning of "idealism," if idealism is not to collapse into scepticism or psychologism.

[34] Cassirer glosses "things in themselves" as things "as they are assumed to be in themselves separate from all the conditions of experience" (*SF*, pp. 277–8; cf. *ETR*, p. 387).

[35] This point makes clear how Cassirer's view differs from Kant's. Kant distinguishes sharply between the regulative principles of reason, which give the norms of a fully systematic science but do not constitute conditions of the possibility of experience, and the constitutive principles of the understanding (principles like the law of causality and the permanence of the magnitude of substance), which are conditions of the possibility of experience. Kant thinks that it is perfectly coherent to think that nature should fail to be systematic. Cassirer, on the other hand, thinks that the validity of what Kant calls the principle of the purposiveness of nature is a condition of the possibility of experience.

Michael Friedman has claimed that Cassirer's blurring of the lines between regulative and constitutive principles is a symptom of his rejection of Kant's strict division between the understanding and sensibility (2000, p. 117). I would add that the elevation of regulative principles of reason to conditions of the possibility of experience directly results from the recognition of fundamental shifts in ontology from one theory to the next, and from the real possibility of inequivalent but equally empirically adequate physical theories.

objects of science in no way depends on there being minds to think them. The point is only that there are principles that are used as essential premises in science, and these principles are not empirical and cannot be sensibly questioned without undermining the whole project of asking sensible questions about physical objects.[36]

This dependence of objects on the mind, then, amounts simply to the claim that our best scientific theories presuppose as premises certain invariant a priori principles. But there is another kind of dependence that Cassirer speaks of besides the logical dependency of a conclusion on a premise or theorem on an axiom. According to the functional theory of knowledge, the highest concepts of the *philosophy* of science stand in a definite order of dependence: the concept <object> depends on the concept <knowledge>, which depends on the concept <objectivity>, since our best philosophical explanation of objecthood (as that which is known by fully objective knowledge) employs the concepts <objective> and <knowledge>, but not vice-versa. <Objectivity> is explained in terms of the systematic unity of science, a systematic unity that is made possible by the invariant a priori elements in science. Given Cassirer's peculiar non-psychological use of "mind" to mean "science," this conceptual dependency of the concept <object> on the concept <unity of science> is a kind of dependence of objects on the mind (1955 [1923], p. 79; cf. Natorp, 1981 [1887], §6).

I noted at the beginning of this section the affinity that some readers have alleged between Cassirer's philosophy of science and contemporary structural realism. It should now be clear that Cassirer would have sympathized with its epistemic realism. But Cassirer certainly would have balked at one core commitment of structural realists and most contemporary realists: the "no miracles" argument that the success of science can only be explained by its truth. Cassirer rejects any global attempt to explain the "unity of science" in any other way than by simply articulating its constitutive concepts, principles, and norms. (If it makes no sense to question the whole of experience all at once, it makes no sense to explain it either.) Indeed, to explain the success of science as a whole by its truth would be to fall back into the point of view of "Substanzbegriff," which (wrongly, on Cassirer's view) tried to explain the concepts <unity of science> and <objectivity> in terms of <truth> and <object>. Indeed, to explain the success of science, say, in giving explanations of physical phenomena by invoking the truth-tracking character of scientific norms would require having some notion of what it is to track the truth that is independent of our notion of giving a good scientific theory. In this way, Cassirer's idealism, like his theory of the a priori, flows out of his core contrast between substance-concepts and function-concepts.

[36] *SF*, p. 298: "The ideality, which is here alone asserted, has nothing in common with the subjective 'presentation', it concerns merely the objective validity of certain axioms and norms of scientific knowledge. […] It is true that there results, strictly speaking, no absolute, but only a relative being. But this relativity obviously does not mean physical dependency on particular thinking subjects, but logical dependency on the content of certain universal principles of all knowledge. The proposition, that being is a "product" of thought, thus contains no reference to any physical or metaphysical causal relation, but signifies merely a purely functional relation, a relation of superordination and subordination in the validity of certain judgments. If we analyze the definition of the "object," if we bring to clear consciousness what is assumed in this concept, we are led to certain logical necessities, which appear as the inevitable constitutive "factors" of this concept. Experience and its object are conceived as dependent variables, which are successively reduced to a sequence of logical "arguments"; and it is this pure dependence in content of functions on their arguments, which is characterized in the language of idealism as the dependence of the "object" on "thought.""

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